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Pressure Dependence of Electron Drift Velocity in Hydrogen at 77.8°K

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(Received 28 October 1971)

Our measurements show at $E/P < 0.03$ V/Torr cm a decrease of the drift velocity in hydrogen with increasing pressure (up to 50 000 Torr), which cannot be explained by an electron trapping mechanism. It is shown that this effect is in agreement with Legler's theory which takes into account multiple scattering. For $E/P > 0.03$ V/Torr cm, the observed decrease of the drift velocity can be explained by an electron trapping mechanism.

Grünberg[1] and Huber[2] observed a pressure dependence of the electron drift velocity in the gases H_2, N_2, ethane, and propane at room temperature. They found a decreasing drift velocity with increasing pressure at a given E/P [in the following P is given by $P = N/(3.30\times10^{16}\ \mathrm{cm^{-3}\ Torr^{-1}})$, with N the density]. The decrease of the drift velocity at high pressures could be explained by the hypothesis that the electrons are captured by molecules for a short time τ. Ritchie and Turner[3] showed that electrons drifting a distance d need a time

$$t_d = (1+\tau\nu)d/v_-(0), \tag{1}$$

where ν denotes the frequency of collisions which produce a trapped electron and $v_-(0)$ the drift velocity without electron capture. Grünberg and Huber showed that $\tau\nu$ is proportional to P in the gases mentioned above. That means $\tau\nu = \tau\nu_1 P$, where $\tau\nu_1$ is pressure independent. From this it follows that

$$v_-(P) = v_-(0)(1+\tau\nu_1 P)^{-1}. \tag{2}$$

Frommhold[4] tried to explain the electron capture in H_2 and N_2 as a rotational resonance.

In recent experiments Crompton and Robertson[5] described the pressure dependence of electron drift velocity in normal hydrogen and parahydrogen at 77°K and gas densities up to $10^{20}\ \mathrm{cm^{-3}}$ (about 3000 Torr; see also Robertson[6]) and found that the drift velocity decreases up to approximately 1.5% in the range $0.018 < E/P < 0.15$ V/Torr cm. They showed that their results of the pressure dependence are consistent with Eq. (2) and with Frommhold's hypothesis that the electron capture is associated with the rotational excitation of the hydrogen molecules.

We have measured the drift velocity in hydrogen at 77.8°K at high pressures up to 50 000 Torr ($N = 1.65\times10^{21}\ \mathrm{cm^{-3}}$). A time-of-flight method has been used similar to that described in Ref. 1. The E/P range is 0.001–0.25 V/Torr cm. The possible error is 2%. The results at low pressure (2000 Torr) agree to within 2% with those of Lowke.[7] The agreement is within the combined error limits.

Figure 1 shows the quotient $q = v_-(P)/v_-(2000\ \mathrm{Torr})$ taken from our measurements: q decreases with decreasing E/P, passing through a minimum between 0.1–0.5 V/Torr cm, and then increases to a maximum. The value of E/P correlated with the maximum is different for different pressures. Then q falls monotonically with decreasing E/P to a constant value: The electrons are in thermal equilibrium with the gas (to be seen in curve 1 of Fig. 1).

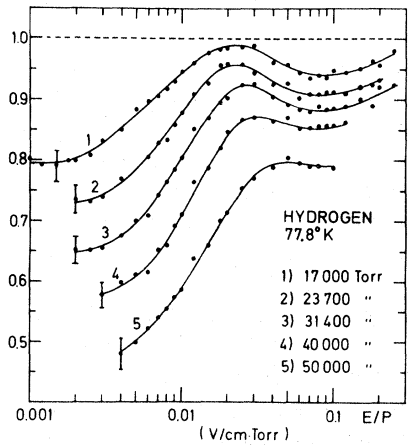

FIG. 1. The quotient q of the drift velocity at high pressures and the drift velocity at low pressure (here 2000 Torr) as a function of E/P.

In order to test whether the results are in agreement with Eq. (2), we must show that $\tau\nu_1$ is independent of pressure. Eliminating $v_-(0)$ in Eq. (2), we obtain

$$\tau\nu_1 = (1-q)(Pq - 2000\ \text{Torr})^{-1}. \quad (3)$$

Instead of plotting $\tau\nu_1$ as a function of E/P we introduce eD/μ (D is the diffusion coefficient and μ the mobility), the characteristic energy of the electrons, in order to compare our results with those of Crompton and Robertson. Since up to now no pressure dependence of eD/μ has been observed, we used values of eD/μ measured at low pressures by Crompton, Elford, and McIntosh.[8]

Figure 2 shows this plot: For higher eD/μ, $\tau\nu_1$ converges to a pressure-independent quantity. In the range of lower eD/μ (< 20 meV; this corresponds to $E/P = 0.03$ V/Torr cm in Fig. 1), $\tau\nu_1$ splits into pressure-dependent curves. This is a new result because until now in hydrogen only a pressure-independent $\tau\nu_1$ has been found.[1,5]

With higher pressure, the pressure independence begins at higher eD/μ: for the pressures 17 000–31 400 Torr, above 22 meV ($E/P = 0.04$ V/Torr cm); for 40 000 Torr, above 27 meV ($E/P = 0.07$ V/Torr cm). At 50 000 Torr this point is outside the measured range.

The results of Crompton and Robertson, who observed at $P < 3000$ Torr (dots in Fig. 2), are in satisfying agreement with our pressure-independent curve of $\tau\nu_1$ (the asymptotic curve in the region of higher eD/μ). For lower eD/μ, Crompton and Robertson did not find any pressure dependence below 15 meV ($E/P \sim 0.018$ V/Torr cm).

The model of temporary trapping of electrons cannot be used to explain the pressure dependence of $\tau\nu_1$. The effect is similar to that found by Grünberg[9] in helium at room temperature where electron-drift-velocity measurements in the high-pressure range (up to 32 600 Torr) show a pressure dependence, while measurements by Crompton and Robertson in the pressure range below atmospheric pressure show no effect. Moreover one can neglect in helium a trapping of slow electrons. An explanation for this pressure dependence at low E/P might be given by multiple scattering; see Legler.[10] He discussed the influence of multiple scattering on electron drift velocity and found that the quotient of the drift velocity at pressure P and the drift velocity at $P \to 0$ depends only on the density (here replaced by P) divided by the characteristic energy eD/μ of the electrons. We therefore plot our values of q as a function of $P/(eD/\mu)$; see Fig. 3. The values of the curves 1–5 in Fig. 2 on the left-hand side of their minimum—the region where the trapping mechanism is not valid—lead in Fig. 3 to one curve within the error limits, now on the right-hand side because the abscissa is divided into units of the inverse energy. That means q depends only on $P/(eD/\mu)$. On the other hand in the region where the model of electron trapping is valid—the range of high values of eD/μ—the plot splits into pressure-dependent curves. This effect cannot be explained by multiple scattering.

The dashed line in Fig. 3 shows the theoretical function $F^{(+)}$ calculated by Legler for electrons with a Maxwellian energy distribution. Taking into account that Legler in his theory replaces the molecules by point scatterers with the use of the scattering-length approximation, there is a reasonable agreement with experiment. In a detailed discussion of the results we must also take into account that $F^{(+)}$ is the ratio $v_-(P)/v_-(0)$: The

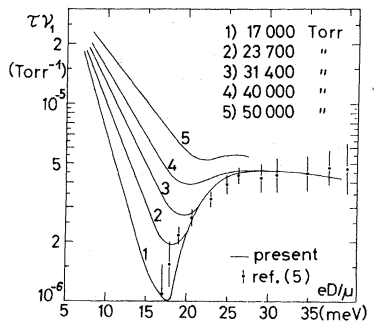

FIG. 2. The product of collision frequency and capture time of the electron as a function of eD/μ. In the range where the curves do not split, the curve represents the mean values of $\tau\nu_1$ with a possible error of $(0.6–0.8)\times 10^{-6}$ Torr^{-1}.

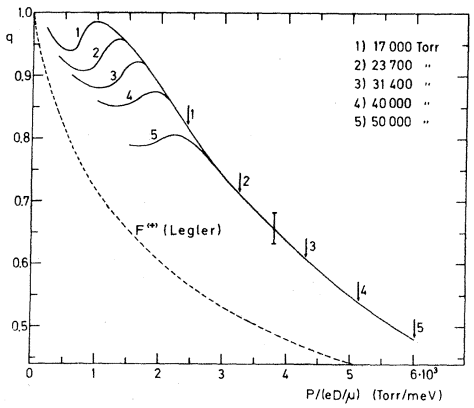

FIG. 3. $q = v_-(P)/v_-(2000$ Torr) as a function of $P/(eD/\mu)$. The arrows show the end of the curves for the indicated pressures.

reference drift velocity $v_-(0)$ should be measured at a pressure $P \to 0$ and not at $P = 2000$ Torr as we did.

In summary, the decrease of the drift velocity in hydrogen at 77.8°K needs at low eD/μ a different model for its explanation than at high eD/μ. The effect at low eD/μ can be explained reasonably by a theory of multiple scattering. The effect at high eD/μ is in agreement with the model of electron capture.

We note that in a recent paper Harrison and Springett[11] described measurements of the electron mobility in hydrogen for $5\times10^{19} < N < 5\times10^{21}$ cm^{-3} and $26 < T < 32$°K. They showed that their results are in agreement with a formation of bubbles (see also Levine and Sanders.[12] It is difficult to compare our results with those of Harrison and Springett because of the difference in temperatures. It is an open question to what extent the bubble formation contributes to our results.

The author thanks Professor Dr. H. Raether for his suggestions and support of these experiments, and Dr. W. Legler for many discussions.

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Modulational Instabilities Due to Trapped Electrons*

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As a simple model for investigating the stability of an electrostatic plasma wave with trapped electrons, the plasma is regarded as a nonlinear, dispersive dielectric. A nonlinear Schrödinger-type equation is found, which predicts instability for both electron plasma waves and ion acoustic waves. These instabilities do not depend on resonance with the bouncing motions of the trapped electrons, and thus have a different physical origin than those predicted by bunched-beam models.

In recent years it has come to be recognized that many waves are unstable against the growth of sidebands, or equivalently, when the sidebands are close to the carrier in frequency, against growth of modulations. In nonlinear optics[1] (and equivalently in water-wave theory[2]), the medium is regarded simply as a nonlinear continuum for which a nonlinear wave equation may be found. In collisionless plasmas, on the other hand, the principal nonlinearity often comes from the trapping of particles in the potential troughs of the waves.[3] In this case the usual approach has been to examine the linear stability of a prescribed periodic equilibrium state, and here too it is found that sidebands are unstable. However, the theory of the stability of a periodic equilibrium is very complicated, except for the case when all trapped particles are concentrated at the bottoms of the potential troughs.[3] This model is plausible when the sideband separation is of the order of the bounce frequency, as deeply trapped particles may respond more or less coherently. However, it is not adequate for calculating the frequency shift of the carrier wave due to trapped particles,[4] nor for treating the stability of close sidebands, when the frequency separation is much less than the bounce frequency. It is the purpose of this Letter to propose a simple model