Effect of Thermal Phonon Pair Breaking on Superconducting Fluctuations*

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Previous experimental measurements of the paraconductivity for various materials show varying degrees of agreement with the Aslamazov-Larkin theory. Our measurements indicate that this behavior can be understood if one includes the Maki-Thompson fluctuation term with a thermal pair-breaking interaction plus an additional pair-breaking parameter proportional to the resistance per unit area of the film.

The excess conductivity or paraconductivity above the transition temperature of superconducting films has shown what appears to be diverse behavior.[1] That is, in some materials, notably relatively clean Pb, Sn, and Al, the paraconductivity is well fitted if one includes both the Aslamazov-Larkin (AL)[2] and the Maki-Thompson (MT) fluctuation terms[3] with the role of the latter diminishing as a pair-breaking interaction increases,[4] while other materials such as low-temperature–deposited Pb, Bi, and Ga[5] appear to be well described by the AL term alone. We believe a satisfactory theory can encompass these apparent differences if one includes both the AL and MT terms in the excess conductivity with the inclusion of an intrinsic pair-breaking interaction due to thermal phonons, as suggested by Appel[6] and by Swihart,[7] plus an additional pair-breaking mechanism of as yet unknown origin which is proportional to the resistance per unit area of the film, $R_\square$.

The AL theory[1] for the paraconductivity of a thin superconducting film of thickness $d \ll \xi(T)$ (the temperature-dependent coherence length) may be written as

$$\sigma_{\mathrm{AL}}/\sigma_N = \tau_0/\tau, \qquad (1\mathrm{a})$$

where $\tau_0 = 1.52\times10^{-5}/\sigma_N d$, $\tau = (T - T_c)/T_c$, and σ_N is the normal-state conductivity. In addition to this term one must include a contribution to the excess conductivity which as its origin in the inclusion of the interaction of the normal electrons with the fluctuating pairs. This term, first specified for thin films by Thompson,[3] takes the form

$$\sigma_{\mathrm{MT}}/\sigma_N = [2\tau_0/(\tau - \delta)]\ln(\tau/\delta), \qquad (1\mathrm{b})$$

where $\delta = (T_{c0} - T_c)/T_c$ is the normalized shift in the transition temperature due to some pair-breaking interaction. The introduction of such an interaction was necessary to remove the long-wave-length divergence in the fluctuation spectrum. Thus, the total paraconductivity is a sum of these two contributions:

$$\frac{\sigma'}{\sigma_N} = \frac{\tau_0}{\tau} + \frac{2\tau_0}{\tau - \delta}\ln\left(\frac{\tau}{\delta}\right). \qquad (2)$$

A cursory examination of the relative contribution from these two terms reveals that as δ increases the significance of the MT term diminishes. This has been verified elsewhere by varying δ with a magnetic field and concentrations of paramagnetic impurities.[4] Thus, if in low-temperature–deposited Pb, Ga, and Bi films the pair breaking were sufficiently high, one could obtain good agreement using the AL term alone. Since no magnetic field or magnetic impurities were present in the experiments of Glover,[5] one has to ask about the origin of this strong pair breaking.

In an attempt to clarify this point, we have measured the paraconductivity of Al, Sn, and Pb films and analyzed these experimental results using Eq. (2) to deduce the variation of δ with $R_\square = 1/\sigma_N d$ and with other material parameters, namely, the strength of the electron-phonon interaction. The films used in this experiment were prepared and measured in the same way as described previously.[4] It should be pointed out

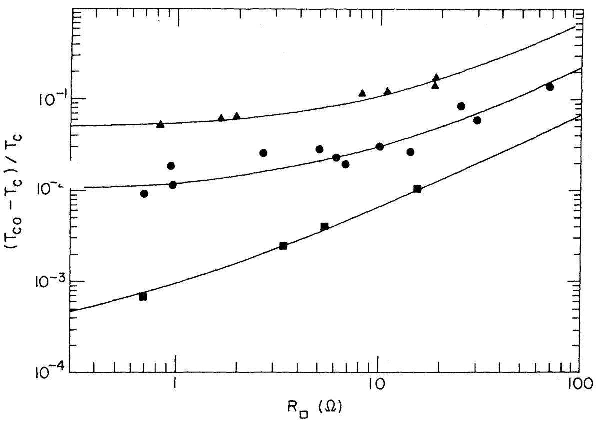

FIG. 1. The variation of $\delta = (T_{c0} - T_c)/T_c$ versus sheet resistance $R_\square$ for Al, Sn, and Pb. Smooth curves are approximations to $\delta = \delta_{th} + \mathrm{const}R_\square$. Squares, Al; circles, Sn; triangles, Pb.

that the most important parameter for an accurate determination of the temperature dependence of the paraconductivity is the normal-state conductivity $\sigma_N(T)$. Extensive measurements of the resistance versus temperature in magnetic fields exceeding 40 kG were conducted to reduce to a minimal amount the uncertainty in $\sigma_N(T, H = 0)$. Both fluctuation effects in a perpendicular field plus magnetoresistance effects were present and corrected by assuming $\rho \propto (H - H_{c2})^{-1}$ for the fluctuation contribution[8] and $\rho \propto H^2/\rho_N$ for the magnetoresistance (Kohler's rule).[9]

The variation of δ vs $R_\square$ for Al, Sn, and Pb is shown in Fig. 1. δ was obtained from a least-squares fit of $\sigma_N/\sigma'(T)$ vs T in the region $10 \leq \sigma_N/\sigma'(T) \leq 1000$ with T_c and T_{c0} adjustable, while holding τ_0 equal to the AL value. The best-fit curves shown in Fig. 1 correspond to

$$\begin{aligned} \delta_{\mathrm{Pb}} &\simeq 0.05 + 6\times10^{-3}R_\square, \\ \delta_{\mathrm{Sn}} &\simeq 0.01 + 2\times10^{-3}R_\square, \qquad (3) \\ \delta_{\mathrm{AL}} &\simeq 3\times10^{-4} + 6\times10^{-4}R_\square. \end{aligned}$$

It should be emphasized that the theoretical expression, Eq. (2), is not very sensitive to the value of δ, thus leading to some uncertainty as reflected by the scatter in δ vs $R_\square$. Clearly some justification for assuming this variation and discussion of the value of the parameters is necessary. From an examination of the experimental data for Sn and Pb, it is quite apparent that δ is not going to zero as $R_\square$ approaches zero, i.e., bulk behavior. This certainly is much less apparent for Al because of the extremely small δ as $R_\square \to 0$.[10] Thus the first implication is that δ has some intrinsic term which we will call δ_{th}. Strong experimental evidence exists for assuming a pair-breaking parameter proportional to $R_\square$.[11-13] However, at present there are no theoretical guidelines for such a term.

Before discussing this latter term, let us consider the first term, δ_{th}. Appel[6] and Swihart[7] have calculated the effect of the occupation of low-energy thermal phonon states on the superconducting transition temperature. From the renormalization of the imaginary part of the electron-phonon self-energy, Appel predicts a temperature-dependent pair-breaking interaction which results in a shift in T_c proportional to $1/\tau_{\mathrm{el-ph}}$, where $\tau_{\mathrm{el-ph}}$ is the electron-phonon lifetime which is closely related to, but not to be confused with, the electron transport time due to phonon scattering.[6] It should be pointed out that the renormalization of the real part of the electron-phonon self-energy would also lead to a shift in T_c; however, this contribution does not limit the lifetime of the paired state and will not contribute to the measured δ_{th}. It is very difficult to make a reliable estimate of $\tau_{\mathrm{el-ph}}$ from low-temperature resistivity because of large uncertainty of the relative contribution due to normal and umklapp processes and phonon-drag effects. However, Appel's calculation indicates that δ_{th} will vary approximately as $\lambda(T/\theta_{\mathrm{D}})^2$,[6] where λ is the electron-phonon coupling constant and θ_{D} is of the order of the Debye temperature; this estimate is semiquantitatively consistent with the measured values of δ_{th} in Pb, Sn, and Al. Using Appel's predicted[6] $\delta_{th} \simeq 0.04$ for Pb plus electron-tunneling measurements, which indicate that λ for low-temperature–deposited Ga and Bi is nearly two times that of Pb, one would obtain a δ_{th} in these materials as large as 10^{-1}.[14] δ_{th}'s of this magnitude are sufficiently large to obscure the MT term unless very careful σ_N measurements are made, which was not done in Glover's experiments.[5]

Now let us discuss the empirical result, $\delta \propto R_\square$. Strongin *et al.*[11] have conducted a rather extensive investigation of the transition temperature of low-temperature–deposited films. They demonstrated that, for very thin films, T_c of the films was rapidly depressed as the thickness decreased and concluded that this depression correlated best with $R_\square$ rather than thickness or mean free path. Similar experiments by Naugle and Glover[12] on Ga and Bi indicated a similar linear depression of T_c with increasing $R_\square$. The rate of depression, δ, in all these studies was

$\sim 10^{-3}R_\square$ which is within an order of magnitude of those values obtained from the fluctuation measurements. It is reasonable to assume that this depression which is manifestly visible in high-resistance films, would persist as $R_\square$ is reduced. Using the AL and MT terms, Kajimura and Mikoshiba[13] have found that $\delta \propto 6\times 10^{-4}R_\square$ $(1 < R_\square \lesssim 10^3)$ for Al which is consistent with this investigation. It has been pointed out that at present there does not exist an adequate explanation for this pair breaker. Patton and also Keller and Korenman[15] have recently calculated the effect of vertex corrections to the MT term which removes the low-momentum divergence even in the absence of pair-breaking interactions. The results of their calculation resembles Eq. (2) with δ being replaced by $\delta_v \sim 10^{-5}R_\square T_c/(T-T_c)$ which seems to be in fair agreement with our and Kajimura and Mikoshiba's results for the $R_\square$ variation of δ observed in Al for the temperature region $\tau \sim 0.1$ which is the region where the experimental data were fitted by Eq. (2). However, this δ_v differs from the usual pair-breaking δ in that it is not related to a shift in T_{c0} and therefore cannot explain the observed depression in T_c at large $R_\square$ reported by Strongin *et al.*[11] It appears that this correction plays a contributory role in the approach of σ' to the AL value as $R_\square$ is increased. This point and further analysis of fluctuation data for Sn and Pb in relationship to recent calculations will be presented in a more expanded discussion of our fluctuation data in two-dimensional systems.

In summary, the observed shift in T_c deduced from fluctuation-conductivity measurements on Pb, Sn, and Al agrees well with that expected from pair breaking arising from thermal phonons. The inclusion of this temperature-dependent pair-breaker δ_{th} could account for the apparent diverse behavior for fluctuations above T_c in films with small $R_\square$. Also, analysis of the data shows that δ will increase with $R_\square$ at a rate closely resembling the sum of values indicated by measurements on the depression of T_c previously reported and the contribution from the vertex correction recently published by Patton.

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