calculating vibrational excitation. The centrality of the fixed-nuclei theory is thereby further established. However, our major additional point is the adequacy of the derivative approximation. Fixed-nuclei calculations require target wave functions, which are generally available at $R=R_0$ and within the derivative approximation may readily be derived at a nearby values of R by perturbation theory. However, at widely different R, as would be necessary in a literal application of the adiabatic-nuclei theory, this would entail a major set of additional bound-state calculations, which we believe can be avoided. We emphasize the practicality of this approximation, particularly for many-electron diatomics, some of which are currently under experimental investigation.[14]

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Parametric Instability of Plasma Waves in a Magnetic Field, Due to High-Frequency Electric Fields*

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We have observed experimentally the parametric excitation of ion acoustic waves and cyclotron harmonic waves by a high-frequency electric field with frequencies near the harmonics of the cyclotron frequency. We have verified both the wave-vector and the frequency selection rules. The measured threshold fields and growth rates are in good agreement with theory.

The understanding of the collective behavior of plasmas in the presence of high-frequency electromagnetic fields is of importance because of possible applications to heating, probing, and rf control of plasmas.[1] Recently a number of papers reported experimental observation of anomalous microwave absorption and/or parametric instabilities in both laboratory and space plasmas.[2]

In this Letter we wish to present preliminary experimental results on the parametric excitation of ion acoustic waves and electron-cyclotron harmonic waves. In contrast to previous reports,[2] in our experiments both the ion acoustic waves and the electron waves propagate almost perpendicularly to the external magnetic field and damp mainly by electron Landau damping and electron-cyclotron damping. Under these conditions, in addition to hydrodynamic mechanisms,[1,2] parametric coupling due to kinetic effects may also occur.[3,4]

For the first time, to our knowledge, we have succeeded in measuring the wavelengths and the direction of propagation of all parametrically excited waves and thus were able to verify the following selection rules:

$$\vec{k}_U - \vec{k}_A = \vec{k}_0, \quad \omega_U - \omega_A = \omega_0,$$
$$\vec{k}_L + \vec{k}_A = \vec{k}_0, \quad \omega_L + \omega_A = \omega_0. \tag{1}$$

Here $(\omega_U, \vec{k}_U)$ and $(\omega_L, \vec{k}_L)$ are the frequencies and the wave vectors of the upper and the lower sidebands, respectively, both of which were identified to be cyclotron harmonic waves[5]; $(\omega_A, \vec{k}_A)$ designates the ion acoustic wave; and $(\omega_0, \vec{k}_0)$ represents the electromagnetic pump field. In our experiments $k_0 \ll k_A$ and $k_A \approx k_U \approx -k_L$. The threshold fields and the growth rates for the instability have also been measured and were found to be in good agreement with our theoretical predictions.[4]

The experiments were carried out in a helium plasma produced by a hot-cathode discharge at one end of a linear machine, with the magnetic field in the experimental region uniform to 0.1%. The typical plasma parameters are the following: $T_e \simeq 5$ to 7 eV, $B \simeq 50$ to 100 G, $n_e \simeq 2\times10^{10}$ cm^{-3}, and $\nu_{e0} \simeq 5\times10^6$ sec^{-1}. The plasma column was approximately 12 cm in diameter and was substantially uniform 3 cm radially. Radially and axially movable, capacitively coupled, T-shaped coaxial probes and/or grids were used to launch or to detect waves. A calibrated interferometer was employed to study the wave dynamics. A detailed description of the experimental setup has been given previously.[5]

By injecting into the plasma an rf signal with frequency $\omega_0 > \omega_{ce}$ and with the electric field mainly perpendicularly to the external magnetic field (which has been ensured by proper probe geometry), we observe frequency spectra such as shown in Fig. 1. In Fig. 1(a) we exhibit the ion acoustic spectrum, peaking near 3.2 MHz, and in Fig. 1(b) the high-frequency spectrum is shown with the sidebands peaking at $f_0 \pm 3.2$ MHz. With increasing pump fields we observe the appearance of the harmonics of the sidebands and a spreading of the spectrum. At the highest powers tested, both the low- and the high-frequency spectra became broad, and components up to the ion plasma fre-

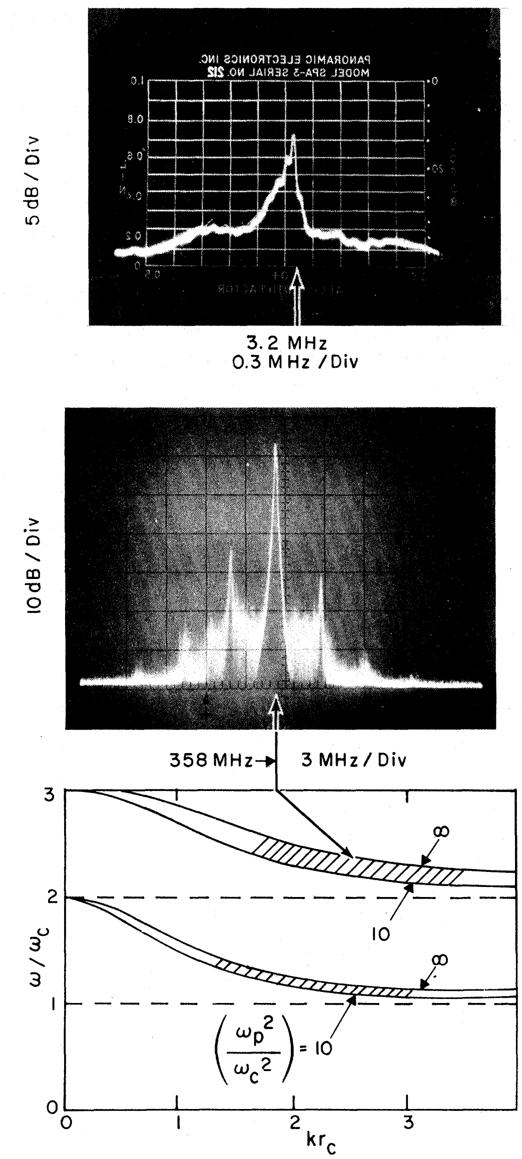

FIG. 1. (a) Spectrum of the ion acoustic waves (above threshold. (b) Spectrum of the high-frequency waves. $\omega_{ce}/2\pi = 148$ MHz, $\omega_0/\omega_{ce} = 2.41$ (above threshold). The arrow shows the pump frequency and its location in the dispersion curve. (c) Dispersion curve of cyclotron harmonic waves. The shaded regions show regions where parametric instability has been observed.

quency have been observed. From the frequency spectrum we note that the frequency selection rules of Eq. (1) are obeyed. In Fig. 1(c) we show the dispersion relation of cyclotron harmonic waves with $k_\parallel \ll k_\perp$, and also the location of the pump (and hence the sideband) frequencies. The associated perpendicular wavelengths determine the ion acoustic frequencies, and thus select the dominant components of the low-frequency spectrum (and thus the sidebands). The shaded regions show the frequency regimes where similar parametric instabilities have been observed in our experiments.[6] We note that density fluctuations of the order of $\Delta n/n_0 \lesssim 0.1$ have been observed.

Typical interferometer outputs of the waves observed above threshold are given in Fig. 2. In taking the traces for the sidebands we have eliminated the strong pump signal by a double-heterodyne detection system. From phase-shift measurements we have determined the directions of the wave propagation, and these are shown by the arrows in Fig. 2. We note that, in agreement with theory, the lower-sideband wave propagates toward the transmitting probe while the acoustic and the upper-sideband wave propagate in the opposite direction. Using the plasma parameters, we have identified the sidebands as cyclotron harmonic waves, and the low-frequency waves have been identified as ion acoustic waves. Additional small signal, linear propagation experiments were also carried out to confirm this. In Fig. 2(d) we show a recorder trace of the detected pump field and in Fig. 2(e) the radial distribution of the ion saturation current is given. Near threshold the electron temperature is nearly constant within 3 cm, and thus we conclude that the radial density profile is sufficiently uniform so that parametric coupling due to density gradients may be neglected. The pump electric field ($\propto dV_0/dr$) is damped for the following reasons: (a) ω_0 is in the cutoff regime for the extraordinary mode ($\omega_{uh} > \omega_0 > 2\omega_{ce}$, ω_{uh} being the upper-hybrid frequency); (b) the probe excites waves which move in a cylindrical wave front; (c) the length of the antenna is finite (finite parallel wavelength). Because of the nonuniformity of the pump electric field, the estimated (by measuring the e-fold radial damping length of the pump electric field) pump wave number is $k \simeq 1.5$, which is approximately 10% of k_A. We note from Figs. 2(a)–2(c) that in the interaction region (the first 2 cm) $k_U \simeq 1.1 k_A$ and $k_L \simeq -0.9 k_A$. Thus, by taking the finite value of k_0 into account, the selection rules [Eq. (1)] for the dominant frequency components are satisfied within experimental error. In addition, we note that because of their stronger relative damping, beyond 2 cm the cyclotron harmonic waves damp out whereas the ion acoustic waves propagate further out (in this case the pump field decreased below threshold at 2 cm). We have also measured the wave number of the second harmonic of the acoustic frequency, and found that $k_\perp(2\omega_A) \simeq 2 \times k_\perp(\omega_A)$, in agreement with theoretical predictions.

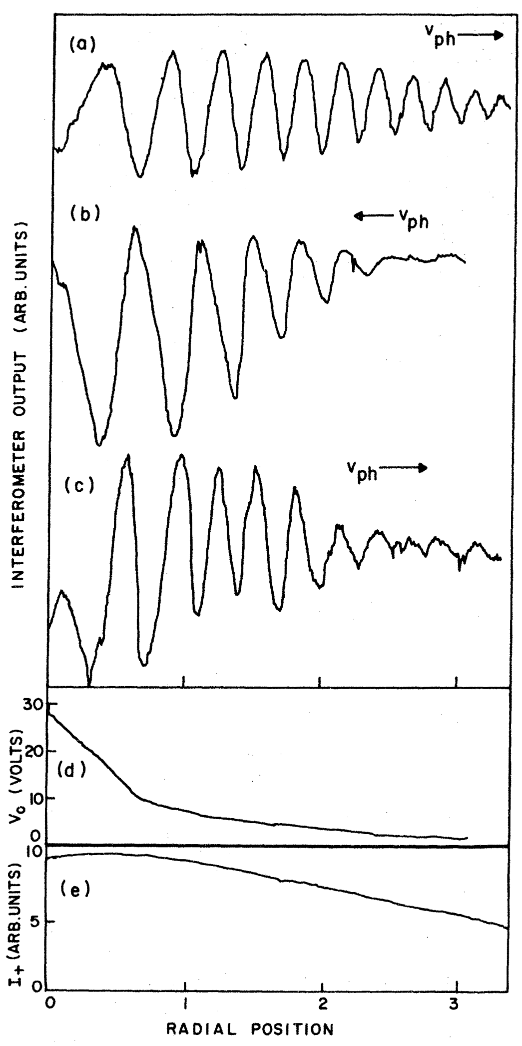

FIG. 2. Interferometer traces of waves above threshold. (a) Ion acoustic wave, $\omega_A/2\pi = 3.2$ MHz, $k_\perp \simeq 20$ cm^{-1} (at $r \approx 1.5$ cm); (b) lower sideband, $\omega_L/2\pi = 354.8$ MHz, $k_\perp \simeq 18$ cm^{-1}, $k_\perp r_{ce} \simeq 2.2$; (c) upper sideband, $\omega_U/2\pi = 361.2$ MHz, $k_\perp \simeq 21$ cm^{-1}, $k_\perp r_{ce} \simeq 2.5$; (d) rectified pump voltage V_0 versus radius; (e) ion saturation current I_+ versus radius.

By using rf probe techniques,[7] we have been able to determine the instability threshold fields from plots such as shown in Fig. 3. In the present case this was found to be approximately 10 V/cm, giving $\mu \simeq 0.08$, in good agreement with the theoretically estimated value of $\mu \simeq 0.05$ to 0.1, where $\mu = eE_0 k_\perp / m(\omega_0^2 - \omega_{ce}^2)$. The theoretical values, which were obtained by numerical analysis,[4] depend on a precise knowledge of the linear damping rates of the electrostatic waves. The damping rates were both calculated (using

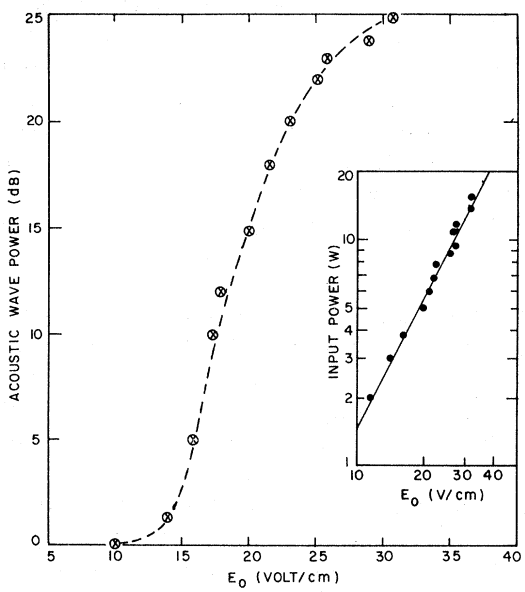

FIG. 3. Maximum wave power versus E_0 the rf field in the plasma. The inset shows that $P_{input} \propto E_0^2$.

measured values of $k_\parallel$ and known collision rates) and also measured. Using interferometry, we found that $\lambda_\parallel$ varied between 4 and 8 cm (i.e., 1 to 2 probe lengths), both for the ion waves and for the cyclotron harmonic waves. This value is considerably less than the collision mean free path of electrons ($\lambda_{mfp} \simeq 40$ cm), and thus collisionless Landau damping dominates. We have also performed ion-wave propagation experiments in four different gases and confirmed that $\omega_A/k_\perp \propto (T_e/M_i)^{1/2}$ and $k_\parallel \ll k_\perp$.[8] In addition, we found that $\omega_A/k_\perp$ was independent of the magnetic field or the density, in agreement with the ion acoustic dispersion relation.

As further checks on the instability mechanism, we have been able to amplify (by several orders of magnitude) an externally launched ion acoustic wave at a frequency close to the observed unstable acoustic wave frequency. In such experiments two separate probes were used to excite the acoustic wave and the high-frequency field, and a third probe was used as the interferometer receiver. In addition, by turning on the pump field by a fast electronic switch, we have directly measured the instability growth rates. The growth was found to be exponential for approximately 6–8 e-folds and then began to saturate. The measured growth rates were in reasonably good agreement with theory (within a factor of 2).

In conclusion, in addition to verifying the fundamental aspects of the parametric instability of ion acoustic waves and cyclotron harmonic waves, we believe that the present experiments suggest a new way to interact with, and/or heat, both laboratory and space plasmas. In particular, near cyclotron harmonics the plasma is often accessible to the extraordinary mode of electromagnetic waves. Thus, heating and/or direct interaction (such as feedback control or scattering) from ion acoustic waves (or other low-frequency modes) should be feasible through coupling with cyclotron harmonic waves. Even in the cutoff regimes, parametrically excited ion acoustic waves with sufficiently low damping rates could propagate into otherwise inaccessible regions. Further investigations of these phenomena are in progress.

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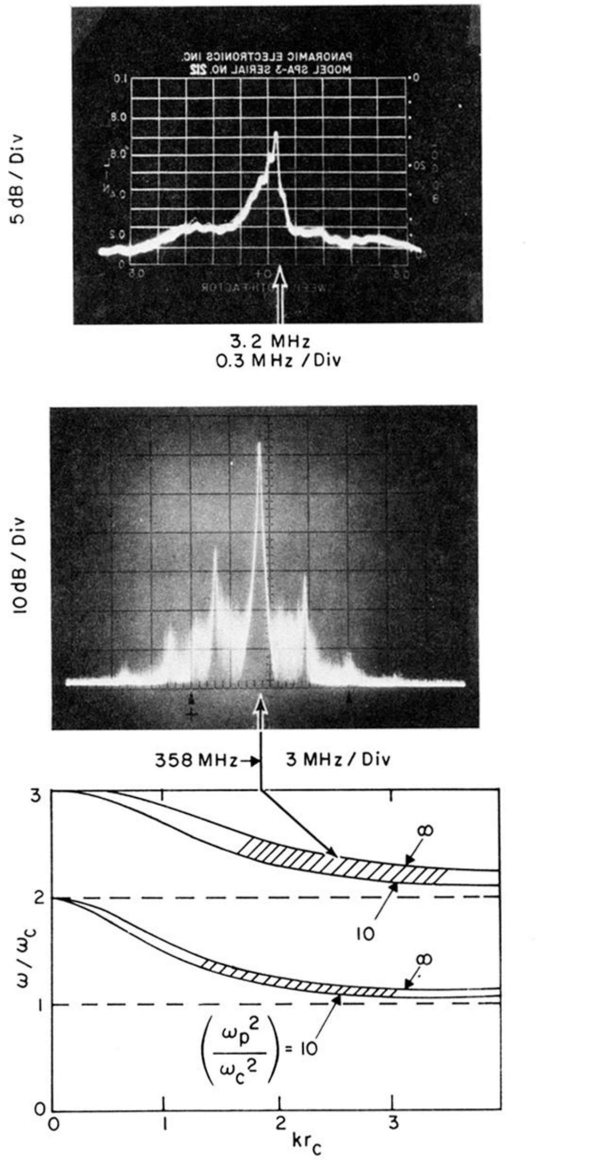
[7]J. C. Hosea, Ph.D. thesis, Stanford University, Stanford, California, 1966 (unpublished).

[8]We note that since $\omega_A/k_\parallel v_{the} \ll 1$, $\omega_A \gg \omega_{ci}$, $\mathrm{Im}(\omega_A) > \omega_{ci}$, the low-frequency waves are ion acoustic waves rather than lower hybrid waves.



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