Single-Particle Effects on Fission Probabilities for the Lighter Elements*

R. Vandenbosch and U. Mosel
University of Washington, Seattle, Washington 98195
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The excitation energy dependence of the competition between neutron emission and fission has been calculated for nuclei in the vicinity of the ^{208}Pb closed shell. A two-center model has been used to calculate single-particle effects on the nuclear level density as well as on the barrier heights. Comparison with experiment demonstrates the importance of including the dependence of the single-particle level density on deformation.

Single-particle effects influence the probability for fission in two ways. In the first place, they can give rise to significant modifications in the potential-energy surface defining the fission barrier. This has been strikingly demonstrated for the heavy elements where Strutinsky[1] and Nilsson *et al.*[2] have been able to account for the double-humped barriers responsible for spontaneous fission isomerism in terms of single-particle corrections to the liquid-drop–model potential-energy surface. Second, the deformation dependence of the single-particle level density can affect the number of channels and hence the relative probability of fission and neutron emission for excited nuclei. The purpose of the present note is to present calculations of the fission probabilities for the lighter elements including the single-particle effects on both the fission barriers and on the level densities affecting the competition between fission and neutron emission for excited nuclei. In these calculations we have not varied parameters to fit the data but have instead tried to see to what extent theory is capable of reproducing the experimental observations.

Before presenting the results of the calculations, we review briefly the evidence for single-particle effects on the competition between neutron emission and fission. The rapid increase in the fission probability with increasing excitation energy for the lighter elements allows a quantitative determination of the energy dependence of Γ_f/Γ_n, the ratio of the probability for fission to the probability for neutron emission. Soon after experimental information on the excitation energy dependence of Γ_f/Γ_n became available, it was noted that the rate of increase of Γ_f/Γ_n with excitation energy was larger than expected.[3,4] The observations could be reproduced if the level density at the fission saddle point, a_f, was larger than the value characterizing the level density for normal undistorted nuclei, a_n. This result was not too surprising as the nuclei involved were all close to the $Z=82$, $N=126$ closed shells where the level-density parameter for the normal undistorted nuclei was known to be unusually small. Later results, however, indicated that a_f/a_n remained slightly greater than 1 for nuclei both considerably lighter[5] and considerably heavier[6] than nuclei in the vicinity of the closed shell at lead.

This behavior may now be understood from the general effects of nuclear shell structure on the fission process. Strutinsky[1] has shown the close relationship between the sign and magnitude of the shell correction energy and the local single-particle level density near the Fermi energy. (The Fermi energy is assumed to be halfway between the last filled level and the first unfilled level in this discussion.) If the single-particle level density is unusually low, as is the case for closed-shell nuclei, the shell correction is negative and the nucleus is unusually stable. The ground states of heavier nuclei are stabilized at a nonspherical equilibrium deformation because the local single-particle level density near the Fermi energy is sufficiently lower than the average at this deformation. On the other hand, a higher-than-average single-particle level density is associated with a positive shell correction. Thus in the heavy elements the double barriers result from a positive shell correction, while the isomeric minimum is associated with a negative shell correction. These results suggest a possible explanation for the systematic tendency for a_f to be larger than a_n. The deformation appropriate to neutron emission corresponds to the ground-state deformation which is a minimum in the potential energy and is likely to have resulted from a lower-than-average single-particle level density. The situation for the fission barrier is somewhat more complex. In the heavy elements the first (inner) barrier is stable with respect to asymmetric distortions, and the resulting saddle point is associated with a positive shell effect. The nucleus at a symmetric distortion corresponding to the second (outer) barrier is, however, unstable with respect to asymmetric distortions, and the second saddle is not associated with a large positive shell correction. Thus the largest ratio for a_f/a_n is expected for those heavy elements where the first barrier is higher in energy than the second barrier. This appears to be the case for plutonium, americium, and curium isotopes.[7]

Recent calculations of the fission barriers for the lighter nuclei using a shell correction derived from a two-center model[8] indicate a positive shell correction at the saddlepoint for $A \gtrsim 200$ and a negative shell correction at the saddlepoint for $180 \leqslant A < 200$. In a recent calculation[9] of Γ_f/Γ_n, the shell effects on the neutron width have been treated explicitly, while the saddlepoint parameters have been deduced from a fit to excitation functions. The barriers obtained in this study, after correction for ground-state shell effects, appear to fluctuate about the liquid-drop–model predictions by no more than an MeV. This has led to the suggestion that shell effects at the saddle are small. The possible magnitude of shell effects may be somewhat larger than apparent because of the fluctuations in the saddle-point pairing gap parameter Δ obtained simultaneously in the fits. Since the pairing strength G is expected to vary smoothly with mass number, variations in the gap parameter imply shell effects in the single-particle level spacings at the saddle deformation. If the single-particle spacing is constrained to be uniform,[9] the shell correction is absorbed in the saddle pairing energy associated with the gap parameters. The fluctuations of this pairing energy (correlated with the fluctuations of the gap parameter) together with aforementioned discrepancies between the liquid-drop–model predictions and the fitted barriers are consistent with shell effects at the saddle of several MeV. It should also be remembered that the liquid-drop model has been fitted to reproduce the experimental barrier of ^{201}Tl and thus contains the saddlepoint shell effect for this nucleus.

We have used the single-particle energies of the two-center model to calculate the total nuclear level densities at the equilibrium ground-state deformation and at the fission barrier deformation for a number of lighter nuclei. The nuclear shape corresponding to the potential employed[10] is that of two equal overlapping spheroids joined by a smoothed neck. In the following we first want to illustrate a few of the points discussed above for the level-density parameter a. For this discussion pairing effects have been neglected for the sake of clarity. For purposes of illustration we have parametrized these level densities in the usual Fermi-gas form, $\rho(E) \propto \exp(2\sqrt{aE})$. The shell effects result in the level-density parameter a no longer being a constant, as would be expected for uniformly spaced single-particle levels. Alternatively, the shell effects could have been parametrized by redefining E in terms of an energy-dependent reference level. For the elements in the immediate vicinity of the $Z=82$, $N=126$ closed shell, a_n is indeed much lower than average, and a_f is appreciably larger than average. This is illustrated in Fig. 1. As the excitation energy is increased, the single-particle states farther away from the Fermi surface play a role, and the shell effects tend to disappear. It must be remembered when looking at Fig. 1 that the average excitation energy at the saddle is considerably less than that of the residual nucleus following neutron emission, since the fission barrier is much larger than the neutron binding energy. Thus the effective a_f/a_n ratio is larger than that obtained by looking at the values of a_f and a_n at a common excitation energy. It is also clear that a_f/a_n is strongly dependent on excitation energy.

We have calculated the ratio of the level widths for fission and neutron emission by numerical integration of Eqs. (1) and (2) of Ref. 4. The only quantities for which theoretical predictions are required are the fission thresholds and the nu-

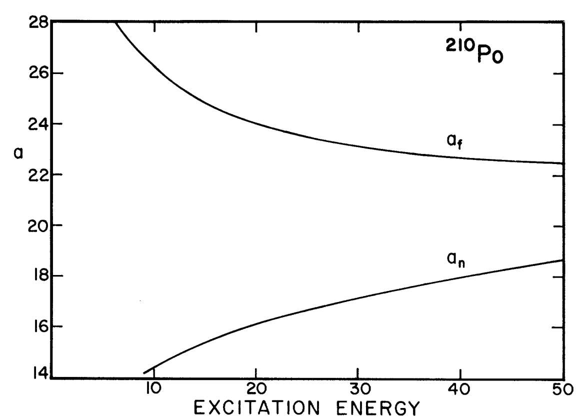

FIG. 1. Level-density parameters a_n and a_f characterizing the level densities calculated from single-particle energies for the spherical ground-state deformation and the saddlepoint deformation, respectively. The excitation energy is measured from the true shell-affected ground or saddlepoint energy surface. Pairing effects have been neglected in this particular illustration.

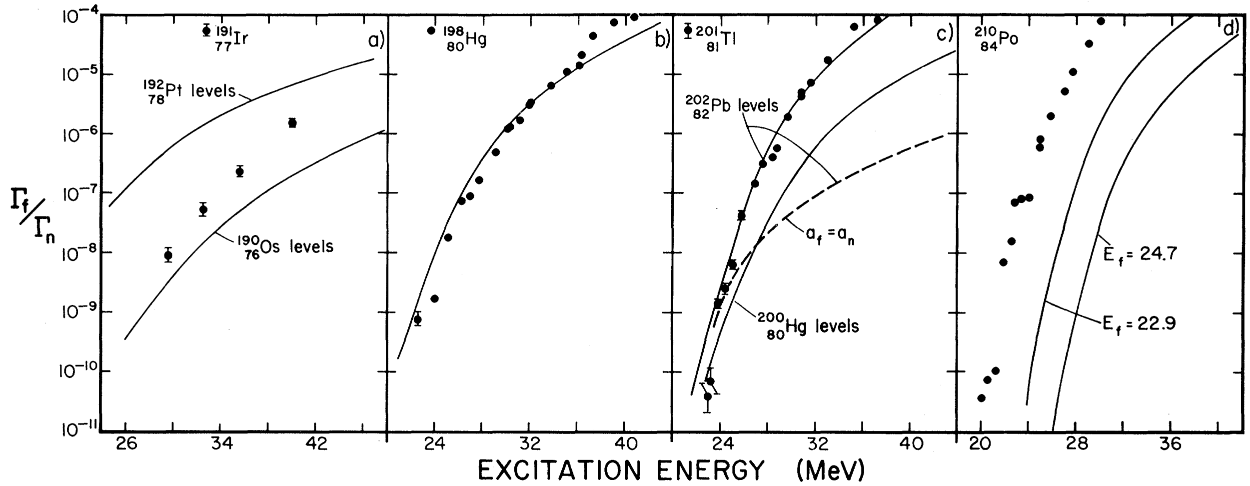

FIG. 2. Comparison of calculated and experimental values of Γ_f/Γ_n. The experimental values for ^{191}Ir are from Ref. 5, for ^{198}Hg and ^{210}Po from Ref. 14, and for ^{201}Tl from Ref. 13.

clear level densities. The fission thresholds have been taken from the calculations of Mosel and Schmitt[8] utilizing the aforementioned two-center model. The total nuclear level densities have been generated from the single-particle energies for the appropriate deformation using the level-density formalism of Decowski *et al.*[11] The actual calculation of the total nuclear level densities, which include pairing effects, have been performed using a computer code written by Bolsterli.[12] The results for four representative nuclei are compared with the experimental data of Burnett *et al.*,[13] Khodai-Joopari,[14] and Raisbeck and Cobble[5] in Fig. 2. The inclusion of pairing effects are of relatively minor importance, the principal effect being a flattening of the excitation energy dependence in the region of the barrier for the heavier elements. This is a consequence of the larger pairing gap at the saddle point than at the ground state when a_f is larger than a_n. Small corrections have also been made for angular-momentum effects and for penetration of and reflection by the fission barrier at energies in the vicinity of the barrier. The level-density calculations were only performed for even-even nuclei, so that two curves are shown for odd-Z fissioning nuclei, corresponding to calculations based on the level densities for even-even nuclei with one more and with one less proton than the desired number, together with an appropriate odd-even level-density correction.

The qualitative features of the experimental fission probabilities are reproduced quite well by the calculations. The improvement due to the inclusion of the dependence of the single-particle level densities on deformation is indicated by comparison with the dashed line in Fig. 2(c). This line was obtained by using the same level densities for Γ_f and for Γ_n, which is equivalent to having $a_f = a_n$. The displacement between the calculated and experimental curves in the case of ^{210}Po is due to the theoretical barrier height being 4 MeV higher than the emprirical value. 2 MeV of this can be attributed to a failure of the model to reproduce the ground-state shell correction. The remaining discrepancy may be a result of the restriction to reflection-symmetric saddle-point shapes. There is an indication in the calculations of Pashkevich[15] that the fission threshold for $A = 210$ is reduced slightly when asymmetric distortions are allowed. Calculations show, however, that the lighter nuclei ^{188}Oc[16] and ^{202}Pb[17] are stable with respect to asymmetric distortions, consistent with the indications in Fig. 2 that the barrier height is given correctly for the lighter elements. It is also interesting to note that for ^{191}Ir, the equilibrium ground state is slightly deformed, and that the saddle is associated with a slightly negative shell correction also. As a result a_f and a_n are very nearly equal at comparable excitation energies for this particular nucleus.

There is evidence in Fig. 2 that the calculated values of Γ_f/Γ_n fail to keep increasing with energy as fast as the experimental values. We believe this difficulty may originate in the approximate manner in which volume conservation is maintained as the potential is distorted. This difficulty is avoided in the calculation of the deformation energy surface and barrier heights by the use of the Strutinsky normalization procedure. It does, however, enter rather directly in the level-density calculation. In the present calculations as in previous calculations with the two-center model, the volume of the equipotential surface for $V = \frac{1}{2}m\omega^2R^2$ with $\hbar\omega = 41A^{-1/3}$ MeV and $R = r_0A^{1/3}$ fm is maintained. For the large saddle-point distortions the volume contained in other equipotentials is not necessarily conserved. It should be noted, however, that this is a general difficulty that appears in all deformed single-particle calculations that are more general than the simple harmonic-oscillator potential of the Nilsson model. There is also an indication that volume conservation is inadequately treated in the latter potential in that the Strutinsky normalization procedure is necessary to keep the deformation energy well behaved at large deformations. We have tried to qualitatively account for proper volume conservation by requiring the Fermi energy to be invariant with deformation. Some improvement is obtained, but not sufficient to reproduce observations. This problem is being pursued further.

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Charge-Dependent Effects in the Photodisintegration of ^{4}He†

J. T. Londergan

Department of Physics, Indiana University, Bloomington, Indiana 47401, and Department of Physics, Case Western Reserve University, Clevelend, Ohio 44106*

and

C. M. Shakin

Department of Physics, Case Western Reserve University, Cleveland, Ohio 44106

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^{4}He$(\gamma,p)^3$H and ^{4}He$(\gamma,n)^3$He cross sections have been calculated for photon energies from the (γ,n) threshold to 34 MeV. Effects of Coulomb interactions, channel spin mixing, and mass differences have been taken into account using a coupled-channel continuum-shell-model calculation of p-^{3}H and n-^{3}He elastic and charge-exchange scattering. The ratio $\sigma(\gamma,p)/\sigma(\gamma,n)$ is predicted to be close to 1, except at energies very close to the (γ,n) threshold.

Recent experiments[1,2] of the photodisintegration reaction ^{4}He(γ,n) showed a striking behavior for the neutron total cross sections. In particular, the (γ,n) measurements of Berman, Fultz, and Kelly[1] (BFK) for photon energies 21 MeV $\leq E_\gamma \leq$ 31 MeV were significantly smaller than the