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Test of Limiting Behavior for Λ and K_1^0 Produced in Inclusive pp Interactions

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New data are presented on longitudinal and transverse momentum spectra of Λ and K_1^0 produced in $pp\to\Lambda$ + anything and $pp\to K_1^0$ + anything at 13, 18, 21, 24, and 28 GeV/c. The Λ spectra show little variation with energy. The K_1^0 inclusive cross section increases by ~ 50% over our energy range, indicating that for this process the threshold for scaling and limiting fragmentation may be above 30 GeV/c. Tests of Mueller-Regge phenomenology are discussed.

Experimental studies[1] demonstrate that single-pion inclusive spectra become energy independent, or nearly so, at lab momenta well below 30 GeV/c. This success at low energy has led to broad acceptance of the concept of limiting behavior in inclusive processes.[2,3] Based on the work of Mueller,[4] more detailed Regge-pole analyses have also been proposed.[5] Estimates have been made for the rate at which limiting behavior should be attained; these predictions have met with success, again for pion production.[1,6] However, little information has been available up to now on energy dependence or shapes of inclusive spectra of K, Λ, or of other heavy produced hadrons.[7-9] In this note, we present new data on the longitudinal and transverse momentum spectra of Λ and K_1^0 produced in reactions $pp\to\Lambda$ + anything[10] and $pp\to K_1^0$ + anything at 13, 18, 21, 24, and 28 GeV/c. From the point of view of testing limiting behavior[2,3] and Mueller-Regge phenomenology,[4,5] the value of a systematic comparative study of single-particle distributions for several incoming energies cannot be overemphasized.[3]

Data were obtained from a total of 75 000 pictures taken at the Brookhaven National Laboratory 80-in. hydrogen bubble chamber. At each momentum, the beam flux corresponds to about 0.5 events /μb. Events with one or more visible vees, independent of the primary interaction topology, were scanned and measured. The resultant efficiencies for scanning and measuring were 98 and 87%, respectively. Of the final sample of vees, after kinematics and ionization comparisons were applied, 97% of the Λ sample and 95% of the K_1^0 sample were identified uniquely. Utilizing the forward-backward symmetry in the c.m., we determined that 13 and 17% of the forward Λ's were lost at 13 and 28 GeV/c, respectively, principally because of the peripheral nature of Λ production. The Λ data presented here, with the exception of Fig. 3, are only for events with $x<0$ ($x=2p_{L\,\mathrm{c.m.}}/\sqrt{s}$, where $p_{L\,\mathrm{c.m.}}$ is longitu-

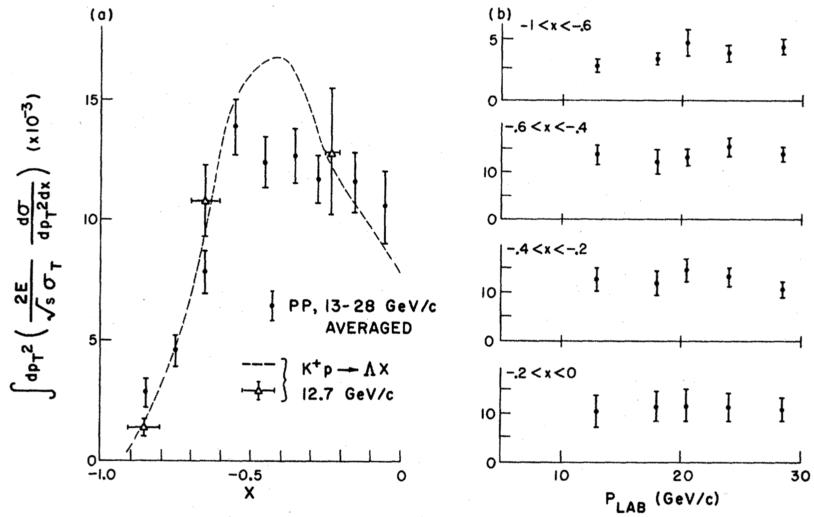

FIG. 1. (a) Normalized, Lorentz-invariant inclusive distribution for Λ from $pp\to\Lambda X$ plotted versus $x=2p_{L\,\mathrm{c.m.}}/\sqrt{s}$, where $p_{L\,\mathrm{c.m.}}$ is the Λ's longitudinal momentum in the c.m. system. Data are averaged over the energy interval 13 to 28 GeV/c and summed over all p_T^2. Shown for comparison are results of Ref. 9 for $K^+p\to\Lambda X$ at 12.7 GeV/c. The dashed curve is a polynomial fit to the Kp data; only three representative Kp points are shown. For pp data, $\sigma_T=40$ mb; for Kp data, $\sigma_T=17.4$ mb. (b) Energy dependence of Λ inclusive cross section in four selected intervals of x.

dinal momentum in the c.m. system). In Fig. 3, corrections have been made for these losses, and cross sections quoted are for both hemispheres in the c.m. system. No similar effects were noted for the K_1^0 sample; consequently, distributions have been folded about $x=0$.

In Fig. 1(a), we display the normalized, Lorentz-invariant x distribution of Λ from[10] $pp\to\Lambda X$. In an effort to increase statistics, we average events from all five values of incident momentum. As is shown in Fig. 1(b), we observe no change in either shape or normalization of the distribution in x as energy is varied. We remark also that the laboratory longitudinal momentum distribution $d\sigma/dq_L$, advocated in Ref. 3, also shows no energy variation, within our statistics, for $q_L<4$ GeV/c. Not shown here are distributions in p_T^2, but we remark that for both Λ and K_1^0 the shape of $F(p_T^2)$ is represented well over the range $0<p_T^2<1$ (GeV/c)^2 by $\exp(-4p_T^2)$. Here $F(p_T^2)=\int dp_L(E\,d\sigma/dp_L\,dp_T^2)$ averaged over s.

Spectra from $pp\to K_1^0X$ are given in Fig. 2. In Fig. 2(a) we show the folded x distributions for events from all five momenta. These data may be parametrized roughly by the form $\exp(-5|x|)$. Energy dependence is shown in Fig. 2(c). For essentially all $|x|$, we observe that the K inclusive spectrum increases by ~50% in magnitude as energy is increased from 13 to 28 GeV/c. This shows that for energies up to 30 GeV/c and values

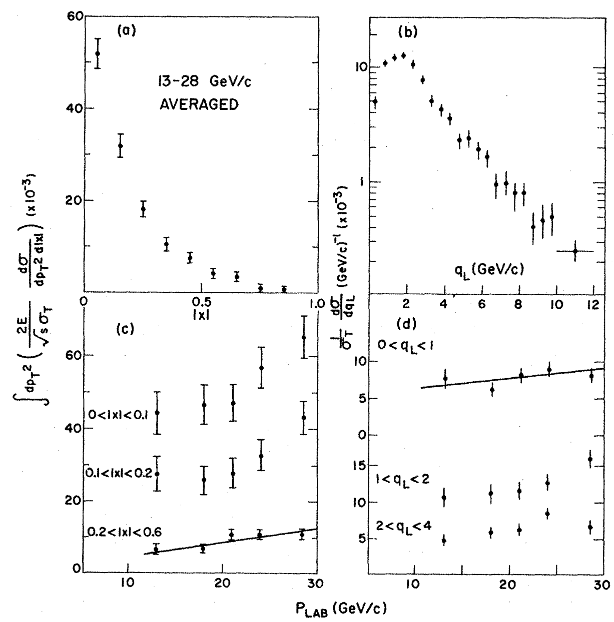

FIG. 2. (a) Normalized, Lorentz-invariant inclusive distribution for K_1^0 from $pp\to K_1^0X$ plotted versus $|x|$. Folded data from both the $x>0$ and $x<0$ hemispheres are shown on the plot. (b) Normalized K_1^0 inclusive distribution plotted versus q_L, the longitudinal momentum of K_1^0. Each event is plotted twice, once in the lab frame and once in the projectile frame. (c), (d) Energy dependence of K_1^0 inclusive cross sections in selected intervals of $|x|$ and q_L, respectively. Solid lines are best linear fits to data; fitted expressions are (c) $(0.1\pm0.4)+(0.21\pm0.04)P_{\mathrm{lab}}$ and (d) $(5.5\pm0.5)+(0.1\pm0.03)P_{\mathrm{lab}}$.

of $|x| \lesssim 0.6$ the Feynman scaling limit has not been achieved for $pp \to K_1{}^0X$. Moreover, the data suggest that, if a limit exists, the inclusive cross section *rises* to meet this limit as s is increased. In other words, scaling may be approached from below. These two facts differ from single pion production.[1] Over energy intervals comparable with ours, pion spectra are observed either to be independent of s for $|x| > 0.1$, or to fall by $\sim 20\%$.

Qualitatively different s dependence of K spectra is observed when data are plotted versus lab longitudinal momentum, as shown in Figs. 2(b) and (d). For $q_L > 1$ GeV/c, a 50% increase with energy is evident in Fig. 2(d). However, for $0 < q_L < 1$ GeV/c, the rise is more gradual and, indeed, perhaps absent. For q_L small (i.e., $q_L \ll \frac{1}{2}\sqrt{s}$), the hypothesis of limiting fragmentation[3] (LF) asserts that as s increases, $d^2\sigma/dq_L dp_T{}^2$ should approach an energy-independent limit. In our experiment, the region of small q_L may be taken to include events which fall below the peak in the q_L distribution of Fig. 2(b). Taking the more conservative cut $0 < q_L < 1$ GeV/c, we observe that our data are superficially consistent with the assertion that LF for $pp \to K_1{}^0X$ is achieved for $p_{\rm lab} \simeq 10$ GeV/c. With χ^2 probability of $\approx 50\%$, an energy independent horizontal line may be drawn through the small-q_L points in Fig. 2(d). On the other hand, we remark that statistics are improved if we compare averages of results obtained by combining data at neighboring energies. We define low (L) and high (H) energy intervals by combining data at 13 and 18 GeV/c and at 21, 24, and 28 GeV/c, respectively. For $0 < q_L < 1$ GeV/c, $\sigma_T{}^{-1}d\sigma/dq_L$ has average values 7.0 ± 0.8 and 8.6 ± 0.6 (GeV/c)$^{-1}$ in the L and H intervals. The difference is 1.6 ± 1.0. Although by no means conclusive, this suggests that $pp \to K_1{}^0X$ does not exhibit LF below 30 GeV/c. In any case, analysis in terms of q_L shows that whereas constant limits may not have been achieved, LF is a better interpretation of the K data than Feynman scaling.

According to Mueller-Regge analysis,[5] the invariant inclusive cross section for $a + b \to c + X$ is predicted to have s dependence of the form

$$E\frac{d^3\sigma}{dp_T{}^2 dq_L} = A(p_T{}^2, q_L) + B(p_T{}^2, q_L)/s^{1/2}. \qquad (1)$$

The functional forms and relative magnitudes of A and B are not specified. However, for cases in which the quantum numbers of $(ab\bar{c})$ are exotic, duality is used to predict that B is zero.[5] This model interprets nicely the observed energy independence[1] of *pion* inclusive cross sections for exotic $pp \to \pi^{\pm}X$ and $\pi^+p \to \pi^-X$; it accommodates also the 20% fall of spectra from nonexotic $\pi^-p \to \pi^-X$ and $\pi^+p \to \pi^+X$.[1] Moreover, it relates correctly the relative normalization[6] of $pp \to \pi^-X$, $K^+p \to \pi^-X$, and $\pi^+p \to \pi^-X$. For both $pp \to K^0X$ and $pp \to \bar{K}^0X$, the quantum numbers of $ab\bar{c}$ (and ab) are exotic. Therefore, current versions[5] of the Mueller-Regge model predict energy-independent inclusive cross sections for both, and for any linear combination of the two. In particular, the prediction of an energy-independent $pp \to K_1{}^0X$ inclusive cross section appears to disagree with our data.

From the point of view of Mueller-Regge phenomenology, based on the three-to-three forward elastic $ab\bar{c}$ amplitude, it is difficult to see why there should be much difference between $pp \to \pi X$ and $pp \to KX$ over our energy range. The difference in mass between π and K might account through kinematic effects for a difference of 1 or 2 GeV/c in threshold value for onset of limiting behavior, but our data suggest that for K production the threshold for scaling behavior is above 30 GeV/c, a factor of at least 4 in lab beam momentum above where limiting behavior sets in for π. Strangeness may be playing a curious role, although, in the Mueller-Regge approach, a strong strangeness-related difference between energy dependences of $pp \to \pi X$ and $pp \to KX$ is not expected. On the other hand, differences between π and K production may be expected from multiperipheral model arguments, based simply on the substantial mass difference between $(K\bar{K})$ and $\pi\pi$. Inasmuch as mean multiplicity grows slowly, the available peripheral phase space for $K\bar{K}$ production increases considerably over the range 10 to 30 GeV/c.

Although perhaps inconsistent with $pp \to K_1{}^0X$, the Mueller-Regge approach predicts no energy variation of the (exotic) $pp \to \Lambda X$ spectra, in agreement with our data. Moreover, the model may be used, along with factorization arguments, to predict

$$\frac{1}{\sigma_{K^+p}}\frac{2E}{\sqrt{s}}\frac{d\sigma(K^+p \to \Lambda X)}{dx\,dp_T{}^2} = \frac{1}{\sigma_{pp}}\frac{2E}{\sqrt{s}}\frac{d\sigma(pp \to \Lambda X)}{dx\,dp_T{}^2}, \qquad (2)$$

valid for $x < 0$, in the proton-fragmentation region. Here σ_{K^+p} and σ_{pp} are the asymptotic total cross sections for K^+p and pp scattering, respectively. This equality is satisfied, as we show in Fig. 1(a).

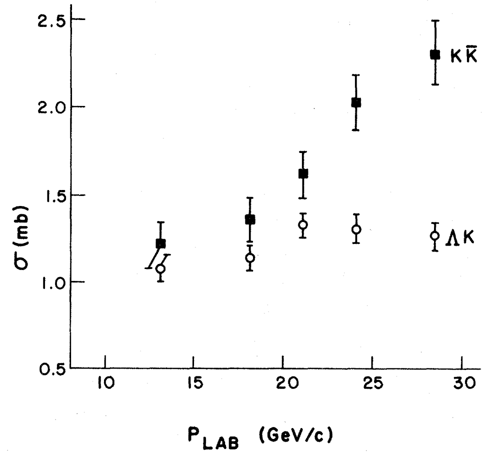

FIG. 3. Total cross sections for $pp\to \Lambda K$ + anything and $pp\to K\bar{K}$ + anything at 13, 18, 21, 24, and 28 GeV/c. See Ref. 11 for details on how these are obtained.

In Fig. 3, we show that the cross section[11] for production of $K\bar{K}$ + anything rises rapidly in the energy range of our study, whereas $\sigma(\Lambda K$ + anything) changes little. This behavior appears to hold also for $\pi^- p$ reactions.[12] It is obvious that if we were to divide our inclusive K_1^0 distribution, at a given energy, by the total cross section for K_1^0 production at that same energy, then the resulting normalized inclusive distribution would be nearly independent of energy.

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[7]C. W. Akerlof *et al.*, Phys. Rev. D 3, 645 (1971); J. V. Allaby *et al.*, CERN Report No. 70-12 (unpublished). In these papers, data are reported for $pp\to K^\pm X$ at 12.5 and 19.2 GeV/c, respectively. Unfortunately, cross sections are not measured at the same values of kaon momenta (p_T^2, p_L) at the different energies. The range of values of K momenta is also very limited.

[8]J. V. Beaupre *et al.*, Nucl. Phys. B30, 374 (1971).

[9]S. L. Stone *et al.*, University of Rochester Report No. UR-875-349 (to be published); 12.7 GeV/c data are presented on shapes of $K^+p\to K^0X$, $K^+p\to\Lambda X$, and $K^+p\to\bar{\Lambda}^0 X$.

[10]In all inclusive reactions $ab\to cX$, there is an unanswerable question at the inclusive level as to whether c is produced directly or whether c is the product of decay of another particle. In $pp\to\Lambda X$, Λ may arise from $\Sigma^0\to\Lambda^0\gamma$, an electromagnetic decay. It may be argued in principle that only Λ's which come from predominantly hadronic production *and* decay should be accepted for $pp\to\Lambda X$, and that events in which Λ comes from Σ decay should be excluded. This electromagnetic difficulty arises in all inclusive reactions; e.g., in $\pi p\to\pi X$, in some small fraction of the events, the final π comes from η or $\eta'\to\pi^+\pi^-\gamma$. For $ab\to\Lambda X$, however, Σ decay presumably accounts for a significant proportion of Λ's. At low energies, in fitted exclusive channels where the separation can be made, $\sigma(\Sigma)/\sigma(\Lambda)\lesssim\frac{1}{3}$ [cf. W. Chinowsky *et al.*, Phys. Rev. 165, 1466 (1968); S. Klein *et al.*, Phys. Rev. D 1, 3019 (1970)]. This fraction may also be typical of high-multiplicity processes at high energy. In our events, we cannot separate Λ and Σ^0. Thus the Λ distributions we present include Λ from both hadronic and electromagnetic processes. We take this as *de facto* definition of inclusive production of Λ.

[11]$\sigma(\Lambda K$ + anything) and $\sigma(K\bar{K}$ + anything) were obtained as follows: $\sigma(\Lambda K + \text{anything}) = \frac{3}{2}[\sigma(\Lambda, \text{ seen alone}) + \sigma(\Lambda K_1^0, \text{ both seen})]$, $\sigma(K\bar{K} + \text{anything}) = 4[\sigma(K_1^0, \text{ seen alone}) - \sigma(K_1^0 K_1^0, \text{ both seen})] - \frac{1}{2}\sigma(\Lambda K_1^0, \text{ both seen})$. Equal cross sections for $K^+\bar{K}^0$, K^0K^-, $K^0\bar{K}^0$, and K^+K^- were assumed in the derivation of the second equation.

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