the sound beam, and (3) the attenuation of the sound. Since (1) and (2) are known, it is in principle possible to determine (3). For the low-angle scattering of these experiments, however, all three factors contribute about equally to the observed width, making it very difficult to extract a good measurement of attenuation.

[5]P. Debye and F. W. Sears, Proc. Nat. Acad. Sci. Wash. 18, 409 (1932).

[6]I. Rudnick, to be published.

[7]G. Jacucci and G. Signorelli, Phys. Lett. 26A, 5 (1967), who first observed light scattering from driven second sound in He II, obtain somewhat different results from ours, especially about 2°K. They used driving powers 500 times larger than ours, however, which may have caused nonlinearities to occur in their sound waves near the λ point.

[8]M. J. Buckingham and W. M. Fairbank, *Progress in Low-Temperature Physics*, edited by J. C. Gorter (North-Holland, Amsterdam, 1961), Vol. III, p. 80.

[9]By using this same technique, we have also observed light scattered from second and first sound in 11% and 20% superfluid He^3-He^4 mixtures. See D. Petrac, Ph.D. thesis, University of California, Los Angeles, 1971 (unpublished). Brillouin scattering has been observed from second sound in mixtures by E. R. Pike, J. M. Vaughan, and W. F. Vinen, Phys. Lett. 30A, 373 (1969).

Parametric Suppression of a Large-Amplitude Plasma Instability

J. F. Decker
Bell Telephone Laboratories, Whippany, New Jersey 07981

and

A. M. Levine
Bell Telephone Laboratories, Whippany, New Jersey 07981, and Richmond College of the City University of New York, Staten Island, New York 10301

(Received 29 September 1971)

The intensity of a large-amplitude, unstable plasma mode has been reduced by applying a pump signal to parametrically couple it to another damped, natural mode of the plasma column. A suppression of the mode is observed when the frequency of the pump ω_0 is equal to the difference between the damped-mode frequency ω_a and the unstable-mode frequency ω_b ($\omega_0 = \omega_a - \omega_b$). The observed behavior of the mode energies is compared with a weak parametric-mode-coupling theory for lumped circuits, which includes nonlinear saturation terms for the unstable mode.

Many observations of parametric excitation of initially stable plasma modes have been reported.[1] It has been suggested theoretically[2] that it is also possible to suppress an unstable mode by parametrically coupling it to a naturally damped mode. In addition to its fundamental interest, this process has possible application as an active means for controlling plasma instabilities, a subject of current interest.[3]

It has been demonstrated by Hai and Wong[1] that azimuthally propagating modes of a magnetized plasma column can be actively coupled, producing the familiar parametric excitation of initially stable modes. In those experiments, a pump excitation applied at the sum frequency $\omega_0 = \omega_a + \omega_b$ (with a similar matching condition for the azimuthal wave numbers) produced parametric growth of two modes A and B. In our experiments, to achieve parametric suppression the pump was applied at frequency $\omega_0 = \omega_a - \omega_b$. This passively coupled[4] an unstable mode B to a damped mode A, and thereby reduced the amplitude of mode B.

To model this situation theoretically, we consider the equations with weak parametric mode coupling for lumped circuits[4] including nonlinear saturation terms for the initially unstable mode:

$$da/dt = (i\omega_a - \nu_a)a + C_{ab}e^{i\omega_0 t}pb, \tag{1}$$

$$db/dt = (i\omega_b + \nu_b)b + C_{ba}e^{-i\omega_0 t}pa - \alpha|b|^2 b - \delta|b|^4 b + \cdots, \tag{2}$$

$$\omega_0 = \omega_a - \omega_b; \tag{3}$$

a, b, and p are the amplitudes of mode A, mode B, and the pump; the $C_{ab,ba}$ are the coupling coefficients. For $p=0$, the uncoupled mode A has a damping rate ν_a, and mode B has a growth rate ν_b and nonlinear saturation parameters α and δ. We seek solutions of the form $a = A(t)e^{i\omega_a t}$ and $b = B(t)e^{i\omega_b t}$ and assume $A(t)$ and $B(t)$ have time dependence e^{st}. Neglecting nonlinear saturation,

$$s_\pm = \tfrac{1}{2}\{\nu_b - \nu_a \pm [(\nu_a + \nu_b)^2 - 4|C_{ab}C_{ba}|p^2]^{1/2}\}$$

since $C_{ab}C_{ba}$ must be negative from energy-flow considerations. Stability is obtained ($\mathrm{Re}\, s<0$) when $\nu_a > \nu_b$

$$p^2 \geq \nu_a \nu_b / |C_{ab}C_{ba}|. \tag{4}$$

In our experiments we start with a large-amplitude instability, which is nonlinearly saturated, and make measurements of the steady-state mode energies as a function of the pump amplitude and frequency. Including the lowest-order nonlinear saturation term for the large-amplitude, unstable mode B, we obtain

$$\dot{A} + \nu_a A = C_{ab} p B, \tag{5}$$

$$\dot{B} - \nu_b B + \alpha |B|^2 B = C_{ba} p A. \tag{6}$$

The steady-state mode energies ($\dot{A} = \dot{B} = 0$) are then

$$E_b = |B|^2 = \frac{1}{\alpha}\left[\nu_b - \frac{|C_{ab}C_{ba}|}{\nu_a} E_0\right], \tag{7}$$

$$E_a = |A|^2 = \frac{|C_{ab}|^2 E_0}{\nu_a^2 \alpha}\left[\nu_b - \frac{|C_{ab}C_{ba}| E_0}{\nu_a}\right], \tag{8}$$

where $E_0 = p^2$. In this approximation, Eq. (7) predicts a linear decrease in the energy of mode B as the pump energy is increased. If higher-order saturation terms are important, then for small E_0, E_b will begin to decrease as some fractional power of E_0. As E_0 is increased and E_b reduced, then it is expected that there will be some range of E_0 for which Eq. (7) is valid. Equation (8) indicates that the energy of mode A increases for small E_0, reaches a maximum, and then can decrease for large E_0 ($E_a = 0$ when $E_b = 0$).

The experiments were performed in the Bell Telephone Laboratories cyclotron-resonance plasma device.[5] Typical operating conditions were charged particle density $\sim 10^{10}$ cm^{-3}, electron temperature ~ 4 eV, background pressure $\sim 10^{-5}$ Torr of argon, and magnetic field ~ 575 G. A strong $m = 1$ (azimuthal mode number) instability having a long axial wavelength was observed to occur naturally in the frequency range $f_1 = 1.5 - 4$ kHz. (For the remainder of this paper, the notation $f_n = \omega_n/2\pi$ will be used.) When this instability occurs, the space potential, which is negative with respect to the wall, exhibits a minimum at a radius of ~ 1 cm from the column center. At this position, the $\vec{E} \times \vec{B}$ drift changes directions, producing a velocity shear that appears to be responsible for driving the instability.[6] The growth rate of this mode is large, and the oscillation is observed in a nonlinearly saturated state having very distorted wave forms.

In the suppression experiments, the pump signal was applied by amplitude modulation of the microwave power, $P = P_0[1 + \beta \cos\omega_0 t]$, used to produce the plasma. This resulted in a plasma density modulation $N = N_0[1 + \epsilon \cos\omega_0 t]$ which we believe is responsible for coupling the modes. The fluctuation energy in narrow frequency ranges was measured using a tuned amplifier with variable Q followed by an rms voltmeter. For each frequency, the probe was located at a radial position where the maximum fluctuation intensity was observed. As the modulation frequency f_0 was varied, the fluctuation energy E_1 in the unstable mode was observed to be reduced over certain ranges of f_0. Data of this type are shown in Fig. 1 with $\beta = 0.19$. The corresponding ϵ varies somewhat with f_0 over the range of f_0 shown but

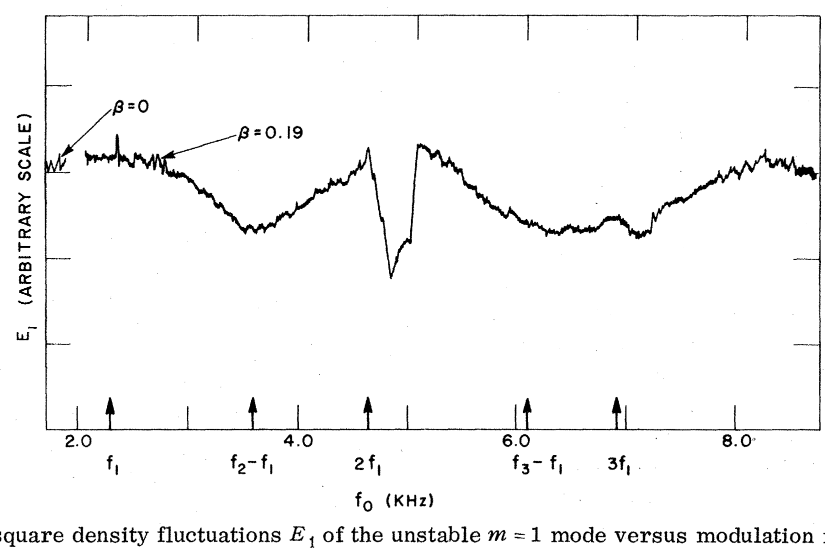

FIG. 1. Mean square density fluctuations E_1 of the unstable $m = 1$ mode versus modulation frequency f_0 for a modulation index $\beta = 0.19$. f_2 and f_3 are the frequencies of the $m = 2$ and $m = 3$ modes, respectively.

was ~0.1. The frequency of the unstable $m=1$ mode was $f_1 = 2.3$ kHz. As f_0 was increased above f_1, a region of suppression was observed with a minimum in E_1 at ~3.5 kHz. Another large but narrow dip occurred near $2f_1$ followed by a broad suppression region with a minimum in E_1 at ~6.1 kHz. Another small dip superimposed on the latter region is seen near $3f_1$.

Also shown is the location of the frequencies $f_2 - f_1$ and $f_3 - f_1$, where $f_2 \sim 5.7$ kHz and $f_3 \sim 8.3$ kHz are the frequencies of the $m=2$ and $m=3$ modes. These frequencies were determined from measurements of the response of the plasma to an applied signal. Minima in E_1 occur when $f_0 \approx f_a - f_1$, where f_a is equal to the frequency of a naturally damped mode of the plasma. The width of these suppression regions will be discussed later. The sharp dip near $f_0 = 2f_1$ and the dip near $3f_1$ appear to result from harmonic-frequency entrainment and suppression effects that often occur in driven, self-excited, nonlinear oscillators. This will be the subject of another report.

The suppression regions of interest do not appear to result from changes in the zeroth-order plasma produced by the modulation. Probe measurements of the radial density and potential profiles in the presence of the modulation show no significant changes from those obtained in the absence of modulation. Also one would not expect the effect of changes of the zeroth-order parameters induced by low-frequency modulation to show frequency resonances.

The mode energies as well as the ratio of the mode energies (E_a/E_b) are shown as functions of the pump energy with $f_0 = 3.5$ kHz and $f_0 = 6.1$ kHz in Figs. 2(a) and 2(b). The behavior of the mode energies in both cases is similar. As the energy in the pump increases, the energy E_1 in the $m=1$ mode (mode B of the theory) decreases gradually at first and then for intermediate pump energies, E_1 decreases linearly with pump energy. This behavior is in qualitative agreement with the theoretical model, as was discussed above. The damped modes (mode A of the theory) are the $m=2$ mode for Fig. 2(a) and the $m=3$ mode for Fig. 2(b). E_2 and E_3 also show the qualitative behavior expected from the model, increasing for small E_0 and then approaching a maximum. From our model, it is seen that

$$E_a/E_b = (|C_{ab}|^2/\nu_a^2)E_0,$$

independent of the type of nonlinear saturation

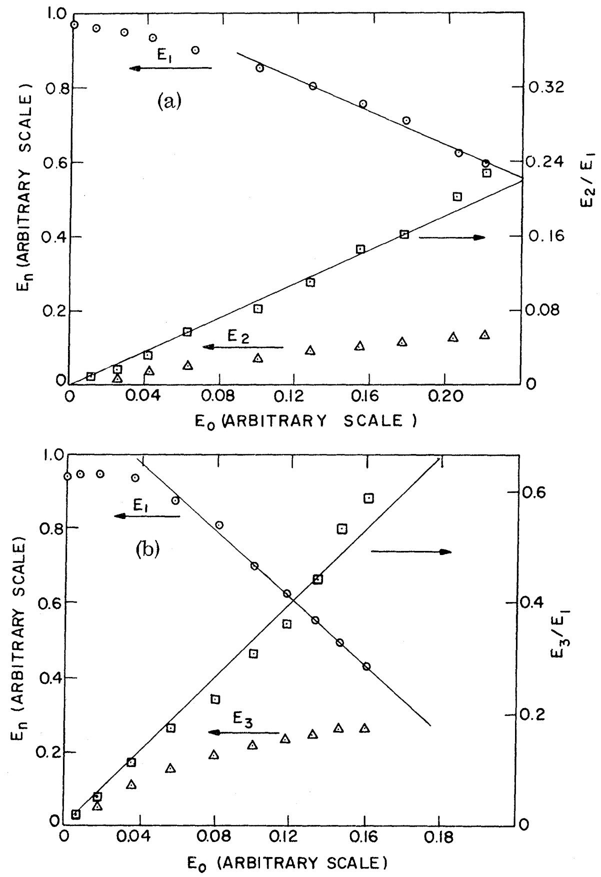

FIG. 2. (a) Observed behavior of the mode energies E_1 and E_2 and the ratio E_2/E_1 as functions of the pump energy E_0 with $f_0 = 3.5$ kHz. The corresponding range of β was $\beta = 0$ to $\beta = 0.3$. (b) Observed behavior of the mode energies E_1 and E_3 and the ratio E_3/E_1 as functions of the pump energy E_0 with $f_0 = 6.1$ kHz. The range of β is the same as for (a).

present in mode B. The plots of E_2/E_1 and E_3/E_1 as functions of E_0 shown in Figs. 2(a) and 2(b) indicate that this linear relationship is obeyed.

We return to a discussion of the width of the suppression regions. For coupling the unstable mode to the $m=2$ mode, the pump is not applied near a resonant mode frequency of the plasma. Since the spectral widths of the pump signal and of the unstable fluctuations are much smaller than the width of the damped mode, it is expected that the width of the suppression region will be determined by the latter. The measured width of the suppression was 0.9 kHz while the width of the $m=2$ mode response was 0.8 kHz. For coupling to the $m=3$ mode, the width of the suppression region was 1.9 kHz while the width of the $m=3$ mode response was 1.2 kHz. In this case, the pump frequency was applied near f_2, and there was evidence from the radial dependence of the density fluctuations at the pump frequency that the $m=2$ mode was being excited. Since $f_2 < f_3 - f_1$, this may account for some of the additional broadening.

An attempt was made to measure the growth rate of the instability by switching off the pump signal and observing the rise time of the instability. These observations indicate that the growth rate was very large, at least as large as the observed instability frequency.

In conclusion, we have observed the suppression of a large-amplitude instability by parametric coupling to a damped normal mode of the plasma column. We obtained qualitative agreement between the experimental observations and a simple theory based on parametric coupling of lumped circuits.

This technique may be applicable to other instabilities and plasma devices. When a large-amplitude oscillation is detrimental to device operation, by using parametric suppression its energy may be transferred to a less detrimental, damped mode. In addition, the energy dissipated in the damped mode may result in particle heating. In contrast to linear feedback stabilization this technique may be used on high-growth-rate, nonlinearly saturated modes. Also since it is a resonant process, it should require less energy than other nonresonant types of dynamic stabilization.

It is a pleasure to acknowledge many useful conversations with R. A. Stern and the technical assistance of C. V. D'Amico.

[1]R. A. Stern and N. Tzoar, Phys. Rev. Lett. 17, 903 (1966); A. Y. Wong, M. V. Goldman, F. Hai, and R. Rowberg, Phys. Rev. Lett. 21, 518 (1968); S. Tanaka, R. Sugaya, and K. Mizuno, Phys. Lett. 28A, 650 (1969); R. A. Stern, Phys. Rev. Lett. 22, 767 (1969); M. Porkolab and R. P. H. Chang, Phys. Rev. Lett. 22, 826 (1969); F. Hai and A. Y. Wong, Phys. Fluids 13, 672 (1970); A. F. Bakai, Zh. Eksp. Teor. Fiz. 59, 116 (1970) [Sov. Phys. JETP 32, 66 (1971)].

[2]A. Hasegawa, R. Davidson, and R. Goldman, Appl. Phys. Lett. 14, 325 (1969), and Phys. Fluids 12, 1247 (1969).

[3]*Proceedings of the American Institute of Physics Conference on Feedback and Dynamic Control of Plasmas, Princeton, New Jersey, 1970,* edited by T. K. Chu and H. W. Hendel (American Institute of Physics, New York, 1970).

[4]W. H. Louisell, *Coupled Mode and Parametric Electronics* (Wiley, New York, 1960).

[5]J. F. Decker and C. V. D'Amico, Rev. Sci. Instrum. 41, 1431 (1970).

[6]L. Enriques, A. M. Levine, and G. B. Righetti, in *Proceedings of the Third International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Novosibirsk, U. S. S. R., 1968* (International Atomic Energy Agency, Vienna, Austria, 1969), Vol. I, p. 641; D. L. Jassby and F. W. Perkins, Phys. Rev. Lett. 24, 256 (1970); F. W. Perkins and D. L. Jassby, Phys. Fluids 14, 102 (1971).

Interpretation of the Nuclear Spin-Lattice Relaxation-Time Behavior in the Nematic Mesophases*

Assis F. Martins

Laboratoire de Résonance Magnétique, Centre d'Etudes Nucléaires de Grenoble, 38 Grenoble-Gare, France

(Received 3 January 1971)

We give a coherent quantitative interpretation of both the frequency and temperature behavior of the proton spin-lattice relaxation time T_1 in the nematic phase of *p*-azoxyanisole. The frequency dependence of T_1 is dominated by the long-wavelength thermal fluctuations in the orientational order. The temperature dependence of T_1 has an important contribution from short-range phenomena involved in the motion of methyl protons. The activation energy for this motion is $W \approx 9$ kcal/mole.

About two years ago, Pincus[1] predicted that in nematic liquid crystals the nuclear relaxation arising from the modulation of the dipolar intramolecular energy due to the long-wavelength orientational modes of motion should be competitive with the relaxation arising from the modulation of the intermolecular spin-spin coupling via the translational diffusion. His calculation, as improved by Lubensky,[2] gives the following expression for the relaxation rate:

$$\frac{1}{T_{1B}} = \omega_D^2 \frac{k_B T S^2}{K} \left[\omega_0 \left(D + \frac{K}{\gamma}\right)\right]^{-1/2} + \frac{1}{T_1'}, \qquad (1)$$

where ω_D is a dipolar frequency, ω_0 is the nuclear Larmor frequency, and K, γ, and S are parameters characteristic of the liquid crystal (see