the boundary conditions $f(\epsilon_i)=0$, ($\epsilon_i=16$ eV), and that each electron attaining the ionization energy ϵ_i disappears and is replaced by $2p_a$ electrons at zero energy. This is appropriate for SF_6, in which attachment occurs mainly in a narrow band[6] below about 0.1 eV, where p_a is the probability of escaping the attachment band, $(2\alpha-\beta)/2\alpha$. For the numerical example $p_a=0.6$ which, based on Pederson's empirical equation,[7] corresponds to $E/p=126.5$ V/cm Torr. We use the low-frequency Eq. (9) for K, and take ν proportional to ϵ (a relationship suggested by matching the SF_6 radius to the sum of theoretically calculated S and F radii[8]). The figures are plotted with energy in eV and time measured in units of $1/K_1$, where K_1 is the value of K at 5 eV. Numerically, $1/K_1$ is 4.1×10^{-13} sec and $\nu=5.4\times10^{12}\epsilon$ when p is 1520 Torr of SF_6 and $E/p=126.5$ V/cm Torr with ϵ in eV.

Although our derivation proceeded independently from the Boltzmann equation, Eq. (14) is consistent with Allis's equation[3] for $f_0{}^0(v)$ (where $f_0{}^0$ is the first term in the Legendre expansion) provided we recall that $f(\epsilon)$ is proportional to $vf_0{}^0(v)$, that our derivation assumes $Y_j=1$, and that the Legendre methods assume $f_0{}^0$ independent of time. Although no precise criterion for truncating the Legendre expansion has been given,[3] if Eq. (15) were violated the distribution function would be highly anisotropic. Including time dependence allows us to treat arbitrary initial energy distributions and rapidly exponentiating avalanches. For cold background gases,[9] we can remove the restriction to zero energy loss by averaging Y_j in Eq. (4) to obtain $\overline{Y}_j=1-2m/M$.

The author has benefited from discussions with Dr. M. Bass and Dr. H. Barrett.

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Light Scattering from Density Variations Excited in He^4 by a Thermal Transducer*

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We have observed light scattering from the density variations in the first- and second-sound modes generated simultaneously by a thermal transducer in superfluid He^4. Measurements of the temperature dependence of the density variations in both sound waves agree with the theoretical treatment by Lifshitz of the density-temperature coupling.

Recent reports[1] of measurements of the pressure variations in the sound waves generated by a thermal transducer in He^4 are in conflict with the theory of Lifshitz.[2] In this Letter, we report direct measurements of the density variation in such sound waves by light-scattering techniques. These measurements are found to agree with the Lifshitz theory.

The hydrodynamic equations for superfluids predict that two wave modes, each with its own characteristic velocity, are generated by a periodically heated, stationary plane surface (thermal transducer). Most of the energy is in the second-sound mode (velocity u_2) which, although it is predominantly a temperature or entropy wave, carries some density variations (ρ_2') as well. In addition to second sound, a low-intensity first-sound wave (velocity u_1) is simultaneously generated, which also carries density variations (ρ_1'). In pure He^4, the coupling between temperature and density variations is due primarily to the small, but finite, thermal expansion coefficient.

There are therefore two density-wave fields present in the superfluid. Since light is scattered by the periodic spatial modulation of the dielectric constant caused by density variations in the medium, we felt that this would be a good method by which to probe the temperature-density coupling. Because $u_1 > u_2$, different scattering regimes are employed to observe the two sound modes at a given frequency. The detection scheme, however, is the same in both cases. The thermal transducer, a gold film evaporated on a quartz substrate, is driven by a gated sinusoidal current at frequency $F/2$ (F ranges from 0.06 to 2.2 MHz) and at power levels of 1 mW/cm^2. The scattered light, shifted in frequency by twice the driving frequency, beats with unshifted local oscillator light in a silicon photodetector. The chopped rf signal from the photodetector is heterodyne detected, rectified, and fed to a lock-in amplifier. The output of the amplifier drives the Y axis of an X-Y recorder. The X axis of the recorder plots the temperature of the superfluid which, in these experiments, ranges from 1.3°K to the λ point.

To observe second sound, we use the Bragg scattering technique[3] in which He-Ne laser light is incident on the transducer at the appropriate (Bragg) angle. The very weak scattered light beats with the collinear local oscillator beam reflected from the surface of the transducer. Measurements are made by choosing a driving frequency $F/2$, setting the scattering angle θ by the direction of incident light and the location of the photodetector, and then slowly drifting the temperature until a peak in the scattered light is recorded. At the temperature of the peak T_p, the Bragg condition $u_2 = \lambda F(2\sin\frac{1}{2}\theta)^{-1}$ is satisfied and determines $u_2(T_p)$ (λ is the wavelength of the light in the medium). Figure 1 shows a temperature sweep in pure He^4, where F and θ are such that

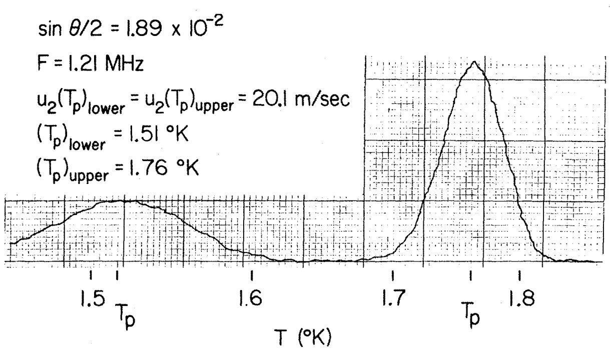

FIG. 1. Recorder trace of light scattering amplitude from second sound in He^4. The two peaks are characteristic of a temperature sweep when the Bragg condition is satisfied by a value of u_2 near the maximum of the $u_2(T)$ curve.

the Bragg condition is satisfied for $u_2 = 20.1$ m/sec, very close to the local maximum of u_2, so that two peaks are seen.[4] Using different combinations of $F/2$ and θ, we have measured u_2 throughout the temperature range. Agreement with previously published measurements is very good. To measure the temperature dependence of ρ_2' the amplitude of the density variation in the second-sound wave, frequency and local oscillator light level are kept fixed, and θ is varied. The height of the scattering peak is then proportional to $\rho_2'(T_p)$.

To observe the weak first sound, which has long wavelengths at these frequencies, we use Debye-Sears scattering.[5] Here, the incident light beam is parallel to the sound wave fronts and is scattered into many orders on either side of the main beam. Diffraction from the edges of the transducer provides local oscillator light to beat with the shifted first-order scattered light. An acoustic cavity is formed by the transducer and the opposite parallel wall of the scattering cell, enhancing the sound level in the superfluid at resonance. As the temperature drifts, changes in $u_1(T)$ cause resonances at T_n [where $u_1(T_n) = 2FL/n$ and L is the length of the cavity]. Peaks in the scattered light occur at these resonance temperatures, and this allows measurements to be made of the change in u_1 with T. The good agreement with previously published measurements of $u_1(T)$ indicates that we are indeed observing first sound. The scattering peaks are proportional to the amplitude of the first sound in the cavity, allowing measurement of the relative temperature dependence of the density variation $\rho_1'(T)$.

The consequences of the hydrodynamic equations in superfluid He^4 for various boundary conditions have been worked out by Lifshitz.[2] Of particular interest for these experiments are the expressions for the amplitude of the density variations in the second and first sound generated by a thermal transducer:

$$\rho_2' = (\alpha / C_p u_2) q', \tag{1}$$

$$\rho_1' = (\alpha / C_p u_1) q', \tag{2}$$

where q' is the periodic heat flux, α is the thermal expansion coefficient, and C_p the specific heat per unit mass of liquid He^4. The expressions ignore effects of attenuation and are valid for $u_1^2 \gg u_2^2$. The dashed curve in Fig. 2 represents $\rho_2'(T)/q'$, calculated from (1) using the known temperature of α, C_p, and u_2.

We wish to compare (1) with the experimentally

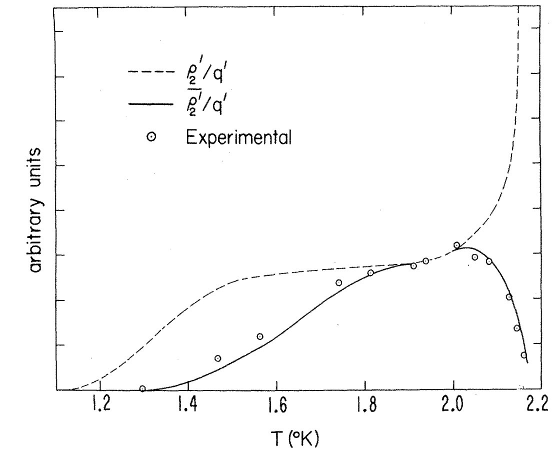

FIG. 2. Temperature dependence of the density variation in 1.46-MHz second sound in He^4 at a constant driving power of 1 mW/cm^2.

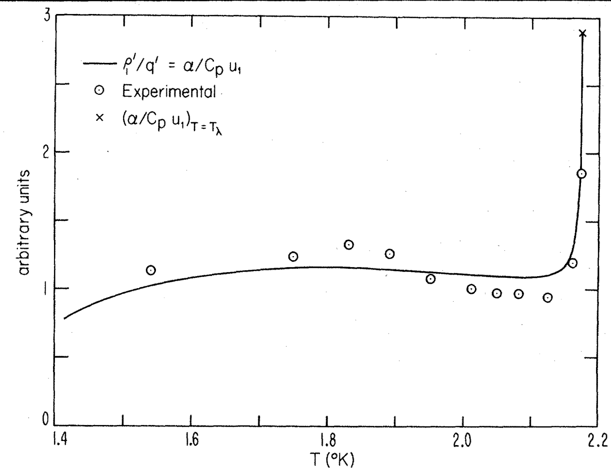

FIG. 3. Temperature dependence of the density variation in the first sound generated by a thermal transducer.

determined temperature dependence of the relative amplitude of light scattered from second sound. To do so, we must correct (1) for the fact that the attenuation β of second sound is large at these frequencies and very temperature dependent. The solid curve in Fig. 2 represents $\bar{\rho}_2'(T)/q'$, the average theoretical density variation in the scattering volume, calculated from (1) by taking into account the exponential decay of the sound amplitude across the incident light beam (diameter d) at different temperatures:

$$\bar{\rho}_2'(T) = \rho_2'(T)(1 - e^{-\beta(T)d})/\beta(T)d.$$

Finally, the circled points in Fig. 2 represent the experimentally measured scattered light amplitude (height of the scattered peaks) at $F = 1.46$ MHz. The three representations are normalized at $T = 1.95$°K where the second-sound attenuation is a minimum. The agreement between $\bar{\rho}_2'(T)/q'$ and the experimental points supports the Lifshitz treatment. Recent reports[1] of measurements of the pressure variations in a second-sound wave conflict with the theory and with each other. It is possible that the mechanical transducers used in those experiments to detect the pressure variations introduce new and complicated boundary conditions for which the Lifshitz theory is inapplicable.[6] Light scattering, however, probes density variations without disturbing the medium, providing a more direct check of the theory.[7]

In Fig. 3, the solid curve represents $\rho_1'/q' = \alpha/C_p u_1$ from (2) as a function of temperature. The Pippard relations[8] are used to evaluate this expression at the λ point, showing that it rises to a finite value (represented by the cross at T_λ). The experimentally determined first-sound light-scattering amplitudes (heights of the scattering peaks at the acoustic resonances) for a typical run are plotted on Fig. 3, arbitrarily normalized to the theoretical curve at 1.9°K. There is no need to correct the theoretical curve here, since the attenuation of first sound is very small at these frequencies. Again, there is good agreement with the theory.

These experiments show that light is scattered from second sound in superfluid He^4 and that the technique provides a direct probe of density-temperature coupling in second and first sound.[9] In addition, the results of the Lifshitz theory for liquid He^4 are in good agreement with the experiments.

We are indebted to Professor I. Rudnick for pointing out the applicability of the Pippard relations for evaluating $\rho_1'(T_\lambda)/q'$.

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the sound beam, and (3) the attenuation of the sound. Since (1) and (2) are known, it is in principle possible to determine (3). For the low-angle scattering of these experiments, however, all three factors contribute about equally to the observed width, making it very difficult to extract a good measurement of attenuation.

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Parametric Suppression of a Large-Amplitude Plasma Instability

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The intensity of a large-amplitude, unstable plasma mode has been reduced by applying a pump signal to parametrically couple it to another damped, natural mode of the plasma column. A suppression of the mode is observed when the frequency of the pump ω_0 is equal to the difference between the damped-mode frequency ω_a and the unstable-mode frequency ω_b $(\omega_0 = \omega_a - \omega_b)$. The observed behavior of the mode energies is compared with a weak parametric-mode-coupling theory for lumped circuits, which includes nonlinear saturation terms for the unstable mode.

Many observations of parametric excitation of initially stable plasma modes have been reported.[1] It has been suggested theoretically[2] that it is also possible to suppress an unstable mode by parametrically coupling it to a naturally damped mode. In addition to its fundamental interest, this process has possible application as an active means for controlling plasma instabilities, a subject of current interest.[3]

It has been demonstrated by Hai and Wong[1] that azimuthally propagating modes of a magnetized plasma column can be actively coupled, producing the familiar parametric excitation of initially stable modes. In those experiments, a pump excitation applied at the sum frequency $\omega_0 = \omega_a + \omega_b$ (with a similar matching condition for the azimuthal wave numbers) produced parametric growth of two modes A and B. In our experiments, to achieve parametric suppression the pump was applied at frequency $\omega_0 = \omega_a - \omega_b$. This passively coupled[4] an unstable mode B to a damped mode A, and thereby reduced the amplitude of mode B.

To model this situation theoretically, we consider the equations with weak parametric mode coupling for lumped circuits[4] including nonlinear saturation terms for the initially unstable mode:

$$da/dt = (i\omega_a - \nu_a)a + C_{ab}e^{i\omega_0 t}pb, \tag{1}$$

$$db/dt = (i\omega_b + \nu_b)b + C_{ba}e^{-i\omega_0 t}pa - \alpha|b|^2 b - \delta|b|^4 b + \cdots, \tag{2}$$

$$\omega_0 = \omega_a - \omega_b; \tag{3}$$

a, b, and p are the amplitudes of mode A, mode B, and the pump; the $C_{ab,ba}$ are the coupling coefficients. For $p=0$, the uncoupled mode A has a damping rate ν_a, and mode B has a growth rate ν_b and nonlinear saturation parameters α and δ. We seek solutions of the form $a = A(t)e^{i\omega_a t}$ and $b = B(t)e^{i\omega_b t}$ and assume $A(t)$ and $B(t)$ have time dependence e^{st}. Neglecting nonlinear saturation,

$$s_\pm = \tfrac{1}{2}\{\nu_b - \nu_a \pm [(\nu_a + \nu_b)^2 - 4|C_{ab}C_{ba}|p^2]^{1/2}\}$$

since $C_{ab}C_{ba}$ must be negative from energy-flow