

has a vanishing frequency. Thus, for every temperature, the thermal population of these modes increases infinitely. Therefore anharmonic processes corresponding to the diagram

$$\text{phonon} \pm \text{magnon} \rightleftharpoons \text{magnon}$$

must become more and more important. Such a situation occurs for  $H \approx H_e'$  in the canted phase as well as in the paramagnetic phase. But, since in the canted phase this effect is superposed on the resonant interaction, it is easier to measure it for  $H \gtrsim H_e'$ . Moreover, one can easily identify on the recordings (Fig. 3) the exact point where  $H = H_e'$ : It corresponds to an abrupt change in the slope of the attenuation curve and reflects a sudden doubling change in the Brillouin magnetic zone because the two sublattices become identical.

In the paramagnetic phase, the anharmonic attenuation of acoustic waves is expressed by

$$\alpha = 4.3 \frac{2\pi}{\hbar^2 v} \sum_k \left[ |g_1(k)|^2 \frac{(1+n_k+n_{k+q})\tau/\pi}{1+\tau^2[\omega_{ac}-\omega_{sp}(k)-\omega_{sp}(k+q)]^2} + |g_2(k)|^2 \frac{(n_k-n_{k+q})\tau/\pi}{1+\tau^2[\omega_{ac}+\omega_{sp}(k)-\omega_{sp}(k+q)]^2} \right],$$

where

$$g_1(k) = \frac{1}{2} i (\hbar \omega_{ac} / 2NMv^2)^{1/2} S \left[ \left( \frac{3}{4} G_{11} - G_{44} \right) \cosh(\xi_k + \xi_{k+q}) - 3 \left( \frac{1}{4} G_{11} + G_{44} \right) \sinh(\xi_k + \xi_{k+q}) \right],$$

$$g_2(k) = i (\hbar \omega_{ac} / 2NMv^2) S \left[ \left( \frac{3}{4} G_{11} - G_{44} \right) \sinh(\xi_k + \xi_{k+q}) - 3 \left( \frac{1}{4} G_{11} + G_{44} \right) \cosh(\xi_k + \xi_{k+q}) \right];$$

$\sum_k$  is the sum over the whole Brillouin zone;  $\xi_k$  is defined by  $\tanh 2\xi \approx H_A / [2H_E(\gamma_k - 1) - H_A + 2H]$ ; and  $n_k$  is the thermal population of magnons with wave vector  $k$ . Numerical calculation gives an order of magnitude compatible with the experimental values of about 10 dB/cm.

In summary, the attenuation curves recorded in the vicinity of the  $H_e'$  point have been explained. They result from the sum of two independent absorption processes. Their different contributions have been schematically drawn in Fig. 4. The explanations, which we have provided here, do not involve the low-frequency motions of dynamic variables which obey conservation laws (longitudinal variables: magnetization and energy density).<sup>10</sup> Their influence has to be detected in experiments with lower-frequency acoustic waves.

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## Subbarrier Fission Resonances in Th Isotopes

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Subbarrier fission resonances have been found in  $^{230,232}\text{Th}(t, pf)$  and  $^{230}\text{Th}(d, pf)$  reactions. This is in disagreement with the fission barriers predicted by the Strutinskiĭ theory.

In a recent article Bolsterli *et al.*<sup>1</sup> have calculated the shapes of the double-humped fission barriers throughout the actinide region. These authors used the Strutinskiĭ procedure<sup>2</sup> of a smooth-

ly varying liquid-drop model with shell corrections calculated from a realistic diffuse-surface single-particle potential. The work concludes that a new island of stability in the region  $Z \approx 114$ ,

$N \approx 184$  probably exists. The purpose of the present study was to test the theoretical predictions for the light actinide elements, namely, the Th isotopes. The barrier shapes for the even Th isotopes calculated by different groups<sup>1,3,4</sup> show a high outer peak combined with a rather shallow second minimum and a very low intermediate barrier between the first and second minimum.

Recently Lynn, James, and Earwaker<sup>5</sup> have analyzed results from a  $^{230}\text{Th}(n, f)$  experiment and derived from these data a fission barrier which indicates roughly equal heights for the two maxima. This result does not agree with the theoretical predictions. However, since the effect of an odd particle on the fission barrier is not well understood, the present study was performed in order to search for subbarrier resonances in the even Th isotopes and thereby remove this ambiguity in the interpretation. The present Letter reports subbarrier resonance structures for the even Th isotopes  $^{234}\text{Th}$  and  $^{232}\text{Th}$ , which have not previously been detected.

Subbarrier resonances were sought for in the following reactions:  $^{230,232}\text{Th}(t, pf)$  and  $^{232,234}\text{Th}(d, pf)$ . The various beams were provided from the Los Alamos tandem Van de Graaff facility and a bombarding energy of 15 MeV was used in both the  $(d, p)$  and  $(t, p)$  reactions. In the experimental setup, coincidences were recorded between fission events from a surface-barrier annular detector and outgoing light particles, which were mass identified by means of a  $\Delta E$ - $E$  counter telescope situated at  $\approx 90^\circ$  with respect to the beam axis. The experimental setup is similar to that described by Gokhberg, Otruschenko, and Shigin.<sup>6</sup> The energy resolution obtained in the telescope system was  $\approx 70$  keV. The targets were vacuum evaporated on carbon backings and were  $\approx 200 \mu\text{g}/\text{cm}^2$  thick. The isotopic purity of the target materials was  $\approx 90\%$  and  $\approx 100\%$  for  $^{230}\text{Th}$  and  $^{232}\text{Th}$ , respectively.

As an example of the quality of the data, the measured fission probability for the  $^{230}\text{Th}(d, pf)$  reaction is plotted in Fig. 1 versus excitation energy in  $^{231}\text{Th}$ . The error bars represent statistical uncertainties and do not include systematic errors in either the fission probability or the energy calibration. The uncertainty in the absolute fission probability is estimated to be  $\pm 20\%$  and the excitation energy uncertainty is  $\pm 30$  keV. The data have been corrected for accidental coincidences, which are shown to scale in Fig. 1. A subbarrier resonance at 5.84 MeV of a width approximately equal to the energy resolution is

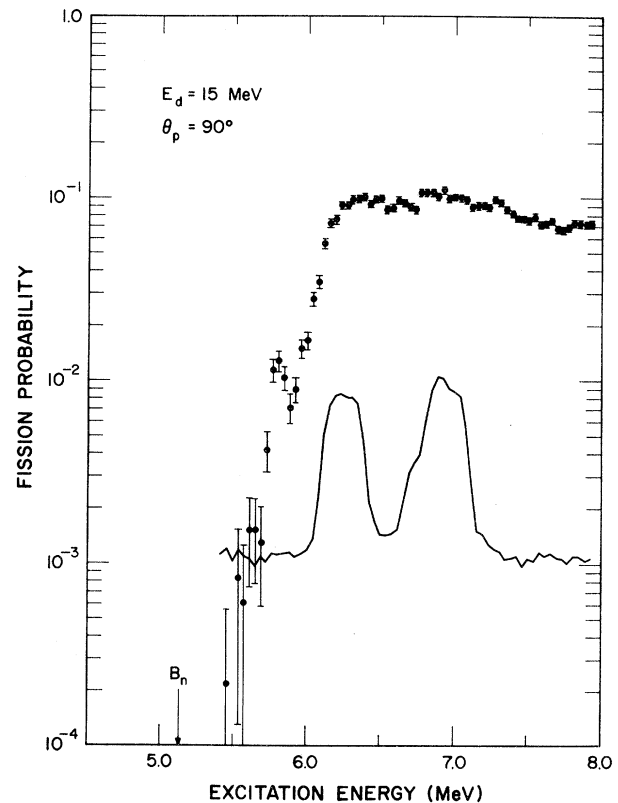


FIG. 1. Semilogarithmic plot of the measured fission probability (dots with error bars) versus excitation energy for the reaction  $^{230}\text{Th}(d, pf)$ . Accidental coincidences are represented by the solid line. The neutron binding energy is indicated by  $B_n$ .

observed in the  $^{230}\text{Th}(d, pf)$  data; this resonance has been reported previously in the  $^{230}\text{Th}(n, f)$  reaction.<sup>5,6</sup> A resonance in  $^{233}\text{Th}$  has probably

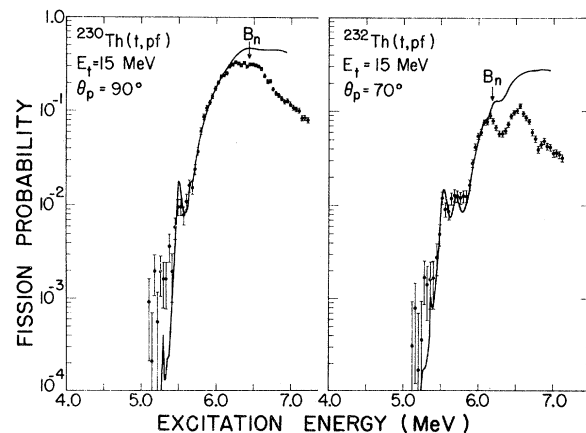


FIG. 2. Semilogarithmic plot of the measured fission probability (dots with error bars) versus excitation energy for the reactions  $^{230,232}\text{Th}(t, pf)$ . Best fits from model calculations corresponding to parameters in Table I are represented by the solid lines. The neutron binding energies are indicated by  $B_n$ .

been identified previously in the  $(n, f)$  reaction.<sup>7</sup>

Figure 2 shows the  $(t, pf)$  results. The broad resonance structure at 5.50–5.85 MeV in  $^{234}\text{Th}$  has not been reported earlier, and it also seems probable that a resonance exists near 5.3 MeV,

although the statistics in this case are poor. On the basis of these data it is not possible to decide whether the sharp decrease in the fission probability which starts at  $\approx 6.15$  MeV is caused by the onset of neutron evaporation alone, ( $B_n = 6.179$

TABLE I. Fission-barrier parameters. The energies of the first maximum, the second minimum, and the second maximum are labeled  $E_A$ ,  $E_{\text{Min}}$ , and  $E_B$ , respectively. The energy of the fission isomer is  $E_{\text{IS}}$ . All energies are relative to the ground-state energy. The curvatures  $\hbar\omega$  are labeled accordingly.

Nucleus	Ref	$E_A$ MeV	$\hbar\omega_A$ MeV	$E_{\text{IS}}$ MeV	$E_{\text{Min}}$ MeV	$\hbar\omega_{\text{Min}}$ MeV	$E_B$ MeV	$\hbar\omega_B$ MeV	$E_B - E_A$ MeV
Theory:									
$^{232}\text{Th}$	a	4.8	-	2.8	-	-	7.3	-	2.5
	b	4.3	-	1.9	1.4	-	8.6	-	4.3
	c	3.4	-	2.3	1.8	-	6.6	-	3.2
$^{234}\text{Th}$	a	5.4	-	2.6	-	-	7.4	-	2.0
	b	4.3	-	2.0	1.5	-	8.6	-	4.3
	c	3.8	-	2.3	1.8	-	6.4	-	2.6
Experiment:									
$^{231}\text{Th}$	d	6.50	0.57	4.5	-	0.6	6.15	0.90	-0.35
$^{232}\text{Th}$	Present	5.50	0.90	4.43	4.25	0.36	6.18	0.47	0.68
$^{234}\text{Th}$	Present	6.05	0.90	4.41	4.12	0.57	6.45	0.70	0.40
Theory:									
$^{240}\text{Pu}$	a	6.3	-	2.5	-	-	5.3	-	-1.0
	e	5.8	-	2.7	-	-	5.6	-	-0.20
	c	5.2	-	2.3	1.8	-	5.5	-	0.30
Experiment:									
$^{240}\text{Pu}$	f	6.05	1.00	2.95	2.35	1.20	5.55	0.70	-0.50
	g	5.50	1.25	2.30	-	-	5.35	0.60	-0.15

<sup>a</sup>Ref. 3. Mass asymmetry at the second maximum is included. Parameters from Table 2A in this reference. It is not clear what value of the zero-point energies in the two minima has been used in this case.

<sup>b</sup>Values from Ref. 7 but corrected for mass asymmetry at the second peak and include hexadecapole deformation at the ground-state minimum. A zero-point energy of 0.5 MeV is assumed for both the ground and isomeric states.

<sup>c</sup>Ref. 4. Mass asymmetry at the second maximum is included. A zero-point energy of 0.5 MeV is assumed for both the ground and isomeric states.

<sup>d</sup>Ref. 5. Unlike these authors we have assumed a smaller and thinner first barrier, which is a qualitative result of the theory.

<sup>e</sup>Ref. 1. A zero-point energy of 0.5 MeV is assumed for both the ground and isomeric states.

<sup>f</sup>Ref. 8.

<sup>g</sup>Parameters extracted from model calculations of the excitation function for populating the fission isomer. H. C. Britt, S. C. Burnett, B. H. Erkkila, J. E. Lynn, and W. E. Stein, Phys. Rev. C 4, 1444 (1971); H. C. Britt, private communication.

MeV) or if it is caused by a fission resonance at  $\approx 6.15$  MeV. A weak resonance possibly occurs at 5.50 MeV in  $^{232}\text{Th}$ , although it is not as pronounced as in the  $^{234}\text{Th}$  case. It is concluded that subbarrier resonances occur systematically in the Th isotopes.

The data have been analyzed using a model for calculating the fission probability which assumes a two-humped fission barrier. This model is essentially the same as previously used in the analysis of  $(d, pf)$  data.<sup>8</sup> The best fits to the data, obtained from these calculations by varying the parameters of the double-humped barrier, are shown in Fig. 2 in comparison with the data. The relative order of the opening of the fission channels has been taken from Cramer and Britt.<sup>9</sup> From the calculations it was found that the difference in height of the two peaks of the fission barrier cannot exceed  $\approx 1$  MeV in the  $^{232}\text{Th}$  and  $^{234}\text{Th}$  cases if the resonances in these nuclei should be reproduced with the observed strength (see Table I). The theoretical calculations give barrier differences between 2.0 and 4.3 MeV. It may be noted that the  $^{231}\text{Th}$  barrier parameters are quite similar to the even Th numbers (Table I).

Theoretical and experimental barrier parameters for the Th and Pu isotopes are shown in Table I. It is seen that the overall agreement between theory and experiment for the Pu isotopes is good, but this is not the case for the Th isotopes. The trend is that the experimental values of the height of the two peaks do not vary much when going from Pu to Th, whereas the theory predicts a strong decrease of the first peak.

It would be desirable to obtain accurate fission

probabilities versus excitation energy for the heavy actinide region in order to learn whether the failure in the theoretical predictions is confined to the Th region or if it persists above Pu. Such experiments, using the  $(^3\text{He}, df)$  reaction are in progress in this laboratory.

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