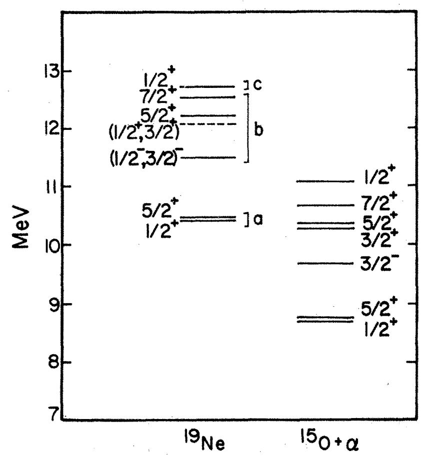

FIG. 2. The α-particle states observed in ^{19}Ne are shown along with the series of levels expected from the α-particle core-excited threshold-states scheme. The experimentally determined spins and parities of the states are also indicated: *a*, taken from Ref. 10; *b*, taken from Ref. 9; *c*, taken from Ref. 11.

action in ^{15}N because of the lower core charge and therefore a more pronounced broadening of the effective core potential leading to compression. These results require much further theoretical study.

A third, somewhat simpler case is observed in ^{19}Ne studied by means of the reactions ^{16}O(^{3}He, α)^{15}O and ^{16}O(^{3}He, ^{3}He)^{16}O.[9-11] In this case the levels parameters were again obtained by means of R-matrix and optical-model-plus-resonance analyses. The results are shown in Fig. 2, along with the spectrum of ^{15}O shifted by the α-particle binding energy in ^{19}Ne. In this case a one-to-one correlation is observed in J, π, and E. So we conclude that the α-particle threshold state must have $L=0$ for this case. A detailed model which is able to predict the L value of the threshold state is not yet available.

These results indicate the importance of α-particle clusters in light nuclei at high excitation energies and provide the first detailed experimental study of these states. Although the simple model applied here should not be taken too literally, it is remarkably successful in accounting for the observed level schemes in addition to predicting the correct level densities of α-particle states at high energies in these light nuclei.

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Intrinsic Quadrupole Moments and Shapes of Nuclear Ground States and Excited States*

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A new method of determining nuclear shapes is proposed which avoids the assumption of a specific nuclear model. The concept of an equivalent ellipsoid, whose charge and moments equal those of the nucleus, is employed. But the method is equally valid for spherical, deformed, and intermediate nuclei. It can be employed for any nucleus (even-even, odd-A, or odd-odd) provided enough $E2$ matrix elements are available. The example of ^{152}Sm is discussed.

A nonzero value of the spectroscopic quadrupole moment (Q^S) implies a nonspherical charge distribution, and the ratio $Q^S/\langle r^2\rangle$ is a measure of nuclear deformation.[1] However, as is well known, a vanishing Q^S does not necessarily imply a spherical charge distribution. The quadrupole moment Q^S vanishes for a nucleus if (a) the total angular momentum is 0 or $\frac{1}{2}$,[2] or if (b) the nucleus has equal probabilities of being prolate and oblate.[3]

The Bohr-Mottelson[4] concept of intrinsic quadrupole moment (Q^i) removes part of the difficulty mentioned above. The total nuclear wave function is written as the product of an intrinsic part (assumed to be independent of angular momentum) and a D function which contains all the angular-momentum dependence. The quadrupole operator is also written as the product of an intrinsic part and a D function. Then the nuclear quadrupole moment Q^S equals the product of Q^i and a CGC (Clesbsch-Gordan coefficient). The $B(E2)$ value for a γ-ray transition equals the product of $(Q^i)^2$ and the square of a CGC. This method allows the determination of Q^i for any nucleus. However, it requires the assumption of a specific nuclear model, namely, the rotational model.

We propose a method which does not require the assumption of a specific nuclear model for determining some quantities which measure the intrinsic nuclear quadrupole moments. For relating these intrinsic moments to nuclear deformations, we do invoke the usual[4,5] concept of an equivalent ellipsoid whose charge, volume (or $\langle r^2\rangle$), and quadrupole moments equal those of the nucleus. But we drop the assumptions of axial symmetry and smallness of deformation, and exploit this concept in its full generality.

Model-independent nuclear moments.—We define an n-body[6] quadrupole moment operator as[7]

$$P^{(n)} = [P_2 \times P_2 \cdot\cdot\cdot \times P_2]_2 \cdot P_2, \tag{1}$$

where P_2 is the one-body electric quadrupole moment operator,

$$P_{2\mu} = \sum_{i=1}^{A} e_i r_i^2 Y_{2\mu}(\Omega_i), \tag{2}$$

e_i being the charge of the ith nucleon. Since $P^{(n)}$ *is defined to be a scalar operator, it can have nonvanishing matrix elements for any nuclear state.* We shall consider only the diagonal matrix elements, which will be denoted by

$$P_s^{(n)} = \langle s, M_s | P^{(n)} | s, M_s\rangle = (2I_s+1)^{-1/2}\langle s \| P^{(n)} \| s\rangle, \tag{3}$$

where $|s\rangle$, with s standing for s, I_s, Π_s, defines a nuclear state.

The reduced matrix element $\langle s \| P^{(n)} \| s\rangle$ of Eq. (3) can be written as the sum of products of n matrix elements of the one-body operator P_2, with the sum running over $n-1$ intermediate states. For this purpose, we employ the expansion[8] of the reduced matrix element of a spherical tensor and the definitions[9]

$$M_{sr} = -\langle r \| P_2 \| s\rangle, \tag{4}$$

$$B(E2; s\to r) = (2I_s+1)^{-1} M_{sr}^2, \tag{5}$$

$$Q_s^S = (16\pi/5)^{1/2}\langle s, M_s = I_s | P_{20} | s, M_s = I_s\rangle = -[16\pi I_s(2I_s-1)/5(I_s+1)(2I_s+1)(2I_s+3)]^{1/2} M_{ss}. \tag{6}$$

The two-body moment is given by

$$P_s^{(2)} = (2I_s+1)^{-1}\textstyle\sum_r M_{sr}^2 \tag{7}$$

$$= \frac{5(I_s+1)(2I_s+3)}{16\pi I_s(2I_s-1)}(Q_s^S)^2 + \sum_{r\neq s} B(E2; s\to r), \tag{8}$$

and the three-body moment by

$$P_s^{(3)} = -5^{1/2}(2I_s+1)^{-1}(-1)^{2I_s}\sum_{rt}\begin{Bmatrix} 2 & 2 & 2 \\ I_s & I_r & I_t\end{Bmatrix} M_{sr}M_{rt}M_{ts}. \tag{9}$$

Clearly, the moments $p_s^{(2)}$, $p_s^{(3)}$, $\cdots$ can be determined directly from the experimental $E2$ matrix elements without assuming anything about the nucleus. The nucleus may be spherical, deformed, or intermediate (transitional). It may be even-even, odd-A, or odd-odd.

The moment $p_s^{(2)}$ is a model-independent measure of the magnitude of intrinsic quadrupole moment or deformation. However, it cannot tell us whether a nucleus is prolate, oblate, or asymmetric. For this purpose, we need the three-body moment $p_s^{(3)}$. These two moments are related below to the intrinsic quadrupole moments and deformation of an equivalent ellipsoid.

The moments $p_s^{(2)}$, $p_s^{(3)}$ give us no indication about the rigidity or softness of nuclear deformation. A measure of fluctuations in the magnitude of nuclear deformation is provided by the quantity

$$B_s = [p_s^{(4)}(p_s^{(2)})^{-2} - 1]^{1/2}, \tag{10}$$

which would be less than 1 for a rigid, well-deformed nucleus, and larger than 1 for a nucleus which is slightly deformable but spherical most of the time. Similarly, fluctuations in the asymmetry of nuclear deformation could be measured by the quantity

$$G_s = [p_s^{(6)}(p_s^{(3)})^{-2} - 1]^{1/2}. \tag{11}$$

Moments and shapes of the equivalent ellipsoid.—In the analysis of the electron scattering or muonic data, one employs a nuclear charge density with a diffuse surface. However, for the sake of convenience and understanding of nuclear size, one employs the concept of an equivalent sphere with uniform charge density whose charge and volume (related to the radius, $R_0 = r_0 A^{1/3}$) equal those of the nucleus. In the same spirit, we employ the concept of an equivalent ellipsoid whose charge, volume, $p_s^{(2)}$, and $p_s^{(3)}$ equal those of the nucleus. A similar procedure has been employed by Ripka[5] to relate the expectation values of the operators $P_{2\mu}$ and r^2 with respect to a Hartree-Fock determinant to the quadrupole moment and deformation of an ellipsoid. However, we do not make the assumption of axial symmetry.

The expectation value of the discrete sum in Eq. (2) over the individual nucleons is replaced by a volume integral of the type

$$Q_{s\mu}^{\,i} = (16\pi/5)^{1/2} \int \rho_s r^2 Y_{2\mu}\, dV, \tag{12}$$

where ρ_s is the charge density, and $Q_{s\mu}^{\,i}$ is the μth component ($\mu = 0, \pm 1, \pm 2$) of the quadrupole moment of the ellipsoid. Because of the reflection symmetry of the ellipsoid, we have

$$Q_{s2}^{\,i} = Q_{s,-2}^{\,i}, \quad Q_{s1}^{\,i} = Q_{s,-1}^{\,i} = 0. \tag{13a}$$

Hence, we can define (without any loss of generality)

$$Q_{s0}^{\,i} = Q_s^{\,i} \cos\gamma_s, \qquad Q_{s2}^{\,i} = Q_{s,-2}^{\,i} = 2^{-1/2} Q_s^{\,i} \sin\gamma_s, \tag{13b}$$

where $Q_s^{\,i}$ is the magnitude of the quadrupole moment and γ_s represents deviations from axial symmetry.

The evaluation of the moments $p_s^{(n)}$ for the equivalent ellipsoid proceeds as follows. The operator $P^{(n)}$ is written, by use of Eq. (1), in terms of the one-body operators $P_{2\mu}$. The expectation value of each operator $P_{2\mu}$ is replaced by the intrinsic moment $Q_{s\mu}^{\,i}$ of Eq. (12). Thus, the moments $p_s^{(n)}$ are expressed in terms of $Q_{s\mu}^{\,i}$ or $Q_s^{\,i}$ and γ_s. Inverting the relations for $n = 2$ and 3, we get

$$Q_s^{\,i} = (16\pi/5)(p_s^{(2)})^{1/2}, \tag{14}$$

$$\cos 3\gamma_s = -(\tfrac{7}{2})^{1/2} p_s^{(3)} (p_s^{(2)})^{-3/2}. \tag{15}$$

A convenient[4,5] measure of nuclear deformation is provided by the ratio of the quadrupole moment to the monopole moment,

$$\beta_{s\mu} = (4\pi/5)(\int \rho_s r^2 Y_{2\mu}\, dV)(\int \rho_s r^2\, dV)^{-1} = (\pi/5)^{1/2} Q_{s\mu}^{\,i} (Z\langle s | r^2 | s\rangle)^{-1}, \tag{16}$$

where the normalization factor has been chosen in such a way that our β equals the β_{BM} of Bohr and Mottelson under certain conditions to be discussed below. Since the proportionality factor between the β and the Q tensors of Eq. (16) is independent of μ, the β tensor obeys relations analogous to Eqs. (13). Thus, instead of $\beta_{s\mu}$, we may employ the magnitude β_s and the asymmetry angle γ_s.

The monopole moment $\langle r^2\rangle$ appearing in Eq. (16) may be taken directly from the electron-scattering or muonic data. However, such data are available for only a few nuclei. If we know the nuclear size (volume) parameter R_0, we can employ the following procedure.

The ellipsoidal surface is governed by the equation

$$R_1^{-2}x^2 + R_2^{-2}y^2 + R_3^{-2}z^2 = 1, \tag{17}$$

where R_1, R_2, R_3 are the three semiaxis lengths. Since the volume of the ellipsoid must equal that of the nucleus, we have the relation

$$R_1 R_2 R_3 = R_0^3. \tag{18}$$

With the assumption that the total charge Z is distributed uniformly over the ellipsoid, one obtains the quadrupole moments and the monopole moment as

$$Q_{s0}^{\,i} = \tfrac{1}{5} Z(2R_3^2 - R_1^2 - R_2^2), \tag{19}$$

$$Q_{s2}^{\,i} = Q_{s,-2}^{\,i} = \tfrac{1}{5}(\tfrac{3}{2})^{1/2} Z(R_1^2 - R_2^2), \tag{20}$$

$$\langle r^2\rangle = \tfrac{1}{5}(R_1^2 + R_2^2 + R_3^2). \tag{21}$$

Using Eqs. (18)–(20), we could write R_1, R_2, R_3 in terms of R_0, $Q_{s0}^{\,i}$, and $Q_{s2}^{\,i}$, and then determine $\langle r^2\rangle$ of Eq. (21) and β_s of Eq. (16). However, it is more convenient to express everything in terms of the deformation parameter,

$$\delta_s = (4\pi/5)^{-1/2} \beta_s, \tag{22}$$

and the modified radius

$$\overline{R}_0(\delta_s, \gamma_s) = R_0(1 - 3\delta_s^2 + 2\delta_s^3 \cos 3\gamma_s)^{-1/6}. \tag{23}$$

TABLE I. Intrinsic quadrupole moments and shapes of ^{152}Sm and ^{131}Cs. Column 7 gives the deformation, obtained by employing the rotational model of Bohr and Mottelson, for the sake of comparison. The same $E2$ matrix elements have been used for columns 4–7. The radius parameter, $R_0 = 1.2A^{1/3}$ fm, has been used.

Nucleus (1)	State (s) (2)	Intermediate states (r or t) (3)	$Q_s{}^i$ (e b) (4)	β_s (5)	γ_s (deg) (6)	$\beta_{\rm BM}$ (7)
$^{152}_{62}\mathrm{Sm}_{90}$ [a]	0^+(g.s.)	1 (2^+)	5.81	0.293	0.0	0.293
		3 ($2, 2_\beta, 2_\gamma$)	5.91	0.298	8.4	···
	2^+(122 keV)	3 (0, 2, 4)	5.89	0.297	1.8	···
		9 ($0, 2, 4, 0_\beta, 2_\beta, 4_\beta, 2_\gamma, 3_\gamma, 4_\gamma$)	6.04	0.304	8.7	···

[a]The $E2$ matrix elements have been taken from experiment, when available (see Ref. 10), otherwise from theory (see Ref. 11).

The final results are

$$R_k(\delta_s, \gamma_s) = \bar{R}_0(\delta_s, \gamma_s)[1 + 2\delta_s \cos(\gamma_s - \tfrac{2}{3}k\pi)]^{1/2} \quad (k = 1, 2, 3), \qquad (24)$$

$$Q_s{}^i = \tfrac{6}{5}Z[\bar{R}_0(\delta_s, \gamma_s)]^2\delta_s = 3(5/\pi)^{-1/2}Z[\bar{R}_0(\delta_s, \gamma_s)]^2\beta_s, \qquad (25)$$

$$\langle s | r^2 | s\rangle = \tfrac{3}{5}[R_0(\delta_s, \gamma_s)]^2, \qquad (26)$$

where the value of δ_s is determined by solving the cubic equation

$$\delta_s{}^3(g_s{}^3 - 2\cos 3\gamma_s) + 3\delta_s{}^2 - 1 = 0, \qquad (27)$$

where

$$g_s = 6ZR_0{}^2[5Q_s{}^i]^{-1}. \qquad (28)$$

As can be checked easily, the relations (24)–(26) reduce to those of Bohr and Mottelson[4] in the limit of small deformation ($\delta_s \to g_s{}^{-1}$, $\bar{R}_0 \to R_0$). However, we have not employed the rotational model (or any other model for nuclear wave functions) at any stage. Also, we have avoided the usual uncertainty in the relation (26) arising from the approximation made about the volume conservation relation (18). Since we have not made an expansion in β (or δ) at any stage, the volume conservation condition (18) is satisfied to all orders in β. *The relations (23)–(26) are exact.*

Convergence tests.—How many data are needed for a reasonably good convergence of the sums in Eqs. (7) and (9)? In order to answer this question, we have applied the above method to ^{152}Sm. The nucleus ^{152}Sm has been chosen because of the availability of a large number of $E2$ matrix elements.[10] The results of Table I indicate that the lowest few states are sufficient for a reasonably good convergence of the sums in Eqs. (7) and (9).

Summary.—The present method helps us understand why the Bohr-Mottelson method, which was designed for well-deformed, rotational nuclei, is applicable to the ground-state nuclear shapes of all even-even nuclei [this point of view was already adopted by Stelson and Grodzins[12] who tabulated the $\beta_{\rm BM}$ values based on the $B(E2; 0 \to 2)$ values for all even-even nuclei; however, no theoretical justification was given]: Since the quadrupole moment $Q_0{}^S$ is zero and the $B(E2; 0 \to 2')$ values for the excited 2^+ states are small, the sum in Eq. (8) for $p_0{}^{(2)}$ reduces essentially to one term. Furthermore, the quantity g_s of Eq. (28) is much larger than 1 (or deformation is much less than 1), hence the usual relation between Q and β has quite general validity.

However, the above conclusions do not apply to the excited states of most even-even nuclei and the ground or excited states of most odd-A nuclei. Here, the intrinsic moments and shapes are determined by a sum over a number of terms of comparable magnitudes. If the intrinsic state corresponding to all these terms is identical (which seems to be the case for a well-deformed nucleus like ^{152}Sm, see Table I), we would get the rotational-model values. However, in general, this is not true.

These questions need to be explored further. Although the present method is no substitute for a detailed, microscopic calculation, it may be useful for studying changes in nuclear shapes as functions of Z, N, and excitation energy. If enough data are available, one can also compute the shape fluctuations and thus determine in a model-independent way (a) whether a nucleus is permanently deformed, (b) whether a nucleus is permanently asymmetric.

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Two Gravity-Wave Detectors: A Comparison*

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The sensitivity of a dumbbell gravity-wave antenna is compared with that for a cylindrical detector. It is concluded that a Weber cylindrical antenna is decidedly the more sensitive at mutually accessible frequencies, particularly if the detector is to operate at higher frequencies in addition to the fundamental. A dumbbell antenna does offer the possibility of sampling the very low-frequency end of the spectrum which is inaccessible to cylinders.

In the current period of activity following the pioneering work of Weber[1,2] in gravity-wave detection, considerable discussion is being generated concerning antenna design.[3-10] As a possible alternative to a cylinder, a dumbbell (two large masses connected by a rod) is considered here as a possible gravity-wave detector.[6] We compare this alternative antenna to the more familiar cylinder without idealizing either of these detectors as two masses connected by a spring. Analysis of longitudinal elastic vibrations resulting from gravity pulses and thermal noise shows superior sensitivity for cylinders, particularly if a single detector is to operate at several frequencies. A dumbbell may, however, provide a way to observe low-frequency (~100-Hz) gravitational radiation which is in practice inaccessible to cylinders.

The dimensional parameters for the detectors are depicted in Fig. 1. All dependence of the elastic oscillations on directions perpendicular to the horizontal symmetry axes of the detectors is ignored. This is equivalent to setting the Poisson ratio equal to zero. Any attempt at a realistic description of the elastic modes in such de-

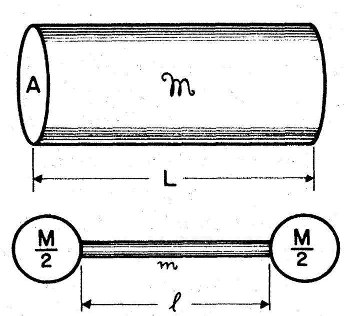

FIG. 1. Two gravity-wave detectors with their relevant parameters.