dent on the exact values of the parameters A_4 and A_6/A_4. Since in the present system A_4 could not be given a definite value, the calculated values of K_1 and K_2 are only approximate.

The theoretical model predicts a first-order phase transition between the regions of different spin orientation. This prediction is in apparent contradiction with the presence of the transition regions in the spin-orientation diagram. The appearance of the transition regions may be due to the following factors:

(1) The inhomogeneity of the samples. This however cannot be the main factor, as some of the experimental spectra (e.g., spectrum f in Fig. 2) cannot be interpreted as a superposition of two spectra corresponding to magnetizations along two different major crystalline directions.

(2) The existence of additional, noncubic anisotropic terms due to distortions, dipolar fields, and anisotropic exchange that have been neglected in Eq. (1). As a result of these terms, the magnetization may not coincide with the direction of one of the major crystalline axes, and in the transition region $\vec{n}$ may deviate significantly from these axes. Thus, for example, spectrum f in Fig. 2 corresponds to a direction of magnetization lying in the (100) plane, but not parallel to one of the major ([100] or [110]) crystalline axes.

In Fig. 1 a slight shift between the experimental and the theoretical boundaries of the three regions is observed. This shift may be due to several factors, the main one being the neglect of an anisotropy term contributed by the Fe-Fe interaction, which favors the [111] direction. Smaller shifts may be caused by the uncertainty in the values of some of the parameters ($\langle r^n \rangle$, antishielding factors) used in the calculations. Furthermore, the possibility of mixing higher ionic J states into the ground state was neglected. As the exchange fields acting on the rare-earth ions in $R\mathrm{Fe}_2$ compounds are quite large, the contribution of this mixing to the anisotropy may be nonnegligible. Finally, the previously mentioned noncubic anisotropic terms, that were not taken into account in Eq. (1), could also contribute somewhat to the shift of the boundaries.

We are carrying out similar measurements and calculations for the systems $R_x^1R_{1-x}^2\mathrm{Fe}_2$ with R^1 = Dy, Ho and R^2 = Er, Tb, and Tm. The experimental results and a detailed comparison with theoretical predictions will be published elsewhere.

---

*Work carried out by the Hebrew University group has been supported in part by the Stiftung Volkswagenwerk.

[1] W. E. Wallace, J. Chem. Phys. 41, 3857 (1964).

[2] G. K. Wertheim and J. H. Wernick, Phys. Rev. 125, 1937 (1962).

[3] G. J. Bowden, D. St. P. Bunbury, A. P. Guimaraes, and R. E. Snyder, J. Phys. C: Proc. Phys. Soc., London 2, 1367 (1968).

[4] G. K. Wertheim, V. Jaccarino, and J. H. Wernick, Phys. Rev. 135, A151 (1964).

[5] R. L. Cohen, Phys. Rev. 134, A94 (1964).

[6] E. Segal, Ph.D. thesis, Hebrew University, 1967 (unpublished).

[7] M. T. Hutchings, in *Solid State Physics*, edited by H. Ehrenreich, F. Seitz, and D. Turnbull (Academic, New York, 1966), Vol. 16, p. 277.

[8] A. J. Freeman and R. E. Watson, Phys. Rev. 127, 2058 (1962).

[9] A. J. Freeman and R. E. Watson, Phys. Rev. 139, A1606 (1965).

Observation of α - Particle Core-Excited Threshold States in Light Nuclei

H. R. Weller
Department of Physics, University of Florida, Gainesville, Florida 32601

(Received 6 December 1971)

The nuclei ^{15}N, ^{15}O, and ^{19}Ne are examined in terms of the appearance of α-particle core-excited threshold states. It is shown that this simple model can account for most of the α-particle states seen above the threshold for α-particle emission. We discuss the most striking features of these states.

Recent work of Gillet[1] and Middleton, Garrett, and Fortune[2] has stressed the importance of quartet structures in the interpretation of nuclear excited states. One expects resonances in the α channel at compound-system energies corresponding to these states. It is the intent of this Letter to summarize and correlate a series of studies which have revealed the existence of α-particle states at high energies in light nuclei. These states have spins, parities, and energies which indicate that they can be interpreted as being built of a core plus α particle with the first state occurring near the threshold for the compound system to emit an α particle and higher states being formed by exciting the core.[3] The nuclei ^{15}N, ^{15}O, and ^{19}Ne will be examined in this regard.

The nucleus ^{15}O has been studied in the energy region of interest by means of the reactions ^{12}C(^{3}He, α)^{11}C and ^{12}C(^{3}He, ^{3}He)^{12}C.[4,5] The principal means of analyzing these data was an optical-model-plus-resonance analysis of the elastic data. Legendre-polynomial analyses of the α-channel data were also employed to obtain spins and parities. A slightly lower energy region of ^{15}O has also been studied by means of the reaction ^{14}N(p, α)^{11}C.[6] The levels seen in this case are correlated with those of the reaction ^{12}C(^{3}He, α)^{11}C where overlap occurs—eliminating questions of statistical fluctuations. Unfortunately the three additional lower levels observed in the reaction ^{14}N(p, α)^{11}C do not have reliable spin and parity assignments.

In the case of the mirror nucleus ^{15}N, the reaction ^{11}B(α, α)^{11}B was used to investigate the level structures. For α-particle energies below 3.8 MeV, a multilevel multichannel R-matrix analysis was employed to obtain level parameters.[7] This analysis employed data in all open channels and simultaneously fit all channels [^{14}C(p, α)^{11}B, ^{14}C(p, n_0)^{14}N, ^{14}C(p, n_1)^{14}N, ^{14}C(p, p)^{14}C, and ^{11}B(α, α)^{11}B]. At higher energies the number of open channels and the complexity of the structure in these other channels made such a detailed analysis impractical (if not impossible). So, for α-particle energies of 3.8 to 5.0 MeV, a modified R-matrix analysis incorporating an optical-model background was employed.[8] This yielded five more levels in ^{15}N, with a successful fit to the ^{11}B(α, α)^{11}B data as a function of angle and energy.

The level schemes of ^{15}N and ^{15}O thus obtained are shown in Fig. 1. This figure also shows the level schemes of ^{11}B and ^{11}C which have been shifted by the binding energy of an α particle in the compound system nuclei ^{15}N and ^{15}O, respectively.

It is clear from these results that the levels in ^{15}N and ^{15}O are well correlated. We see also that the structures observed require that the α-particle threshold state must consist of an $L = 1$ α particle coupled to the core. So the ground state, of, say, ^{11}B coupled to an $L = 1$ α particle results in a $\frac{5}{2}^+$, $\frac{3}{2}^+$, $\frac{1}{2}^+$ triplet. This triplet is observed in ^{15}N and, although the spin assignments are lacking, the corresponding levels in ^{15}O are also present. The next core state, the first excited state of ^{11}B or ^{11}C, has a spin of $\frac{1}{2}^-$. This should give rise to a $\frac{1}{2}^+$, $\frac{3}{2}^+$ doublet. Again this is observed in both ^{15}O and ^{15}N. The higher core states will give rise to triplets. The next triplet in ^{15}O has been observed[3] (although it is barely split), and that in ^{15}N is present and has the correct spin-parity values. Still higher states in ^{15}O have been observed.[4] These states have centers of gravity which follow the pattern of levels in ^{11}C. They are not very wide, indicating that the triplets (presumably present) are very weakly split.

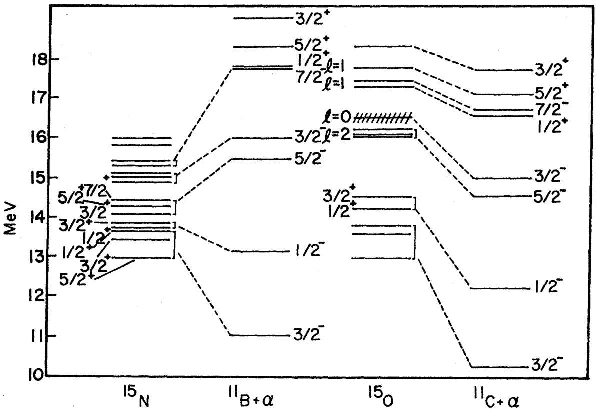

FIG. 1. The α-particle states observed in ^{15}N and ^{15}O are shown along with the series of levels expected from the α-particle core-excited threshold-states scheme. The experimentally determined spins and parities are also indicated.

There are many features of these results which require consideration. Perhaps it is not surprising that the splitting decreases with higher excitation of the core, since the effective core-α-particle distance would be expected to increase. The ground-state triplet has a spacing of levels which suggests that an $\vec{I}\cdot\vec{L}$ interaction is a dominant part of the α-core interaction. If the radial wave functions are not different, the interval rule predicts that the splitting should go as 5:3 for the spacing between the $\frac{5}{2}^+$, $\frac{3}{2}^+$ states and the $\frac{3}{2}^+$, $\frac{1}{2}^+$ states, respectively. This is in rough agreement with observations. In the ground-state triplet the ordering of the levels ($\frac{5}{2}^+$, $\frac{3}{2}^+$, $\frac{1}{2}^+$) is as expected. The next two levels ($\frac{1}{2}^+$, $\frac{3}{2}^+$) appear to be inverted—suggesting deformation. One other striking feature is the large compression of the center of gravity of the core multiplets in ^{15}N relative to that in ^{15}O. This result would be consistent with there being a stronger α-core inter-

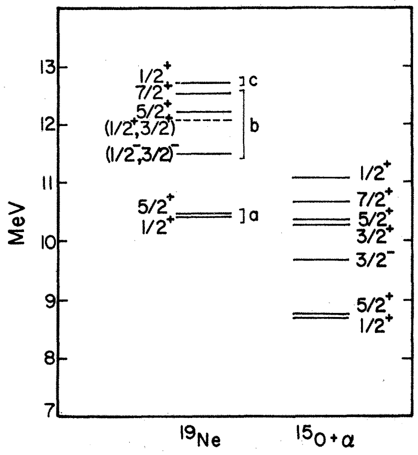

FIG. 2. The α-particle states observed in ^{19}Ne are shown along with the series of levels expected from the α-particle core-excited threshold-states scheme. The experimentally determined spins and parities of the states are also indicated: a, taken from Ref. 10; b, taken from Ref. 9; c, taken from Ref. 11.

action in ^{15}N because of the lower core charge and therefore a more pronounced broadening of the effective core potential leading to compression. These results require much further theoretical study.

A third, somewhat simpler case is observed in ^{19}Ne studied by means of the reactions ^{16}O(^{3}He, α)^{15}O and ^{16}O(^{3}He, ^{3}He)^{16}O.[9-11] In this case the levels parameters were again obtained by means of R-matrix and optical-model-plus-resonance analyses. The results are shown in Fig. 2, along with the spectrum of ^{15}O shifted by the α-particle binding energy in ^{19}Ne. In this case a one-to-one correlation is observed in J, π, and E. So we conclude that the α-particle threshold state must have $L=0$ for this case. A detailed model which is able to predict the L value of the threshold state is not yet available.

These results indicate the importance of α-particle clusters in light nuclei at high excitation energies and provide the first detailed experimental study of these states. Although the simple model applied here should not be taken too literally, it is remarkably successful in accounting for the observed level schemes in addition to predicting the correct level densities of α-particle states at high energies in these light nuclei.

[1] M. Danos and B. M. Spicer, Z. Phys. 236, 320 (1970).
[2] R. Middleton, J. D. Garrett, and H. T. Fortune, Phys. Rev. Lett. 27, 950 (1971).
[3] H. R. Weller, Phys. Lett. 30B, 409 (1969).
[4] H. R. Weller and H. A. Van Rinsvelt, Nucl. Phys. A129, 64 (1969).
[5] R. F. Jackson and H. R. Weller, Nucl. Phys. A160, 247 (1971).
[6] P. N. Shrivastava, F. Boreli, and B. B. Kensey, Phys. Rev. 169, 842 (1968).
[7] J. J. Ramirez, R. A. Blue, and H. R. Weller, Phys. Rev. C (to be published).
[8] W. R. Ott and H. R. Weller, to be published.
[9] W. R. Ott and H. R. Weller, to be published.
[10] D. A. Bromley, J. A. Kuehner, and E. Almqvist, Nucl. Phys. 13, 1 (1959).
[11] H. Roepke, K. P. Lieb, and R. Konig, Nucl. Phys. A97, 609 (1967).

Intrinsic Quadrupole Moments and Shapes of Nuclear Ground States and Excited States*

Krishna Kumar†
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830
(Received 10 May 1971)

A new method of determining nuclear shapes is proposed which avoids the assumption of a specific nuclear model. The concept of an equivalent ellipsoid, whose charge and moments equal those of the nucleus, is employed. But the method is equally valid for spherical, deformed, and intermediate nuclei. It can be employed for any nucleus (even-even, odd-A, or odd-odd) provided enough $E2$ matrix elements are available. The example of ^{152}Sm is discussed.

A nonzero value of the spectroscopic quadrupole moment (Q^S) implies a nonspherical charge distribution, and the ratio $Q^S/\langle r^2\rangle$ is a measure of nuclear deformation.[1] However, as is well known, a vanishing Q^S does not necessarily imply a spherical charge distribution. The quadrupole moment Q^S vanishes for a nucleus if (a) the total angular momentum is 0 or $\frac{1}{2}$,[2] or if (b) the nucleus has equal probabilities of being prolate and oblate.[3]

The Bohr-Mottelson[4] concept of intrinsic quad-