Deformation Parameters of ^{152}Sm by Electron Scattering

W. Bertozzi, T. Cooper, N. Ensslin, J. Heisenberg, S. Kowalski, M. Mills, W. Turchinetz, and C. Williamson
*Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139**

and

S. P. Fivozinsky, J. W. Lightbody, Jr., and S. Penner
National Bureau of Standards, Washington, D. C. 20234
(Received 5 May 1972)

We have measured the cross sections for excitation of the ground-state rotational band in ^{152}Sm by high-energy electrons. The ground-state charge distribution has been determined including the deformation parameters β_2 and β_4 of the nuclear surface.

The determination of intrinsic deformation parameters in nuclei has been of recent interest.[1] These deformation parameters are usually obtained on the basis of the Bohr-Mottelson model which relates them to the transition probability from the ground state to the rotational level of angular momentum L. With strong-interaction probes, coupled-channel analyses using deformed optical potentials are required. In the particular case of electromagnetic transitions, the $B(EL)$ is related to the nuclear electric multipole moment $\langle E_L\rangle$ of the intrinsic state by

$$B(EL)=(2L+1)\langle E_L\rangle^2/16\pi.$$

The derivation of deformation parameters from these $\langle E_L\rangle$ requires a model; they depend on the shape of the charge distribution (mainly on the radius and the skin thickness) and therefore cannot be derived uniquely if the charge distribution is not known. For the same quadrupole moment and $\langle r^2\rangle^{1/2}$, the value of β_2 for a Fermi distribution is about 15% larger than for a uniform distribution.

Electron scattering not only gives information about the transition probabilities; it also allows one to determine the charge distribution of the ground state and therefore allows a very accurate determination of the deformation parameters within a particular parametrization.

In this Letter we report the results of an electron scattering experiment on ^{152}Sm performed at the National Bureau of Standards electron linac. The experiment was performed at incident electron energies (E) between 50 and 105 MeV and at the approximate scattering angles 93.5°, 110°, and 145°. The energy resolution of the experiment was typically 0.08% and therefore allowed a clean separation of all rotational levels (Fig. 1). The areas under the peaks were extracted using a line-shape fitting procedure accounting for radiative effects, Landau straggling, background, and the energy distribution of the incident beam. The cross section was obtained by comparison to a ^{12}C measurement, for which we assumed the charge distribution as given by Sick and McCarthy.[2]

Since the two-parameter Fermi distribution has been very successful for spherical nuclei, a deformed Fermi distribution for the intrinsic shape of ^{152}Sm was used:

$$\rho(r,\theta)=\bar{\rho}(1+\exp\{[r-R(\theta)]/t\})^{-1},$$

with

$$R(\theta)=c[1+\beta_2Y_{20}(\theta)+\beta_4Y_{40}(\theta)+\beta_6Y_{60}(\theta)+\cdots].$$

This charge distribution can be expanded into multipoles,

$$\rho(r,\theta)=\rho_0(r)Y_{00}(\theta)+\rho_2(r)Y_{20}(\theta)+\rho_4(r)Y_{40}(\theta)+\rho_6(r)Y_{60}(\theta)\cdots$$

with

$$\rho_L(r)=\int\rho(r,\theta)Y_{L0}(\theta)\,d\Omega.$$

The multipole moment $\langle E_L\rangle$ is then given by

$$\langle E_L\rangle=[16/(2L+1)]^{1/2}\int_0^\infty\rho_L(r)r^{L+2}dr.$$

In calculating the cross section from the transition charge $\rho_L(r)$ we use a distorted-wave Born approximation code taking into account the distortion of the electron wave as caused only by $\rho_0(r)$.

In this deformed Fermi model the cross sections for the elastic scattering, together with the excitations of the rotational levels, are determined by just five variables: c, t, β_2, β_4, and β_6. The excitation of the 6^+ rotational level was not observed, and the data for the 0^+, 2^+, and 4^+ states are nearly independent of β_6. Therefore

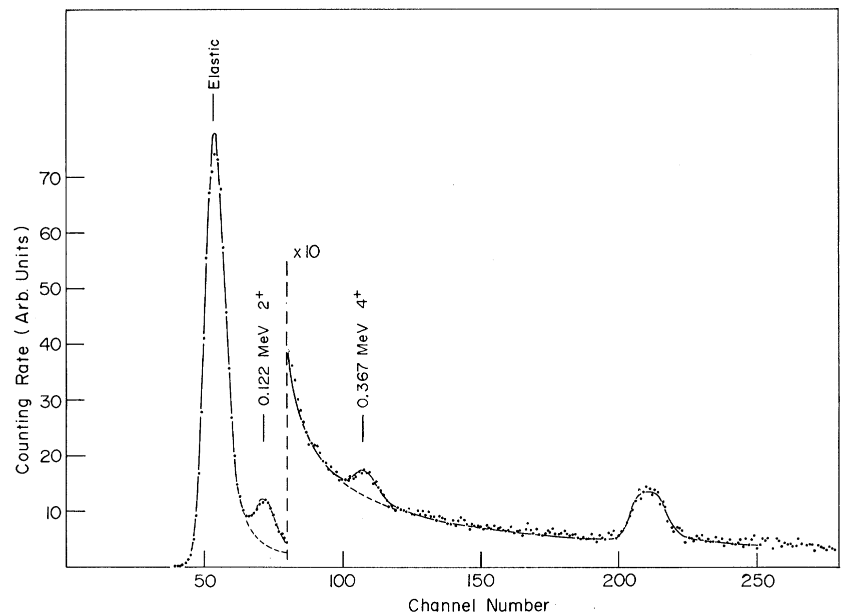

FIG. 1. Spectrum of scattered electrons from ^{152}Sm at 93.5°. Incident electron energy, 76 MeV. Besides the ground-state rotational band, the 3^- level at 1.041 MeV and the 2^+ level at 1.088 MeV are seen (channel 210).

this parameter was taken from the α-scattering results[3]: $\beta_6 = -0.012$. From μ-mesic x rays the rms radius of ^{152}Sm is known[4] to be 5.0922 fm. The value of $B(E2)$ has been measured by several methods[5] for which the average value is $B(E2) = 0.338 \times 10^5 e^2$ fm^4. By assuming these three numbers, we have three constraints on the five parameters, which leaves two degrees of freedom to be determined by this experiment.

The elastic scattering cross section depends only on $\rho_0(r)$. We can approximate ρ_0 by a Fermi distribution,

$$\rho_0(r) \sim \left[1 + \exp\left(\frac{r - c_{\rm eff}}{t_{\rm eff}}\right)\right]^{-1},$$

where $t_{\rm eff}$ is given by[6]

$$t_{\rm eff}^2 = \frac{3}{\pi^2 \rho_0(0)} \left\{ 2\int_0^\infty \rho_0(r) r\, dr - \frac{[\int_0^\infty \rho_0(r)\, dr]^2}{\rho_0(0)} \right\}.$$

Since $\langle r^2 \rangle^{1/2}$ is fixed, $t_{\rm eff}$ is determined by the elastic cross section. Finally, β_4 is fixed by fitting the 4^+ excitation cross section. The best fit is shown in Fig. 2. For the purposes of display we plot only the best-fit form factor for a scattering angle of 93.5°. For those data not taken at 93.5°, the experimental points are multiplied by the theoretical ratio $d\sigma(93.5°)/d\sigma(\theta)$ calculated from the best-fit parameters and plotted at $q_{\rm eff} = q(1 + \frac{4}{3} Z\alpha / \langle r^2 \rangle^{1/2} E)$. The cross sections are completely specified by the following quantities:

$$\langle r^2 \rangle^{1/2} = 5.0922 \text{ fm},$$

$$\langle E2 \rangle = (0.582 \pm 0.006) \times 10^2 e \text{ fm}^2,$$

$$t_{\rm eff} = 0.683 \pm 0.040 \text{ fm},$$

$$\langle E4 \rangle = (0.870 \pm 0.040) \times 10^4 e \text{ fm}^4,$$

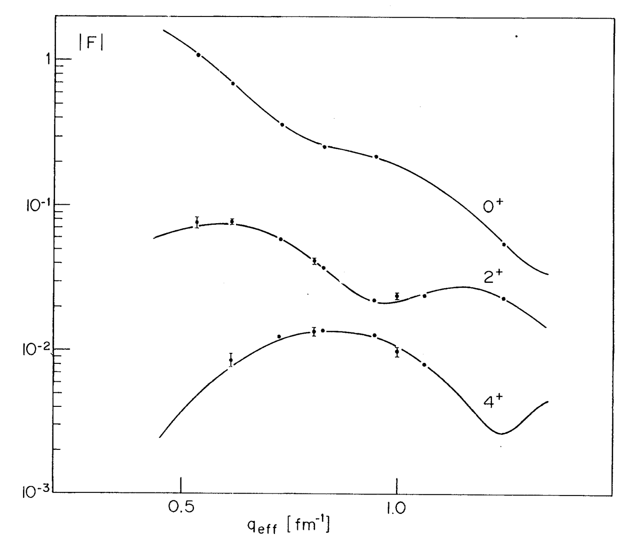

FIG. 2. Form factor F for the excitation of the ground-state rotational band; $F = (\sigma/\sigma_{\rm Mott})^{1/2}$. For $q_{\rm eff} = 1.1$ fm^{-1} the average of two experimental points is plotted. Error bars shown represent standard deviations due to counting statistics. Where not shown, the standard deviations are smaller than the plotted points.

of which the top two were used as constraints. The error in $\langle r^2\rangle^{1/2}$ is assumed to be small enough to be neglected. Using the deformed Fermi shape with $\beta_6 = -0.012$, we find $\beta_2 = 0.287 \pm 0.003$ and $\beta_4 = 0.070 \pm 0.003$. It should be emphasized that the values of $B(E2)$ and $\langle r^2\rangle^{1/2}$ from other experiments together with the elastic scattering completely fix the predictions of our model for the $0 \rightarrow 2^+$ transitions. The good agreement with experiment is an indication that our model is a close representation for these states in Sm^{152}.

The error ascribed to β_4 and thus to $\langle E4\rangle$ comes from the fact that the parametrization of the intrinsic state relates the transition radius of ρ_4 to the very accurately known rms radius and thus provides the extrapolation to the photon point. This fit is not completely satisfactory since for the 4^+ cross section gives $\chi^2 = 16$ with seven degrees of freedom and thus may indicate a systematic discrepancy. With this in mind we note that our value for $\langle E4\rangle$ is about 20% smaller than the results of Greenberg,[7] $\langle E4\rangle = (1.11 \pm 0.12)\times 10^4 e$ fm^4; and of Stephens, Diamond, and de Boer,[1] $\langle E4\rangle = (1.06 \pm 0.21)\times 10^4 e$ fm^4. These latter results are from heavy-particle Coulomb-excitation experiments and the determination of $\langle E4\rangle$ is straightforward. There are several possible causes for the present discrepancy:

(A) In our analysis we neglected dispersive corrections which may play a role because of the strong collective character of the lowest states. This is expected to be a small effect.[8]

(B) We assumed the excited states to be pure rotational states. This is only an approximation. Stokstad *et al.*,[9] show that there are admixtures of vibrational states. The consequences of such admixtures on electron scattering have not been investigated.

(C) The parametrization in terms of a deformed Fermi shape may be wrong. In fact the discrepancy can be resolved by choosing a charge distribution which gives a larger transition radius for the 4^+ excitation. One possible parametrization would be

$$\rho(r,\theta) = \bar{\rho}\left[1 + \exp\left(\frac{r - R(\theta)}{t[1 + \gamma_4 Y_{40}(\theta)]}\right)\right]^{-1}.$$

The additional parameter, γ_4, allows the transition radius of $\rho_4(r)$ to be adjusted. The best fit gives $\chi^2 = 9$ for six degrees of freedom. This fit ($\gamma_4 = 0.065 \pm 0.030$) yields $\langle E4\rangle = (0.956 \pm 0.050)\times 10^4 e$ fm^4, which agrees within the errors with the Coulomb-excitation results. Parametrizing $R(\theta)$ as before, we find that c, t, β_2, and β_4 are not changed significantly. Our results show that the $L = 4$ deformation of the nuclear surface is given by the amplitude of ρ_4 which can be measured directly by electron scattering. On the other hand, a knowledge of $\langle E4\rangle = \int \rho_4 r^6\, dr$ emphasizes the deformation at large radii and yields a deformation that depends on the specific model used.

From α-particle scattering,[3] values of $\beta_2 = 0.246$ and $\beta_4 = 0.048$ have been reported. Although these are considerably smaller than the values we infer from our data, it is not clear that a simple comparison is appropriate because the α particle probes the nuclear potential distribution, while electron scattering and Coulomb excitation probe the charge distribution.

*Work supported in part through funds provided by the U. S. Atomic Energy Commission under Contract No. AT(11-1)3069.

[1] F. S. Stephens, R. M. Diamond, and J. de Boer, Phys. Rev. Lett. 27, 1151 (1971).

[2] I. Sick and J. S. McCarthy, Nucl. Phys. A150, 631 (1970).

[3] D. L. Hendrie *et al.*, Phys. Lett. 26B, 127 (1968).

[4] D. Hitlin *et al.*, Phys. Rev. C 1, 1184 (1970).

[5] Average value taken from Refs. 1 and 4 from E. G. Funk, H. J. Prask, and J. W. Mihelich, Phys. Rev. 141, 1200 (1966).

[6] B. W. Downs, D. G. Ravenhall, and D. R. Yennie, Phys. Rev. 106, 1285 (1957).

[7] J. S. Greenberg, private communication.

[8] C. Toepffer and W. Greiner, Phys. Rev. 186, 1044 (1969).

[9] R. G. Stokstad, J. S. Greenberg, I. A. Fraser, S. H. Sie, and D. A. Bromley, Phys. Rev. Lett. 27, 748 (1971).