[see J. N. Bahcall and S. C. Frautschi, Phys. Lett. 29B, 623 (1969)].

[4]A possible role for a massless pseudoscalar field in CP-violating interactions has been proposed by F. Gürsey and A. Pais, unpublished.

[5]The most severe limit that we know for m is $\lesssim 60$ eV, given by K. Bergkvist, in *Topical Conference on Weak Interactions, CERN, Geneva, Switzerland, 14–17 January 1969* (CERN Scientific Information Service, Geneva, Switzerland, 1969), p. 91.

[6]The lifetime for processes (i) and (ii) would be proportional to $m_\nu^{-4}E$. Specifically, for process (i), with $h=k(\bar{\psi}_\nu\psi_\nu)\varphi_1\varphi_2$, $\tau=3(2^8\pi^3)m_\nu^{-4}k^{-2}E$. In order to have instability in our sense one would need a fairly strong interaction (~ electromagnetic interactions) which, however, cannot be excluded by present data on muon decay. The lifetime for process (iii) is proportional to $m_\nu^{-6}E$, which would require, for instability in our sense, a very strong interaction that can be excluded by present data.

[7]We refer to the accurate e^+ spectra used for a determination of the ρ value. For a list of references, see M. Roos *et al.*, Phys. Lett. 33B, 32 (1970).

[8]The ^{87}Rb experiment was first proposed by A. W. Sunyar and M. Goldhaber, Phys. Rev. 120, 871 (1970); the ^{7}Li experiment by J. N. Bahcall, Phys. Lett. 13, 332 (1964); and the ν-e scattering experiment by F. Reines and W. R. Kropp, Phys. Rev. Lett. 12, 457 (1964). The counting rates given in Table I have been calculated in the usual way by averaging the relevant neutrino absorption cross section over the neutrino spectrum of each β-decay source [see J. N. Bahcall, Phys. Rev. 135, B137 (1964)]; the cross sections used were taken from J. N. Bahcall, Phys. Lett. 13, 332 (1964), and Phys. Rev. Lett. 23, 251 (1969); and G. V. Domogatsky, Lebedev Institute Report No. 153, 1971 (unpublished).

[9]Most of the expected low-energy neutrinos are from the basic reactions $p+p\to{}^2\mathrm{H}+e^++\nu_e$ and $p+e^-+p\to{}^2\mathrm{H}+\nu_e$. The *astrophysical* basis for expecting the calculated number of these low-energy neutrinos is very secure; the calculation depends mainly on the measured solar luminosity and the fact that four protons are ~25 MeV heavier than an α particle (see Ref. 1 and other references cited therein).

[10]This could happen, for example, if $\nu'=\nu_\mu$ (or $\nu'=\bar{\nu}_\mu$) and muon neutrinos are scattered by electrons, as predicted by some versions of weak-interaction theory; see S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970); G. 't Hooft, Phys. Lett. 37B, 195 (1971).

Observation of an S-Wave Resonance in the f^0 Mass Region*

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We study $\pi\pi$ scattering in the reactions $\pi^-p\to n\pi^+\pi^-$ and $\pi^+n\to p\pi^+\pi^-$ at 7 GeV/c using the absorption-modified one-pion-exchange formalism. Fitting the $\pi\pi$ angular distributions we obtain S-, P-, and D-wave $\pi\pi$ phase shifts and inelasticities for $0.6<M(\pi\pi)<1.5$ GeV/c^2. We find an $I=0$ S-wave resonance with $M=1.25\pm0.04$ and $\Gamma=0.3\pm0.1$ GeV/c^2. Support for our phase shifts in the f^0 mass region is obtained from data on $\pi^+n\to pK^+K^-$ and $\pi^+n\to p\pi^0\pi^0$ at 7 GeV/c.

While $\pi\pi$ scattering in the ρ^0 resonance region has been studied by many authors,[1] there have been relatively few such studies in the f^0 mass region. Previously, Oh *et al.*[2] determined $\pi\pi$ phase shifts for $0.6\le M(\pi\pi)\le 1.4$ GeV/c^2 by fitting the $\pi\pi$ angular distribution in $\pi N\to N\pi\pi$ at 7 GeV/c. Beusch *et al.*[3] have studied the $I=0$ S wave ($\delta_S{}^0$) in the $S^*(1060)$ mass region by examing inelastic $\pi\pi$ scattering in the reaction $\pi^-p\to K_1K_1n$ at 4 and 6.2 GeV/c. The Aachen-Berlin-CERN collaboration of Beaupre *et al.*[4] determined $\delta_s{}^0$ near the f^0 peak in $\pi^+p\to\Delta^{++}\pi^+\pi^-$ at 8 GeV/c. In this work we direct our attention to the di-pion mass range 1.0–1.5 GeV/c^2 where we observe an S-wave resonance at 1.26 GeV/c^2 near the f^0 peak.

Our data were obtained from two exposures of the Midwestern Universities Research Association–Argonne National Laboratory 30-in. bubble chamber to 7-GeV/c pions. These experiments yielded 10 845 $\pi^+\pi^-$ events in the channels

$$\pi^-p\to n\pi^+\pi^-,\quad 4191\text{ events};\tag{1}$$

$$\pi^+d\to p_sp\pi^+\pi^-,\quad 6654\text{ events}.\tag{2}$$

The $\pi^- p$ data are from the work of Oh *et al.*[2] The $\pi^+ d$ data have been discussed previously[5-7] (we require $|\vec{P}_s| < 0.3$ GeV/c). The combined $\pi^+\pi^-$ mass distribution for Reactions (1) and (2) shows strong ρ^0 and f^0 resonance production and a weaker g^0 signal. The four-momentum transfer to the nucleon (t) and the $\pi\pi$ angular distributions are practically identical for the $\pi^- p$ and $\pi^+ d$ data. One half of the $\pi\pi$ scattering-angle [$\cos(\theta_{\pi\pi})$] distributions are plotted in Fig. 1. The azimuthal-angle (φ) distributions in the f^0 mass region are isotropic.

For the $\pi^+ d$ experiment the 30-in. bubble chamber had two $\frac{1}{8}$-in. Ta plates mounted at the downstream end of the chamber, allowing study of the reaction

$$\pi^+ d \to p_s p \pi^0 \pi^0. \tag{3}$$

Since most $2\pi^0$ events did not yield four observed γ's (single-γ conversion probability $\simeq 0.7$ for $E_\gamma \gtrsim 1.0$ GeV), we use a procedure for obtaining approximate information on the π^0 directions for events with 1–3 γ's.[7,8] The $\gamma\gamma$ opening angle for π^0 decay satisfies the inequality $\tan\frac{1}{2}\theta \geq \mu/p_\pi$; and the opening angle distribution is sharply

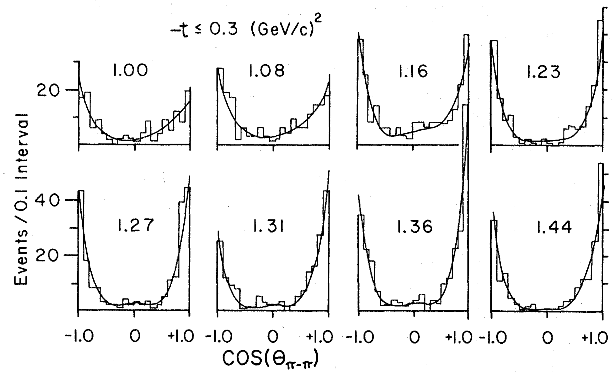

FIG. 1. $\text{Cos}(\theta_{\pi\pi})$ distributions for $\pi^- p \to n\pi^+\pi^-$ and $\pi^+ n \to p\pi^+\pi^-$ at 7 GeV/c. Central values are shown for the 40- (or 20-) MeV/c^2 bins in $M(\pi^+\pi^-)$. The solid curves show the results of the AOPE-model fits. $\text{Cos}\theta_{\pi\pi} = \hat{n}_{\pi\text{-(in)}} \cdot \hat{n}_{\pi\text{-(out)}}$ and is thus the usual definition of the π-π scattering angle or "Jackson" angle in the di-pion rest frame.

peaked near $\theta_{\min}$. We fit the 2–3 γ events using artificial π^0 tracks constrained to lie on cones of half-angle $\sim 1.25\theta_{\min}/2$ about the measured γ directions. This procedure has been verified by Monte Carlo studies.[9] Figure 2(a) shows the dis-

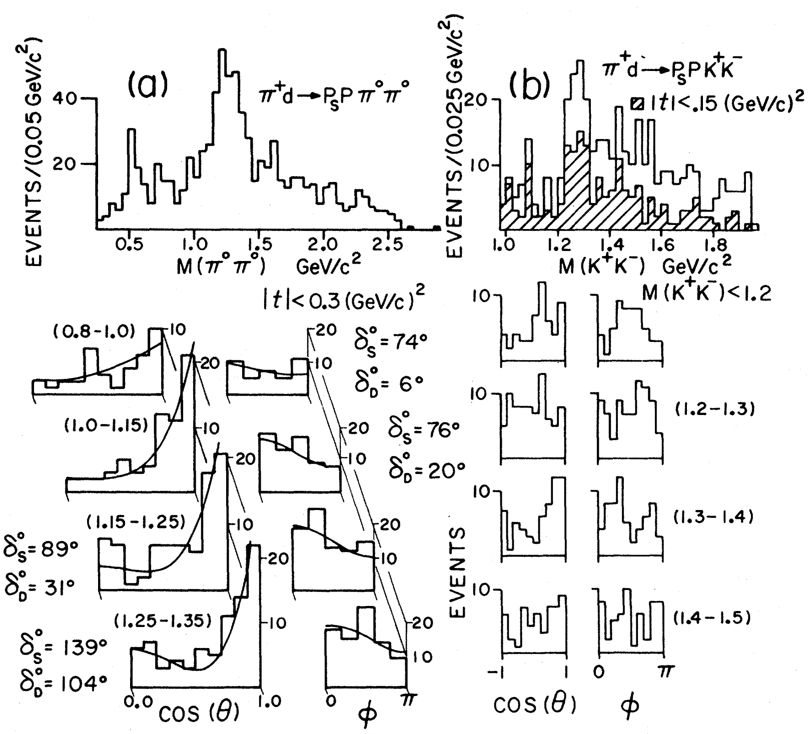

FIG. 2. (a) $M(\pi^0\pi^0)$, $\cos\theta$, and azimuthal angle distributions for $\pi^+ n \to p\pi^0\pi^0$. The solid curves on the angular distributions result from the AOPE-model fit with $\delta_S^{\,0}$ and $\delta_D^{\,0}$ as shown. (b) $M(K^+K^-)$, $\cos\theta$, and Treiman-Yang angle distributions for $\pi^+ n \to pK^+K^-$. $\text{Cos}\theta$ refers to the π-π or $\pi_{\text{in}}^- \to K_{\text{out}}^-$ scattering angle as defined in Fig. 1. The Treiman-Yang angle is the angle between the planes defined by $\hat{n}_{\pi(\text{in})} \times \hat{n}_{\pi(\text{out})}$ and $\hat{n}_{\pi(\text{in})} \times \hat{n}_{\text{nucleon(out)}}$, again in the di-pion rest frame.

tribution of $M(\pi^0\pi^0)$ for fitted events with 1–4 γ's. There is some η^0 and ω^0 signal since our procedure does not discriminate against $3\pi^0$ and $\pi^0\gamma$ for $M(\pi\pi) \lesssim 1.0$ GeV/c^2. The $2\pi^0$ angular distributions from 0.8–1.35 GeV/c^2 are also plotted in Fig. 2(a). The mass bin 0.8–1.0 GeV/c^2 shows some D wave, and there is a strong D-wave signal at the f^0 peak.

We have also studied at 7 GeV/c the reaction

$$\pi^+ d \to p_s p K^+ K^-. \tag{4}$$

A Vidicon device, which measures relative ionization, has been used to enhance the $K^\pm$ identification.[10] The $M(K^+K^-)$ distribution plotted in Fig. 2(b) shows some $S^*(1060)$ structure and a broad peak from 1.225 to 1.325 GeV/c^2 in the f^0-A_2 mass mass region. The Jackson and Treiman-Yang angles for the K^+K^- data are also shown in Fig. 2(b).

The method of our $\pi\pi$ phase-shift analysis is similar to that of Oh *et al.*[2] We use the absorption-modified one-pion-exchange (AOPE) formalism of Durand and Chiu.[11] For the $\pi^+\pi^-$ data with $|t|<0.3$ (GeV/c)^2, the $\cos(\theta_{\pi\pi})$ and φ distributions are fitted simultaneously with the $\pi\pi$ phase shifts and inelasticities $(\delta_l{}^I, \eta_l{}^I)$ as the only free parameters. The overall normalization is fixed so as to maximize the agreement between the resulting values for $\delta_D{}^0$ and the prediction of a Breit-Wigner resonance form near the f^0 peak. We take $M_{f^0} = 1.28$ and $\Gamma_{f^0} = 0.2$ GeV/c^2 as determined from a fit to the $\pi^+\pi^-$ data with the usual relativistic Breit-Wigner resonance forms[12] and a peripheral 2π phase-space background. To obtain values of $\delta_P{}^1$ at the ρ^0 peak in good agreement with a P-wave Breit-Wigner resonance form, we must use a normalization 13% larger than that found at the f^0 peak. The loss of events in the f^0 region probably has to do with the presence of the extra nucleon rather than natural-parity exchange.[13]

In fitting the $\pi^+\pi^-$ data the $I=2$ parameters are fixed at values determined from the reaction

$$\pi^- p \to p\pi^-\pi^0. \tag{5}$$

For $M(\pi\pi) < 1.2$ GeV/c^2 we use the $I=2$ phase shifts of Baton, Laurens, and Reignier[1] obtained from Reaction (5) at 2.77 GeV/c. At larger $M(\pi\pi)$ the $I=2$ parameters have been determined by fitting the $\pi^-\pi^0$ angular distributions in our 7-GeV/c data.[2] The resulting values for $\eta_S{}^2$, $\delta_S{}^2$, and $\delta_D{}^2$ are plotted in Fig. 3 along with the results of Baton, Laurens, and Reigner in the lower-mass region. The data are consistent with

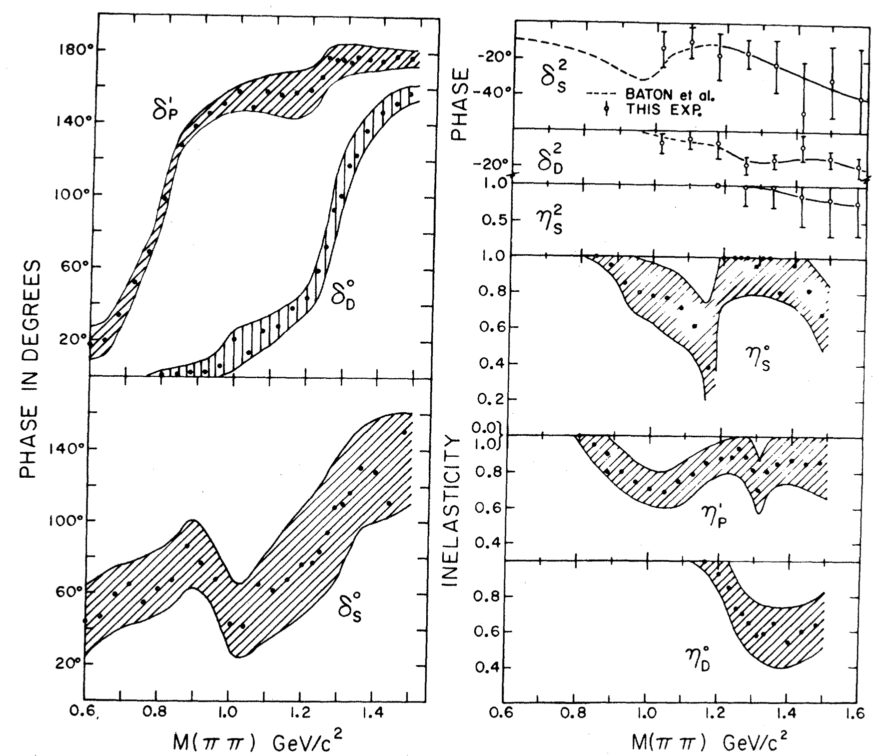

FIG. 3. $\pi\pi$ phase shifts and inelasticities from the AOPE-model fits to the $\pi^+\pi^-$ and $\pi^-\pi^0$ angular distributions. The shaded bands indicate roughly the area between the upper and lower limits.

$\eta_D^2 = 1.0$ below 1.5 GeV/c^2. Our 7-GeV/c data favor the results of Baton, Laurens, and Reigner for $1.0 < M(\pi^-\pi^0) < 1.2$ GeV/c^2 as opposed to a solution with $\delta_S^2 \simeq -30°$ and a smaller value of δ_D^2.

The phase shifts δ_S^0, δ_P^1, and δ_D^0 and the corresponding inelasticities are plotted in Fig. 3 for $0.6 < M(\pi\pi) < 1.5$ GeV/c^2. We find $\delta_F^1 = 1° \pm 5°$ at $M(\pi\pi) = 1.44$ GeV/c^2 and no F wave at smaller $\pi\pi$ masses. The curves in Fig. 1 show a sample of the resulting fits to the $\cos(\theta_{\pi\pi})$ plots. The azimuthal distributions in both the ρ^0 and f^0 mass regions are also described well by the model. The sharp backward peaking in $\cos(\theta_{\pi\pi})$ for mass bins between 0.98 and 1.14 GeV/c^2 is seen to be a P-D interference effect, i.e., a fairly constant P wave interfering with a rising D wave. This same result was found by Oh *et al.*[2] The dip in δ_S^0 at $M(\pi\pi) \simeq 1.0$ GeV/c^2 (near $K\bar{K}$ threshold) is in qualitative agreement with the "down" solution of Baton, Laurens, and Reigner.[1] Our results do not, however, completely rule out the possibility that δ_S^0 rises very rapidly through 90° near 1.0 GeV/c^2 since the statistics are weak in this mass region. We observe δ_S^0 rising through 90° in the f^0 peak region. This result seems to us to be quite unambiguous in spite of the presence of several potentially contributing partial waves. The situation is somewhat simplified in that the P-wave amplitude seems to be very small in the 1.2–1.4-GeV/c^2 mass range. The $I = 0$ D wave must be fairly well described by a Breit-Wigner resonance form in the range below 1.3 GeV/c^2. The lack of the hump in the middle of the angular distribution tells one immediately that the S-wave amplitude is nearly in phase with the D wave and, in fact, close to the unitarity limit.

We have tried to extrapolate the ratio of counts in the polar to equatorial regions of the angular distributions to the pion pole. Within our statistical accuracy (~20%) we find no variation of this ratio between $|t|$ values of 0.02 and 0.15 (GeV/c)^2.

Although our angular resolution is not as good for the $2\pi^0$ system, we find qualitative agreement with the $\pi^+\pi^-$ results. The 2π angular distributions are fit with δ_S^0 and δ_D^0 free to vary, and all other parameters fixed as determined by the $\pi^+\pi^-$ data. The results for δ_S^0 and δ_D^0 are consistent with δ_S^0 rising through 90° near the f^0 peak.

The $I = 0$ D wave is seen to be significantly inelastic at and above the f^0 peak. Similar results were obtained by Oh *et al.*[2] From a study of non-2π f^0 decay modes, we estimate $\eta_D^0 = 0.80 \pm 0.05$ at the f^0 peak, a value within errors of our $\pi^+\pi^-$ fitted results.[7,5] We do not understand the sizable inelasticity in the P wave above the ρ^0 peak. Although the AOPE-model fits persistently gave $\eta_P^1 < 1.0$, the low statistics in the mass range 0.9–1.0 GeV/c^2 do not rule out an elastic P wave.

The K^+K^- mass spectrum [Fig. 2(b)] qualitatively supports the phase-shift analysis. There is a rapid rise at $M(K^+K^-) \simeq 1.225$ GeV/c^2. The Jackson-angle distribution in this mass range is consistent with isotropy. Interpretation of the K^+K^- angular distributions is complicated by contributions from both A_2^0 and f^0 decay. We estimate 1.9 μb (~10 events) and 7.6 μb from A_2^0 and f^0 decay, respectively.[14] These data as well as those of Beusch *et al.*[3] give evidence for a strong S-D interference near the f^0, indicating the importance of the S wave in this mass region. The K^+K^- data show a strong D-wave signal only above 1.3 GeV/c^2. This supports our contention that the $I = 0$ D wave becomes quite inelastic in the mass range above 1.3 GeV/c^2.

In conclusion, we observe an S-wave resonance in the f^0 mass region which seems to be nearly elastic. We note that the mass of this resonance is quite close to that of the f^0 and consequenctly it qualifies as a member of a daughter trajectory.[15]

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[1]J. P. Baton, G. Laurens, and J. Reignier, Phys. Lett. 33B, 528 (1970); E. I. Shibata, D. M. Frisch, and M. A. Wahlig, Phys. Rev. Lett. 25, 1227 (1970); P. Sonderegger and P. Bonamy, in Proceedings of the Fifth International Conference on Elementary Particles, Lund, Sweden, 25 June–1 July, 1969 (unpublished); W. Deinet *et al.*, *ibid.* See B. Y. Oh *et al.*, Phys. Rev. D 1, 2494 (1970), for references to earlier $\pi\pi$ phase-shift studies.

[2]Oh *et al.*, Ref. 1.

[3]W. Beusch *et al.*, Proceedings of the Fifteenth International Conference on High Energy Physics, Kiev, U. S. S. R., 1970 (Atomizdat, Moscow, to be published).

[4]J. V. Beaupre *et al.*, Nucl. Phys. B28, 77 (1971).

[5]J. A. J. Matthews *et al.*, Nucl. Phys. B33, 1 (1971).

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[8]R. M. Morse, Ph.D. thesis, University of Wisconsin, 1969 (unpublished); J. R. Bensinger, Ph.D. thesis, University of Wisconsin, 1970 (unpublished).

[9]The recovery efficiency for $2\pi^0$-2γ events in the f^0 region is 0.9; however, the discrimination against $3\pi^0$-2γ events is only 0.3. For 1-γ events we simply point π_1^0 in the γ direction and calculate the direction of π_2^0 (a zero-constraint fit). Our Monte Carlo study shows that in this case we usually get the π^0 direction to within 2°–3° in the laboratory frame. For additional discussion of the fitting procedure, see Ref. 7.

[10]R. N. Diamond, A. R. Erwin, and M. A. Thompson, Nucl. Instrum. Methods 89, 45 (1970).

[11]L. Durand, III, and Y. T. Chiu, Phys. Rev. 139, B646 (1965).

[12]J. D. Jackson, Nuovo Cimento 34, 1644 (1964).

[13]J. T. Carroll *et al.*, Phys. Rev. Lett. 27, 1025 (1971).

[14]R. N. Diamond, Ph.D. thesis, University of Wisconsin, 1971 (unpublished).

[15]H. Lipkin has proposed the existence of a quark-antiquark resonance which is a mixture of $J=0$ and $J=2$, and which would couple naturally to the $\pi\pi$ system. Such a resonance would account for the observed angular distribution.

Test of Limiting Behavior for Λ and K_1^0 Produced in Inclusive pp Interactions

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New data are presented on longitudinal and transverse momentum spectra of Λ and K_1^0 produced in $pp\to\Lambda$ + anything and $pp\to K_1^0$ + anything at 13, 18, 21, 24, and 28 GeV/c. The Λ spectra show little variation with energy. The K_1^0 inclusive cross section increases by ~ 50% over our energy range, indicating that for this process the threshold for scaling and limiting fragmentation may be above 30 GeV/c. Tests of Mueller-Regge phenomenology are discussed.

Experimental studies[1] demonstrate that single-pion inclusive spectra become energy independent, or nearly so, at lab momenta well below 30 GeV/c. This success at low energy has led to broad acceptance of the concept of limiting behavior in inclusive processes.[2,3] Based on the work of Mueller,[4] more detailed Regge-pole analyses have also been proposed.[5] Estimates have been made for the rate at which limiting behavior should be attained; these predictions have met with success, again for pion production.[1,6] However, little information has been available up to now on energy dependence or shapes of inclusive spectra of K, Λ, or of other heavy produced hadrons.[7-9] In this note, we present new data on the longitudinal and transverse momentum spectra of Λ and K_1^0 produced in reactions $pp\to\Lambda$ + anything[10] and $pp\to K_1^0$ + anything at 13, 18, 21, 24, and 28 GeV/c. From the point of view of testing limiting behavior[2,3] and Mueller-Regge phenomenology,[4,5] the value of a systematic comparative study of single-particle distributions for several incoming energies cannot be overemphasized.[3]

Data were obtained from a total of 75 000 pictures taken at the Brookhaven National Laboratory 80-in. hydrogen bubble chamber. At each momentum, the beam flux corresponds to about 0.5 events /μb. Events with one or more visible vees, independent of the primary interaction topology, were scanned and measured. The resultant efficiencies for scanning and measuring were 98 and 87%, respectively. Of the final sample of vees, after kinematics and ionization comparisons were applied, 97% of the Λ sample and 95% of the K_1^0 sample were identified uniquely. Utilizing the forward-backward symmetry in the c.m., we determined that 13 and 17% of the forward Λ's were lost at 13 and 28 GeV/c, respectively, principally because of the peripheral nature of Λ production. The Λ data presented here, with the exception of Fig. 3, are only for events with $x<0$ ($x=2p_{L\,\mathrm{c.m.}}/\sqrt{s}$, where $p_{L\,\mathrm{c.m.}}$ is longitu-