a zero-order spatially uniform distribution. The characteristic time taken to achieve steady state in a system of length L is of the same order of magnitude as the time taken by a given vortex ring to travel the distance L in the presence of the electric field due to the rest. Since $dE = ne^2 x\,dx$, using (2) we have that $v(x) = \beta/ne^2x^{-2}$. Thus the time to travel the distance L is $t_{\rm eq}(L) = \frac{1}{3}ne^2\beta L^3$. This is to be compared with the time for the steady flow of vortex rings, $t_{\rm transit}(L) = L/v$. Thus for a long enough system so that $t_{\rm eq}(L) \gg t_{\rm transit}(L)$, the distribution will remain approximately uniform.

[8]For example, T. H. Stix, *The Theory of Plasma Waves* (McGraw-Hill, New York, 1962), p. 110.

Growing Collective Modes in Vortex Ring Beams in He II

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Growing collective modes are observed in beams of charged vortex rings in superfluid helium. The growth of the waves is found to be dependent upon the path length of the beam and the modulation frequency, in agreement with theory.

In previous work[1] it has been shown that a pulsed beam of charged quantized vortex rings displays collective behavior and evolves into a steady state. This follows from the counteracting effect of the Coulomb field and negative effective mass of the vortex ring.[2] The mutual interactions that lead to such behavior are conducive to possible excitation of collective modes of oscillation. In the preceding paper[3] Hasegawa and Varma derive the dispersion relation for such waves and predict that the wave amplitude should grow within a certain range of frequencies and be dependent upon the distance travelled. In the present experiment we have observed these collective modes for the first time. To create them we produce a small density perturbation by modulating the energy of the beam and then observe the growth of the density perturbation as a function of frequency and drift space. The results are found to be in agreement with the theoretical predictions.

The experimental cell is immersed in superfluid helium ($T \sim 0.3$ K) and is shown in Fig. 1. Four grids, G_1–G_4, are spaced between a radioactive source and a guarded collector. G_1 is 5.7 mm from the source; G_2 and G_3 are spaced by 0.13 mm and are located between G_1 and G_4, the latter being 1 mm away from the collector C and acting as an electrical guard. Runs were taken with the distance L, between G_3 and G_4, equal to 27.5 and 8.8 mm.

The electrical connections are shown in Fig. 1. G_3 and G_4 are grounded while a small sinusoidal voltage $V_{\rm ac}$ is applied to G_1 and G_2, which are connected together, and a dc voltage $V_{\rm dc}$ is applied between G_1 and the source. Thus, charged vortex rings, created near the source, acquire energy $eV_{\rm dc}$ as they pass through G_1. They continue to move through the field-free region and pass G_2. Because the distance between G_2 and G_3 is very small compared to the distance vortex rings travel in one period of the highest experimentally applied frequency, the electric field between G_2 and G_3 can be considered constant and vortices become velocity modulated as a result of their energy change. Between G_3 and G_4 is again a field-free region where the velocity-modulated beam propagates and then, after passing G_4, arrives finally at the collector. With the aid of a fast electrometer[4] and a signal averager, one can then easily observe the modulation in the signal current. Typically, the dc current $I_{\rm dc}$ is $\gtrsim 5\times10^{-13}$ A while the modulation $I_{\rm ac}$ (peak to

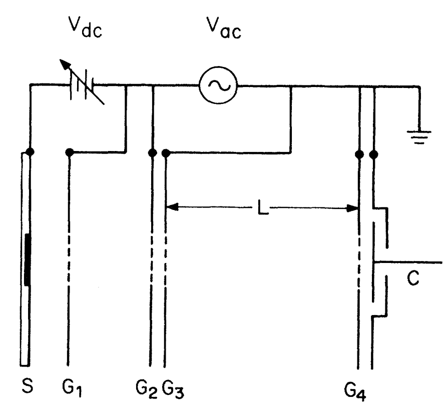

FIG. 1. Schematic diagram of the experimental cell. The cell is cylindrical in shape and is ~4 in. long and 3 in. in diameter.

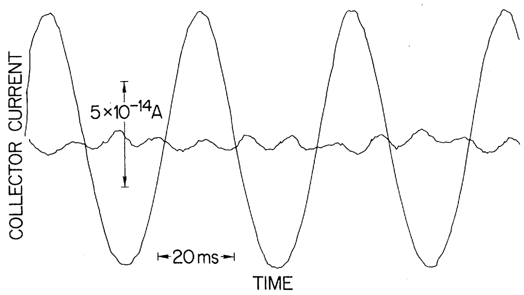

FIG. 2. Typical current signal received at the collector. The trace in the middle is obtained when the vortex beam is off but the ac is on. The sine wave is obtained for a 20-eV beam, $V_{ac} = 0.2$ V and $f = 25.5$ Hz.

peak) is kept less than 30% of I_{dc} to avoid operation in a nonlinear regime due to saturation.[5] In Fig. 2 we show a typical received modulated current signal with the beam on and off. Keeping the ac on but by applying appropriate dc potentials, we can shut off the vortex beam and observe no coherent signal corresponding to the modulation frequency, indicative of good electrical shielding.

We define amplification of the wave by the ratio,

$$A \equiv (I_{ac}/I_{dc})(V_{ac}/V_{dc})^{-1}, \tag{1}$$

and show plots of A versus frequency f for several different energies in Fig. 3. Overall, the data show that as the frequency increases, A first increases linearly, rises to a maximum, then decreases. Curves A and B are obtained for $L = 27.5$ mm and curve C is for $L = 8.8$ mm.

To compare our results with the theory in the preceding Letter, we start with Eq. (9) therein and linearize the expression for the current $J = nev$; then we assume a wavelike form for the current density J, number density n, and velocity of the beam v,

$$\begin{pmatrix} J(x,t) \\ n(x,t) \\ v(x,t) \end{pmatrix} = \begin{pmatrix} J \\ n \\ v \end{pmatrix} e^{i(kx-\omega t)}. \tag{2}$$

Now defining small changes in J, n, and v as J_1, n_1, and v_1 we obtain

$$J_1 = n_1 e\bar{v} + \bar{n} e v_1. \tag{3}$$

Using Eqs. (2) and (3) as well as $\bar{J} = \bar{n}e\bar{v}$ ($\bar{J}$, $\bar{n}$, and $\bar{v}$ are steady-state dc components) we find a relationship between velocity and current-density modulation,

$$\frac{J_1}{J} = \frac{v_1}{\bar{v}} \frac{1}{2} \left| \frac{\exp(ik_1 x)}{(1 - k_1\bar{v}/\omega)} + \frac{\exp(ik_2 x)}{(1 - k_2\bar{v}/\omega)} \right| , \tag{4}$$

where k_1 and k_2 are roots from Eq. (10), Ref. 3.

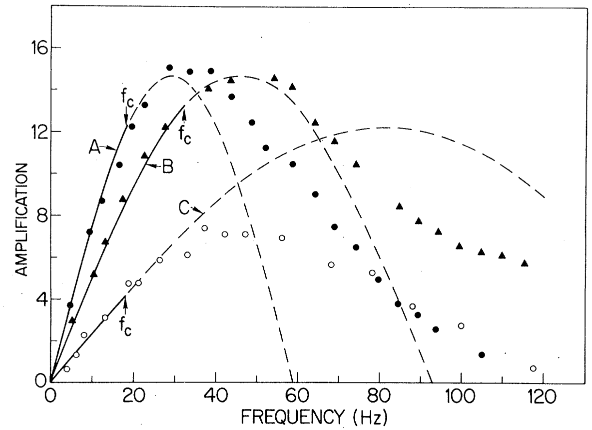

FIG. 3. Amplification versus frequency. The points are experimental and the lines are fits by Eq. (5). Curves A and B are obtained with 20- and 10-eV vortex beams, respectively, and $L = 27.5$ mm, while curve C is for a 20-eV beam but $L = 8.8$ mm. The theoretical curves are fitted with following parameters: $A - x = 17$ mm, $\omega_0 = 9.4$ sec^{-1}, $\omega_c = 113$ sec^{-1}; $B - x = 17$ mm, $\omega_0 = 17.6$ sec^{-1}, $\omega_c = 201$ sec^{-1}; $C = x = 6$ mm, $\omega_0 = 9.4$ sec^{-1}, and $\omega_c = 113$ sec^{-1}. Solid and dashed lines, for frequencies less than and greater than ω_c, respectively.

Unfortunately, at the present time we cannot solve directly Eq. (10) and must use the approximate k values extracted from Eq. (19), Ref. 3. At low frequencies, especially for $\omega < \omega_c$ (ω_c is defined when $k_1 = k_2$), Eq. (19) is probably reasonably accurate; however, for frequencies $\omega > \omega_c$ Eq. (10) must be used since the approximations leading to Eq. (19) are no longer valid. Nevertheless, if we use those roots as well as constants ω_0 and ω_c defined in Ref. 3, we can simplify Eq. (4) to read

$$\left(\frac{J_1}{J}\right) = \frac{v_1}{v} \frac{1}{\Delta} \sinh\left(\frac{\Delta \omega x}{\bar{v}}\right) , \tag{5}$$

where

$$\Delta \equiv (\omega_0/\omega_c)(\omega_c^2/\omega^2 - 1)^{1/2}.$$

Let us now try to fit our data, especially in the linear regime, by letting x, ω_0, and ω_c be adjustable parameters. By virtue of Eq. (1), Ref. 3, the amplification A can be written as

$$A = (J_1/\bar{J})(v_1/\bar{v})^{-1}. \tag{6}$$

The solid lines in Fig. 3 represent our best fits to the data. In each case we find that we can approximate reasonably well the linear portion of each curve with constants that are in accord with theoretical estimates. We find, for instance, that $\omega_0/\omega_c \cong 0.08$, independent of energy and drift space, in agreement with the earlier work found in Ref. 1.[6] The absolute value of the plasma frequency (e.g., taking the 20-eV beam we find $\omega_0 = 9.4\ \mathrm{sec}^{-1}$) agrees with Hasegawa and Varma's estimate of $\omega_0 = 10\ \mathrm{sec}^{-1}$. We also find that $\omega_0 \propto \bar{v}$, in agreement with theory, although it should be pointed out that the fit is only very sensitive to the ratio of ω_0 to ω_c and the value of x, and much less sensitive to ω_0. The values of the drift space x, obtained from the fit, are somewhat smaller than measured: 17 mm compared with 27.5 mm, and 6 mm compared with 8.8 mm. The reason for this difference between measured and calculated values of the drift space can be due to several things. Most important is beam spreading which will tend to limit A to a somewhat smaller value than predicted by the one-dimensional theoretical model. This is borne out by the fact that with the short drift space the fractional discrepancy in x is smaller than for the long space where beam spreading is more significant.

For frequencies much higher than ω_c our data show A slowly decreasing to zero with no apparent periodicity. On the other hand, we note that Eq. (5) becomes a sine function multiplied by a constant factor. (We show with the dashed line only one period of each curve.) The periodicity we believe is an artifact which occurs when we use the wrong k values; nevertheless, we note that the general character of the curve of A versus f within one period is qualitatively similar to that observed. Most of the data were taken with $V_{ac}/V_{dc} = 0.02$; however, similar curves were obtained with values of V_{ac}/V_{dc} from 0.005 up to 0.05, and in all cases A was found to be independent of V_{ac}, indicating that the origin of this phenomenon is not due to klystronlike behavior.

In summary, we have observed growing collective modes of oscillation in vortex-ring beams which are produced by modulating the energy of the beam. The amplitude of these waves as a function of frequency and drift space agrees with theoretical predictions for low frequencies. At higher frequencies, a direct comparison cannot be made until the numerical solution of Eq. (10), Ref. 3, is obtained.[7]

In closing, it would be interesting to investigate this problem with a collimated beam, higher densities, and longer drift space to obtain bigger values of A. At present, the limit in A is not L but probably beam spreading which changes the density as the beam propagates through the field-free region.

I wish to thank Akira Hasegawa and C. M. Varma for many informative discussions and H. W. Dail for collecting the data.

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[1]G. Gamota, A. Hasegawa, and C. M. Varma, Phys. Rev. Lett. 26, 960 (1971).

[2]Negative effective mass rather than negative pressure, which was emphasized in Ref. 1, is more fundamental to the observed phenomena.

[3]A. Hasegawa and C. M. Varma, preceding Letter [Phys. Rev. Lett. 28, 1689 (1972)].

[4]The rise time of our electrometer is ~ 1 msec and the wide-band noise is $\sim 10^{-12}$ A. See W. H. Wing and T. M. Sanders, Jr., Rev. Sci. Instrum. 38, 1341 (1967), for use of operational amplifiers as fast electrometers.

[5]Most of the data were obtained with $V_{ac}/V_{dc} = 0.02$ so that with maximum $A \approx 15$ we would have at most $I_{ac}/I_{dc} \sim 0.3$.

[6]From Ref. 3 we have Eq. (20), $\omega_c = (\omega_0/v_T)\bar{v}/\sqrt{3}$. The quantity ω_0/v_T can be redefined in terms of just one parameter, Δ_f, which is the spatial half-width measured in Ref. 1. From that experiment, we know that Δ_f to first order is insensitive to changes in vortex-ring energy; thus one expects $\omega_0 \propto \bar{v}$ and ω_0/ω_c to be a constant.

[7]A. Hasegawa, to be published.