$K_2{}^0 \to \gamma\gamma$ Decay in a Current-Current Quark Model*

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(Received 27 April 1972)

The fermion-loop model introduced by Steinberger to explain the decay $\pi^0 \to \gamma\gamma$ is suitably modified for weak interactions so that it provides a qualitative explanation for $K_2{}^0 \to \gamma\gamma$ as well. The weak Hamiltonian is phenomenologically constructed from the one-baryon octet matrix elements of the symmetric quark model. Since these parameters are fitted to the nonleptonic decay of hyperons, this is a model for $K_2{}^0 \to \gamma\gamma$ with no adjustable parameters.

For some time now the possible existence of neutral currents in the strangeness-changing weak Hamiltonian for nonleptonic interactions, in addition to the customary charged currents (the so-called "Cabibbo current"), has been the subject of considerable speculation.[1-3] The recent measurements of the decay mode $K_2{}^0 \to \gamma\gamma$,[4] yielding the reported average[5] for the branching ratio of

$$\Gamma(K_2{}^0 \to \gamma\gamma)/\Gamma(K_2{}^0 \to \text{all}) = (5.6 \pm 0.5)\times 10^{-4}, \quad (1)$$

have provided theoreticians with a process in which such a possible enlargement of the usual current-current Hamiltonian can be tested. The most successful of such (current-current) models is that of Moshe and Singer[6]; it is also the "simplest" (i.e., with a minimum of neutral currents) current-current CP-conserving $\Delta S = 1$ Hamiltonian,[7]

$$\mathcal{H}_{\text{MS}} = \sqrt{2}\, G_{\text{nl}} \tfrac{1}{2}(\{J_\mu{}^1, J_\mu{}^4\} + \{J_\mu{}^2, J_\mu{}^5\} - \{J_\mu{}^3, J_\mu{}^6\}), \quad (2)$$

which preserves the $\Delta I = \frac{1}{2}$ rule. It should be noted that an earlier version of this weak Hamiltonian suggested by Sakurai,

$$\mathcal{H}_{\text{S}} = \sqrt{2}\, G_{\text{nl}}\, d_{6ab} J_\mu{}^a J_\mu{}^b, \quad (3)$$

which behaves like a λ_6 vector of an SU(3) octet, fails in explaining the rate by almost 2 orders of magnitude.[8] In *either* case,[9] dominance of the vector (axial) currents by vector (pseudoscalar and axial vector) particles leads one to a description of the process in terms of tree graphs.[10] To be sure, the good agreement with experiment realized by Moshe and Singer requires, in addition, the large SU(3)-breaking effects for amplitudes involving strange mesons predicted by the PVV-coupling model of Brown, Munczek, and Singer,[11] and it is this (broken) SU(3) symmetry of PVV couplings which relates the process $K_2{}^0 \to \gamma\gamma$ to the decay $\pi^0 \to \gamma\gamma$ in the tree-graph description.

On the other hand, it has long been accepted[12] that the calculation of the π^0 lifetime in the (charged) baryon-loop model first proposed by Steinberger[13] is in good agreement with the experimental value[5]

$$\Gamma(\pi^0 \to \gamma\gamma) = 7.2 \pm 1.2 \text{ eV}, \quad (4)$$

for physical values of the nucleon mass and the pion-nucleon coupling. (However, as we note below, this apparent good argument is somewhat vitiated when one includes the contribution from strange-baryon triangle graphs as well.) It is then of interest to inquire whether this alternative description has a viable extension to the process $K_2{}^0 \to \gamma\gamma$ also. In the course of addressing ourselves to this interesting question, we will find that the straightforward extension of Steinberger's Feynman-graph calculation[13] is in essential agreement with a simple soft-kaon current-algebra treatment of this process.

Before sketching these alternative treatments of the extended fermion-loop model, we should remark on the new ingredient in our calculation, deriving from the results of Gronau's[14] recent study of parity-conserving single-baryon octet matrix elements in an application of the symmetric quark model[14] to a (semiphenomenological) current-algebraic approach to nonleptonic hyperon decays. In this model the nonleptonic weak Hamiltonian density relevant for hyperon decays,[15]

$$\mathcal{H}_G = \sqrt{2}\, G \cos\theta \sin\theta\, \tfrac{1}{2}\{J_\mu{}^{(1-i2)}, J_\mu{}^{(4+i5)}\}, \quad (5)$$

yields parity-conserving matrix elements parametrized as[14]

$$\langle B_j|\mathcal{H}_G|B_i\rangle = 2\sqrt{2}\, \bar{u}_j(-if_{6ji}F + d_{6ji}D)u_i, \quad (6)$$

by virtue of the octet property. In the framework of his model, Gronau[14] finds the "symmetry" value $D/F = -1$ for the octet-baryon matrix elements of $\mathcal{H}_G$. A best fit (and the fit is impressive) to the

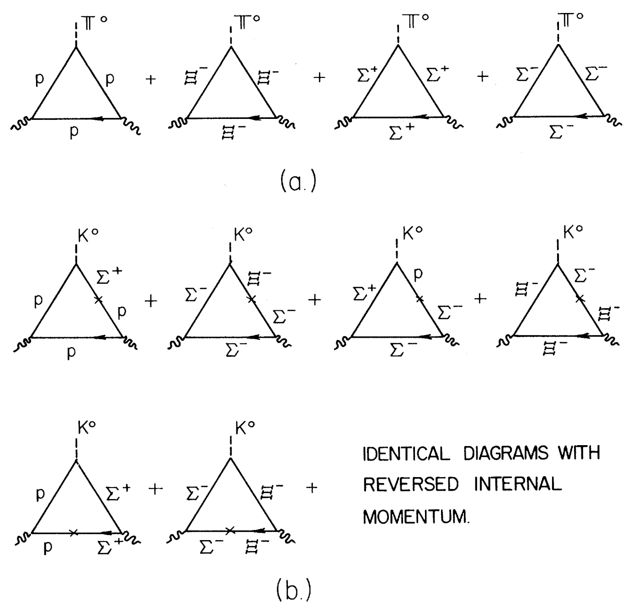

FIG. 1. (a) Four charged-baryon triangle graphs which contribute to the amplitude for $\pi^0 \to \gamma\gamma$ in the case of SU(3) symmetry. (b) Baryon loop graphs which contribute to the amplitude for $K_2^0 \to \gamma\gamma$; a cross on an internal (baryon) line indicates a weak-interaction ($\mathcal{H}_w$) insertion.

experimental amplitudes is obtained with the values $F = 4.7 \times 10^{-5}$ MeV, $D/F = -0.85$, and $d/f = 1.8$, where d and f $(f + d = 1)$ are the symmetric and antisymmetric MBB couplings,[16,17] and these values are adopted in our two-photon-decay calculation. The working assumption in our baryon-loop model for $K_2^0 \to \gamma\gamma$ is the phenomenological replacement of Gronau's one-baryon octet matrix element $\langle B_j | \mathcal{H}_G | B_i \rangle$ by an equivalent weak Hamiltonian,

$$\mathcal{H}_w = 2\sqrt{2}\ \bar{\psi}_j(-if_{6ji}F + d_{6ji}D)\psi_i(x), \tag{7}$$

expressed in terms of physical baryon fields. It is further assumed that $\mathcal{H}_w$ is valid as well for off-baryon-mass-shell calculations such as we mean to pursue below; we also choose to adopt in our calculations the convenient point of view (as does Gronau[14]) which has SU(3) conserved at vertices and broken in hadron masses.

The effective interaction Lagrangian for our extended baryon-loop calculation of the two-photon decay of the K_2^0 is

$$\mathcal{L}_{\rm int} = -\sqrt{2}\ g(1-d)\,\mathrm{Tr}([\bar{B}i\gamma_5, B]M) + \sqrt{2}\ gd\,\mathrm{Tr}(\{\bar{B}i\gamma_5, B\}M) - \tfrac{1}{2}eA_\mu\,\mathrm{Tr}([\bar{B}\gamma_\mu, B]Q) + \mathcal{L}_w, \tag{8}$$

with

$$\mathcal{L}_w = \sqrt{2}\ F\,\mathrm{Tr}([\bar{B}, B]\lambda_6) - \sqrt{2}\ D\,\mathrm{Tr}(\{\bar{B}, B\}\lambda_6), \tag{9}$$

where $B = \lambda_i\psi_i/\sqrt{2}$ is the traceless baryon matrix,[18] $M = \lambda_i\varphi_i/\sqrt{2}$ is the traceless Hermitian meson matrix,[18] and the matrix $Q = \lambda_3 + \lambda_8\sqrt{3}$. While it is straightforward to write down expressions for the allowed loop graphs [see Fig. 1(b)] in the case where the virtual baryons have the physical masses, in terms of the basic Feynman integral,[19]

$$\epsilon_{\mu\nu\rho\sigma}k_\rho k_\sigma'\Delta(m_1, m_2, m_3; p^2 = (k + k')^2) = -\int \frac{d^4t}{(2\pi)^4}\,\mathrm{Tr}\left[\gamma_5 \frac{1}{\gamma\cdot(t+k) - m_1}\gamma_\mu \frac{1}{\gamma\cdot t - m_2}\gamma_\nu \frac{1}{\gamma\cdot(t-k') - m_3}\right], \tag{10}$$

it is probably more instructive to present the expression that obtains in the limit that one neglects mass breaking in the baryon octet as well as the mass of the decaying pseudoscalar meson. In this limit, the amplitude for $K_2^0 \to \gamma\gamma$ is given by

$$\mathfrak{M}(K_2^0 \to \gamma\gamma) = (4k_0k_0')^{1/2}\langle\gamma(k)\gamma(k')\,\mathrm{out}|\mathcal{H}_w(0)|K_2^0(p)\rangle(2p_0)^{1/2} = ge^2 4\sqrt{2}\ (fF + dD)12S$$
$$\times[\tfrac{1}{3}d\Delta(m, m, m; 0)/dm]\epsilon_{\mu\nu\rho\sigma}\epsilon_\mu(k, \lambda)\epsilon_\nu(k', \lambda')k_\rho k_\sigma', \tag{11}$$

where $S = \frac{1}{2}$ is the (equal) unitary spin weight of each graph (with either f- or d-type coupling), and

$$\tfrac{1}{3}d\Delta(m, m, m; 0)/dm = -1/24\pi^2m^2, \tag{12}$$

is the contribution of *each class* of insertion. In this limit one has

$$\Gamma(K_2^0 \to \gamma\gamma) = \tfrac{1}{2}\sum_{\mathrm{pol}\,\lambda,\lambda'}|\mathfrak{M}(K_2^0 \to \gamma\gamma)|^2\frac{1}{16\pi m_K} = \frac{2}{\pi^2}\left(\frac{g^2}{4\pi}\right)\alpha^2\frac{(fF + dD)^2 m_K^3}{m^4}, \tag{13}$$

so that

$$\Gamma_{\text{theo}}(K_2^0 \to \gamma\gamma)/\Gamma(K_2^0 \to \text{all}) = 1.22\times 10^{-4} \quad (D/F = -0.85,\ d/f = 1.8)$$
$$= 1.35\times 10^{-4} \quad (D/F = -1,\ d/f = 1.5), \tag{14}$$

using the value[5] of $\tau_{K_2^0} = 5.17\times 10^{-8}$ sec and a "mean" baryon mass of 1 GeV. Although agreement with the experimental value of 5.6×10^{-4} quoted earlier is not very impressive and worsens in the "exact" calculation alluded to above, where $[\Gamma_{\text{theo}}(K_2^0 \to \gamma\gamma)/\Gamma(K_2^0 \to \text{all})]_{\text{exact}} = 0.78\times 10^{-4}$, it must be emphasized that there are *no* free parameters in this calculation, unlike the analogous calculation in the tree-graph model.[6] In the latter model it is necessary to require a global fit of the model weak interaction *with* SU(3)-breaking parameters[6,11] to a certain class of K-meson radiative decays[6] to determine a value for the $K_2^0 \to \gamma\gamma$ decay rate. Note that our calculation also indicates an interesting qualitative consistency between the nonleptonic hyperon decays and this ($B=0$) decay process; such a connection cannot be made in the framework of the tree-graph model.

Although the quantitative agreement with experiment we find is poor, it is well to bear in mind that the good agreement between theory[12] $[\Gamma_{\text{theo}}(\pi^0 \to \gamma\gamma) = 13.8\ \text{eV}]$ and experiment usually claimed[12] for the baryon-loop (or Steinberger) model of π^0 decay, where

$$\Gamma(\pi^0 \to \gamma\gamma) = \pi^2 (g^2/4\pi)\alpha^2 m_\pi^2 |\Delta(m_p, m_p, m_p; 0)|^2$$
$$= (g^2/4\pi)\alpha^2 m_\pi^3/16\pi^2 m_p^2, \tag{15}$$

applies to its SU(2) description in terms of a nucleon loop only. If one includes the contribution of strange-baryon loops as well (specifically the nonvanishing contribution from the Ξ^- loop) one finds

$$\Delta(m_p, m_p, m_p; 0) \to \Delta(m_p, m_p, m_p; 0) - (1-2d)\Delta(m_{\Xi^-}, m_{\Xi^-}, m_{\Xi^-}; 0)$$
$$+ 2(1-d)\Delta(m_{\Sigma^+}, m_{\Sigma^+}, m_{\Sigma^+}; 0) - 2(1-d)\Delta(m_{\Sigma^-}, m_{\Sigma^-}, m_{\Sigma^-}; 0), \tag{16}$$

with $\Gamma_{\text{theo}}(\pi^0 \to \gamma\gamma) = 20.1$ eV, now almost 3 times too large.

Alternatively, one could approach the evaluation of the matrix element

$$\langle \gamma(k)\gamma(k')\ \text{out}|\mathcal{H}_{\text{w}}(0)|K_2^0(p)\rangle (2p_0)^{1/2} \tag{17}$$

from the standpoint of the current algebra. In the soft-kaon limit, this matrix element is given by the equal-time commutator,

$$\lim_{p\to 0}\langle \gamma(k)\gamma(k')\ \text{out}|\mathcal{H}_{\text{w}}(0)|K_2^0(p)\rangle (2p_0)^{1/2} = -i\int d^3x\, \langle \gamma(k)\gamma(k')\ \text{out}|C_6^{-1}[\mathcal{H}_{\text{w}}(0), A_6^0(\vec{x}, 0)]|0\rangle, \tag{18}$$

where C_6 is defined by

$$\partial_\mu A_\mu^6 = C_6 m_K^2 \varphi_6. \tag{19}$$

In terms of the effective axial-vector current,

$$A_6^0 = \tfrac{1}{2} g_A \{ d\,\text{Tr}(\{\bar{B}\gamma^0\gamma_5, B\}\lambda_6) - (1-d)\,\text{Tr}([\bar{B}\gamma^0\gamma_5, B]\lambda_6)\}, \tag{20}$$

one finds

$$\lim_{p\to 0}\langle \gamma(k)\gamma(k')\ \text{out}|\mathcal{H}_{\text{w}}(0)|K_2^0(p)\rangle (2p_0)^{1/2} = -\sqrt{2}\,\frac{g_A}{C_6}(fF + dD)\langle \gamma(k)\gamma(k')\ \text{out}|\sum_{j=1,2,4,5}\bar{\psi}_j i\gamma_5\psi_j|0\rangle, \tag{21}$$

using the explicit form for $\mathcal{H}_{\text{w}}(0)$ and assuming the photons couple only to charged fermions. This expression simplifies further to

$$\lim_{p\to 0}\langle \gamma(k)\gamma(k')\ \text{out}|\mathcal{H}_{\text{w}}(0)|K_2^0(p)\rangle (2p_0)^{1/2} = -\sqrt{2}\,(fF + dD)\frac{g}{m}\langle \gamma(k)\gamma(k')\ \text{out}|$$
$$\times (\bar{\psi}_p i\gamma_5\psi_p + \bar{\psi}_{\Xi^-} i\gamma_5\psi_{\Xi^-} + \bar{\psi}_{\Sigma^+} i\gamma_5\psi_{\Sigma^+} + \bar{\psi}_{\Sigma^-} i\gamma_5\psi_{\Sigma^-})|0\rangle, \tag{22}$$

on making use of the Goldberger-Treiman relation $g_A m/g \simeq C_6$, and introducing the physical baryon fields. This result is to be compared with the similar expression for the matrix element for π^0 decay in the equivalent fermion-pair model,

$$\langle \gamma(k)\gamma(k')\ \text{out}|J_{\pi^0}(0)|0\rangle = \langle \gamma(k)\gamma(k')\ \text{out}|\,gd(\bar{\psi}_p i\gamma_5\psi_p + \bar{\psi}_{\Xi^-} i\gamma_5\psi_{\Xi^-})|0\rangle. \tag{23}$$

Neglecting baryon mass differences, one finds the ratio of the two (reduced) matrix elements to be the same as was obtained before.

*Work supported in part by the National Science Foundation.

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[9]However, the *good* results of Sakurai (Ref. 1) are still valid when the Hamiltonian of Moshe and Singer (Ref. 6) is used, since the J^6J^8 term does not contribute to the K and hyperon decays which the current-current model fits.

[10]The contribution to $K_2{}^0 \to \gamma\gamma$ from an intermediate $\eta \equiv \pi^8, A^8$ is characteristic of the Sakurai octet form only, neglecting η-π^0 mixing as a perturbation on the result of Moshe and Singer (Ref. 6).

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[15]J_μ is given by $J_\mu{}^a = \frac{1}{2}\bar{q}\gamma_\mu(1+\gamma_5)\lambda_a q$, where q stands for the multicomponent Bose-quark field operator and λ_a is the appropriate 3×3 SU(3) matrix.

[16]We also adopt the value of the pion-nucleon coupling, $g^2/4\pi = 14.6$, used by Gronau (Ref. 14) in this fit.

[17]The fourth parameter in Gronau's fit (his δ/φ, Ref. 14) is the ratio D/F for the γ_μ coupling at the strong VBB vertex in the case of the vector meson pole and does not figure in our considerations. The interested reader should consult Gronau's paper for a more detailed discussion of the theoretical (and mechanical) aspects of this fit.

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Evidence for Stray Baryonic States from a Study of $K^+\Lambda$ Photoproduction

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Strong confirmatory evidence is found for the existence of a D_{13} resonant state around 1670 MeV c.m. energy; and evidence, though somewhat weaker, is obtained for the existence of a stray baryonic F_{15} state recently conjectured by Donnachie.

We have studied the available data[1,2] below 2.25 GeV c.m. energy for the process $\gamma p \to K^+\Lambda$ by use of an energy-dependent multipole analysis. The analysis was done in keeping with the spirit of duality and only direct-channel resonance terms with a phase rotation were included along with background terms in both S and P waves.[2,3]

There are several reasons why it is particularly important to study this process at this time. (1) New data[1] indicate considerable structure in the differential cross section. There seems to be a rather pronounced dip around 1750 MeV c.m. energy and a second rather broad maximum centered at about 1910 MeV. This dip is more pronounced in the backward direction (Fig. 1), but can still be detected in the forward direction. (2) Our recently gained knowledge with regard to $K\Lambda$ and γN partial widths[3-6] makes it possible to determine the approximate contribution of several of the isospin-$\frac{1}{2}$ nucleon resonances to $K^+\Lambda$ photoproduction. (3) A very interesting conjecture made by Donnachie,[7] concerning the possibility of stray baryonic states (in particular the stray F_{15} state) which do not couple significantly to the πN channel, can be tested. (4) New and confirming information can be obtained regarding