Trapping of Electrons in Large-Amplitude Electrostatic Fields Resulting from Beam-Plasma Interaction

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Trapping of beam electrons has been observed in the potential wells of large-amplitude electrostatic waves ($E = 7\times 10^3$ V m^{-1}) resulting from beam-plasma interaction. Trapping is attended with the growth of a red-shifted sideband near the main peak in the frequency spectrum and with spatial oscillations in the amplitude of the main peak.

In a number of quiescent-plasma experiments[1-4] it has been shown that the excitation of large-amplitude plasma waves results in nonlinear phenomena. Some of the nonlinearities are due to the trapping of plasma particles in the wave potential troughs, in the vicinity of the point where the wave is excited in the plasma. The trapped particles cause oscillations in the amplitude of the excited wave and the growth of new waves, the so-called sidebands. Sidebands are found symmetrically placed around the main peak in the frequency spectrum, displaced by an amount $\omega_B/2\pi$. [See Eq. (2).] The nonlinear phenomena are observed when the wave phase velocity $v_f = \omega/k$ is not much larger than the thermal velocity of the plasma particles v_t. In previous experiments[1-4] $v_f \lesssim 5v_t$; measurements were performed on low-density ($n \approx 10^{14}$ m^{-3}) plasmas, with wave electric fields $E \approx 10$ V m^{-1} and energy density $\epsilon_0 E^2/n\kappa T \approx 10^{-2}$.

In the beam-plasma experiment discussed in this Letter we have observed the trapping of beam electrons in the large fields ($E \approx 7\times 10^3$ V m^{-1}) of beam-plasma instabilities, together with the excitation of a red-shifted sideband and with spatial oscillations in the amplitude of the instability. Our measurements therefore extend the wave amplitudes for which sideband generation has been observed by two orders of magnitude. Moreover our plasma is turbulent rather than quiescent and the ratio of phase velocity to thermal velocity is high, $v_f/v_t \approx 10$. The large fields result in a large energy density, $\epsilon_0 E^2/n\kappa T \approx 1$.

In the experiment a monoenergetic beam of 1.5 kV, 20 mA is continuously injected along a homogeneous magnetic field of ≈ 200 G into a beam-created plasma, $n \approx 10^{16}$ m^{-3} and $T_e \approx 3$ eV. A beam-plasma system is linearly unstable and under the said conditions the beam-excited waves are essentially monochromatic and quasistatic in nature. We measured a halfwidth $\Delta\nu/\nu \approx 2\%$, corresponding to a time duration of an instability of some fifty periods as measured with an oscilloscope. It means that every instability occurs as a series of random and spontaneous bursts while the beam is running continuously through the plasma. Bursts of different instabilities alternate. The time-integrated frequency spectrum, Fig. 1, shows that many instabilities are excited in the beam-plasma system. Since the instability discussed here is due to a coupling of the slow beam space-charge wave with the backward plasma wave,[5] the unstable frequencies are found in the interval $\nu_{ce} < \nu < \nu_{uh} \equiv (\nu_{pe}^2 + \nu_{ce}^2)^{1/2}$. The condition for instability is $v_t < v_f < v_{0b}$, where v_{0b} is the injection velocity of the beam. In our experiment[6] $v_f \approx 0.85 \times v_{0b} \approx 2\times 10^7$ m sec^{-1}. So we see that particles with a velocity nearest to the phase velocity are beam electrons and not plasma electrons. Therefore trapping has been found for beam electrons. Calculations show that a large fraction of the beam becomes trapped indeed.[7]

The possibility of trapping of beam electrons seems to have been considered first by Berezin and co-workers.[8] Experimental proof of trapping

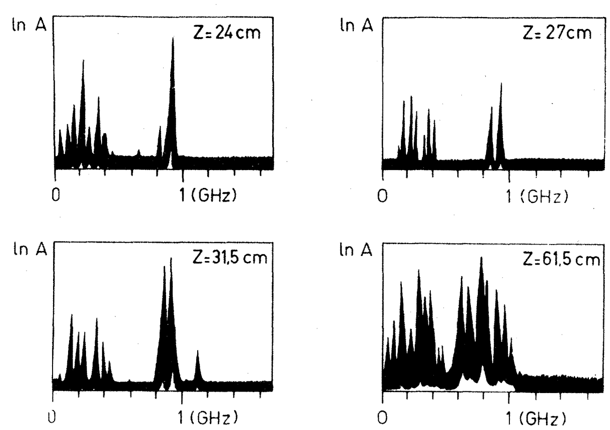

FIG. 1. Measurement of frequency spectrum, showing growth of a red-shifted sideband. Main instability is at 900 MHz; sideband at ≈ 800 MHz. $B_0 = 180$ G; $V_b = 1.5$ kV; $i_b = 1.5$ mA; $p = 7\times 10^{-4}$ Torr; logarithmic detection.

was given only recently.[9] Measurements of the beam velocity distribution $f_b(v)$ when the beam-plasma instability was strong enough showed $f_b(v)$ to be symmetric about v_f, whereas it is symmetric about v_{0b} for weaker instabilities. Under change of the ratio v_f/v_{0b} between 0.8 and 0.9, $f_b(v)$ remained symmetric about v_f.[6] This behavior was considered as a sufficient indication that beam electrons become trapped.

The condition that a particle with velocity v be trapped is[10]

$$\tfrac{1}{2}m(v-v_f)^2 \leq 2eEk^{-1}. \tag{1}$$

The measurement of the extreme values of Δv $v-v_f$ in $f_b(v)$ provides a way to know E, using Eq. (1). The wave number $k(\omega)$ of the spontaneously excited waves is determined with interferometric methods using the plasma as the signal generator and a spectrum analyzer as a narrow-band detector at the frequency ω. Because the instability stops growing after trapping has taken place, the field obtained from $f_b(v)$ is approximately the maximum field amplitude of the wave. For the conditions used in Eq. (1), the field amounts to 7×10^3 V m^{-1}, with $v_f=\omega/k=2\times10^7$ m sec^{-1}, $\omega=5.6\times10^9$ sec^{-1}, and $k=270$ m^{-1}.

The part of the beam that becomes trapped starts to oscillate in the trough with a frequency[10]

$$\omega_B=(eEk/m)^{1/2}=\tfrac{1}{2}k\Delta v. \tag{2}$$

The second equality is obtained after substitution of Eq. (1). After the trapping, the beam electrons are no longer able to give their energy to the unstable wave. The wave amplitude therefore saturates, but now the coherent oscillation of the trapped beam electrons forms an energy source for new instabilities.[11,12] In the spectra in Fig. 1, taken for increasing distances from the electron gun, one can observe some 110 MHz below the main peak at 900 MHz, which is the beam-plasma instability, the growth of a second peak. The sideband starts to grow as soon as trapping has occurred. Its phase velocity, measured with interferometric methods is again 2×10^7 m sec^{-1} $\pm 5\%$; the sideband seems to be supported by the slow beam space-charge wave. The phase velocities of the main wave and its sideband being equal, the theories of Kruer, Dawson, and Sudan[12] and Mima and Nishikawa[11] both give the result that the displacement of the sideband is equal to ω_B. Substituting Δv and k into Eq. (2) gives $\omega_B/2\pi=90$ MHz. Contrary to measurements in quiescent plasmas, we find no upper sideband. Another pecularity is the linear decrease of the sideband displacement with distance, as seen from Fig. 1.

Related to sidebands are spatial amplitude oscillations. Recent calculations by Onishchenko *et al.*[13] and O'Neil, Winfrey, and Malmberg[14] show the saturation of beam-plasma instabilities due to trapping and a periodicity in the saturated amplitude. When trapped particles are at the bottom of the potential well they oppose the space charge of the plasma electrons that maintain the wave with their coherent oscillation. The wave has at this point a smaller amplitude than when the trapped particles are spread out in the trough. Thus it is to be expected that the wavelength of the amplitude oscillation is given by

$$\lambda_B=2\pi v_f/\omega_B. \tag{3}$$

Using the setup shown in Fig. 2, we obtain the amplitude pattern shown in Fig. 3. The wavelength is $\lambda_B=9$ cm $\pm 30\%$. Using the known value of v_f, Eq. (3) gives $\omega_B/2\pi=210$ MHz.

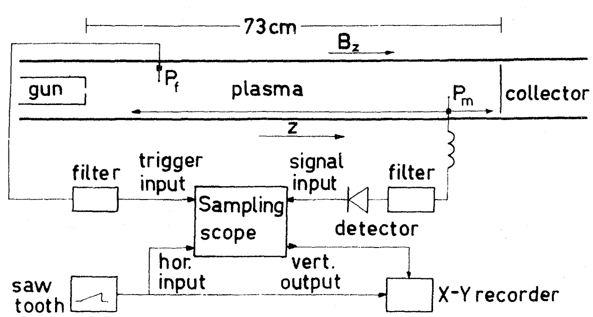

FIG. 2. Setup for measuring the space-time correlation of the spontaneously occurring beam-plasma instabilities.

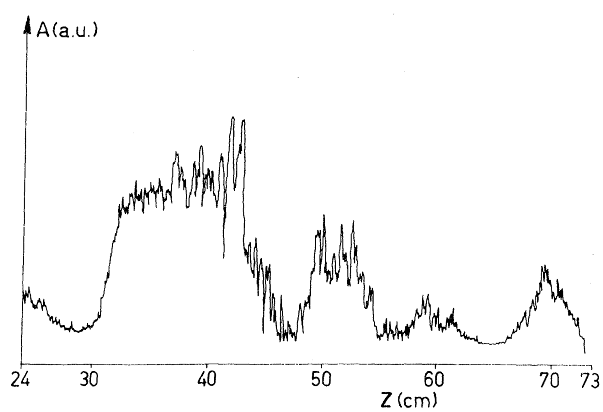

FIG. 3. Momentary amplitude pattern of the instability at 900 MHz, shown in spectrum of Fig. 1. Spectrum is still monochromatic at position of probe P_f at $z=31$ cm, which provides the triggering signal for the sampling scope. Linear detection.

The width of the hump around $z = 35$ cm causes a large uncertainty in the λ_B value. The time history[15] of the instability shows 100 nsec earlier a definite peak at $z = 40$ cm, making a λ_B value of 9 or 10 cm more probable than a larger value that is still compatible with Fig. 3. As the beam continues to feed energy into the instability, the front on the hump at $z = 30$ cm steepens and moves towards the gun. These two effects mask the original maximum at $z = 40$ cm. The time history also shows that the humps in the amplitude are not isolated phenomena but are correlated with each other. They occur at the *same* time, which is surprising for a rather turbulent system.

The different humps might exert different influences on the measured beam distribution function. Change of the system length from 73 to 50 cm did not affect the distribution function significantly. The width Δv is therefore determined by the first hump seen in Fig. 3.

One of the essential points in the setup of Fig. 2 is the triggering of the sampling scope with the signal from probe P_f. This triggering occurs at a fixed phase of the instability. The resulting Fig. 3 is therefore a true momentary picture of the amplitude pattern. The jitter in the triggering is small compared with the wave period. The phase of the signal from P_m has also been measured,[15] thus giving a second measurement of k. The triggering is necessary because the beam-plasma system is noisy, with many instabilities with different amplitudes alternating. Triggering provides a careful selection of a particular state of the system.

From the measured width of the beam distribution function we conclude that the beam-plasma instability saturates and that at that point beam electrons become trapped in the instability. The width of $f_b(v)$ leads to an estimate of the amplitude of the saturated electric field, 7×10^3 V m^{-1}. Nonlinear phenomena following upon trapping are the spatial growth of a red-shifted sideband and spatial amplitude oscillations. The sideband is not due to a decay instability because no peak is present in the frequency spectrum at $\omega_B/2\pi$ though ω_B falls on a propagation band of the plasma. With coincidence measurements we found that instabilities in the low- and high-frequency range of the spectrum are mutually excluding each other[16] and show no correlation.

The generation of sidebands is symmetric with respect to the frequency of the main peak. Both sidebands should obey the linear dispersion characteristics of the beam-plasma system. The absence of a blue-shifted sideband can only be imputed therefore to the dispersion. Simple calculations[5] show that a beam-plasma system is unstable for $\omega_{ce} < \omega < \omega_{uh}$; in case $\omega_{ce} \ll \omega_{pe}$ it will have a maximum growth rate for ω near ω_{pe}. Thus a red-shifted sideband will always be unstable and grow with the linear growth rate. Theory predicts a phase velocity that is slightly less than the beam velocity, as is found. For the blue-shifted sideband it depends on the displacement whether the particular frequency is unstable or not. Preliminary calculations of the beam-plasma dispersion diagram for our conditions show that the sideband stabilizes at a displacement of ≈ 150 MHz. Although the blue sideband is presumably unstable, its growth rate is not large enough to let it grow out of the background noise.

The discrepancy between the measured frequency shift $\Delta\omega$ and the value of ω_B calculated from Eq. (3) can only be discussed when a theory for the present case has been set up. Kruer, Dawson, and Sudan[12] find $\omega_B \approx 3\Delta\omega$. The calculations[13,14] do not show the rapid decay of the amplitude oscillations displayed in Fig. 3. As the sideband grows to the same amplitude as the instability, a destructive interference results in places with very small electric fields. Here the beam electrons become untrapped for a moment and lose their phase relation with the instability, and the amplitude oscillations disappear. (New calculations by Shapiro[7] show a similar behavior.) The interference wavelength is approximately 30 cm, which agrees with the length $40 < z < 65$ cm over which amplitude oscillations are observed. The last hump at $z \approx 70$ cm is not taken into consideration because it is influenced by reflections from the collector at $z = 73$ cm.

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