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Enhanced Scattering Inherent to "Loss-Cone" Particle Distributions*

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Particle scattering due to a Rosenbluth-Post convective loss-cone instability is calculated using a slab model. This collective contribution to the scattering rate is found to be the classical scattering rate increased by a factor $(T_i/T_e)^{3/2}(m_e/m_i)^{1/2}(\ln\Lambda)^{-1}\times$(energy amplification of the convective instability in the finite plasma).

Because of their loss-cone distribution functions, plasmas confined in open-ended configurations have been shown by Rosenbluth and Post (RP)[1,2] to be inherently subject to an instability which convects along the magnetic field $\vec{B}$ with wavelengths perpendicular to $\vec{B}$ short compared to the ion gyroradius. They obtained a stability criterion based on the limit of ten e foldings of the instability in the length of the plasma. Current mirror experiments satisfy this criterion.

In such mirror experiments as 2X[3] and 2X-II,[4] under the most favorable conditions, the decaying plasma is apparently quiescent. However, the density decay rate is always a few times the classical one and is relatively insensitive to ion temperature. A possible cause of this anomalous loss is that, although the machines satisfy the RP ten-e-folding criterion, there may exist a level of fluctuation over the classical value, the source of which is the two-particle scattering process amplified by the appropriate convective growth. Such fluctuations would be expected to be more effective at scattering ions than ion-cyclotron instabilities of the same amplitude because of their shorter wavelength and the occurrence of ion-wave resonance. It is our purpose to describe the fluctuation level and the associated particle scattering rates for this process.

Such a calculation of the fluctuation level and scattering due to a convective instability in a finite plasma differs in principle from quasilinear or other nonlinear theories for absolute (standing wave) instabilities (or convective instabilities in infinite plasmas). In the latter case, growth is stopped by ion scattering modifying the distribution function. In a finite plasma which is only convectively unstable the fluctuation level is automatically limited,[5] and one can envisage (with a source of ions) a steady state with enhanced scattering into the loss cone. The familiar expansion of kinetic theory holds with the small parameter (number of particles in a Debye sphere)$^{-1}$ increased by the energy amplification of a wave in crossing the plasma.

The fluctuation level of an unstable drift-type wave convecting across the magnetic field has been obtained by Kent and Taylor,[6] although they do not obtain the accompanying particle scattering. Because the wave we are interested in has a parallel group velocity greater than that of any particle (which is to say that the wave is convective in the frame of all particles), our result is similar to theirs with alteration in the direction of propagation. The method is based upon constructing a two-particle correlation function by a superposition of uncorrelated but dressed test particles, the principles of which were described by Rostoker.[7-9] In constructing the field due to a test particle, one treats the plasma as locally uniform near the test particle, retains only that part of the field which survives asymptotically (the unstable wave), and then joins this asymptotic solution to the eikonal solution appropriate to the slowly varying equilibrium.[10]

As a model we consider a plasma uniform in x-y and varying in z with $\vec{B}=B(z)\vec{e}_z$. Performing Fourier-Laplace transforms in x-y, t $[\vec{E} \sim \exp(i\vec{k}_\perp\cdot\vec{r}-i\omega t)]$, the eikonal solution in z will yield a local dispersion relation $\epsilon(\omega,\vec{k}_\perp,k_\parallel,z)=0$ which fixes $k_\parallel=k_0(\omega,\vec{k}_\perp,z)$. For the parameters of interest we follow RP in taking the ion (electron) Larmor radius to be infinite (zero). Thus in the wave, the electrons move only along $\vec{B}$, ions move independent of $\vec{B}$, and the local plasma dielectric constant is given by

$$\epsilon(\omega,\vec{k}_\perp,k_\parallel,z)=1+\frac{\omega_{pe}^2(z)}{\omega_{ce}^2(z)}+\frac{\omega_{pe}^2}{k_\perp^2}\int d^3v\,\frac{k_\parallel\partial f_e(\vec{v},z)/\partial v_\parallel}{\omega-k_\parallel v_\parallel}+\frac{\omega_{pi}^2}{k_\perp^2}\int d^3v\,\frac{\vec{k}_\perp\cdot\partial f_i(\vec{v},z)/\partial\vec{v}}{\omega-\vec{k}_\perp\cdot\vec{v}}, \tag{1}$$

where ω_{pi} (ω_{pe}) is the ion (electron) plasma frequency and ω_{ce} the electron gyrofrequency. We neglect parallel ion motion because $k_0 \ll k_\perp$. Because in most mirror machines the ion-electron temperature ratio is large (e.g., $T_i/T_e\sim 30-40$ in 2X and 2X-II, $\mathrm{Re}k_0$ is determined by the electrons. For small $\mathrm{Re}k_0$

$$\mathrm{Re}k_0=k_\perp(\omega/\omega_{pe})(1+\omega_{pe}^2/\omega_{ce}^2)^{1/2}, \tag{2}$$

so that as long as $k_\perp<k_{De}(1+\omega_{pe}^2/\omega_{ce}^2)^{-1/2}$, where k_{De} is the inverse electron Debye length, electron damping may be neglected, and $\mathrm{Im}k_0$ is determined by the ions

$$\mathrm{Im}k_0=-\frac{\pi}{2}\frac{\omega_{pe}\omega}{k_\perp}\left(1+\frac{\omega_{pe}^2}{\omega_{ce}^2}\right)^{-1/2}\sum_i\frac{m_e}{m_i}\int d^3v\vec{k}_\perp\cdot\frac{\partial f_i}{\partial\vec{v}}\,\delta(\omega-\vec{k}_\perp\cdot\vec{v}). \tag{3}$$

For an ion loss-cone distribution, $\mathrm{Im}k_0$ can be negative, so that such waves grow convectively until the density falls sufficiently that electron Landau damping sets in.

In the large-ion-gyroradius limit for general instabilities convecting rapidly down the field line, we find (cf. Ref. 5)

$$\left(\frac{\partial f_i}{\partial t}\right)_{\text{ion-ion collective}}=\frac{\partial}{\partial\vec{v}}\cdot\overleftrightarrow{B}_i(\vec{v},z)\cdot\frac{\partial f_i}{\partial\vec{v}}, \tag{4}$$

where

$$\overleftrightarrow{B}_i(\vec{v},z)=\frac{16\pi^3q_i^2}{m_i^2}\sum_j n_jq_j^2\int\frac{d^2k_\perp}{(2\pi)^2}\frac{\vec{k}_\perp\vec{k}_\perp}{k_\perp^4}\int d^3v'\int dz' f_j(\vec{v}',z')\delta(\omega-\vec{k}_\perp\cdot\vec{v}')$$
$$\times\left.\frac{\exp[-2\,\mathrm{sgn}(z-z')\int_{z'}^{z}\mathrm{Im}k_0(z'')\,dz'']}{|\partial\epsilon(z)/\partial k_\parallel|_{k_\parallel=k_0(z)}|\,|\partial\epsilon(z')/\partial k_\parallel|_{k_\parallel=k_0(z')}|}\right|_{\omega=\vec{k}_\perp\cdot\vec{v}} \tag{5}$$

in which the sum in j runs over ions species. The manner in which this result is obtained from the superposition of two test particles is apparent. We emphasize that this general form holds only if both the test and field particles travel slower than the wave energy. There is no corresponding electron contribution to the enhanced spectrum: Any such waves emitted by electrons are subject to electron Landau damping, and so are stable and just lead to classical electron scattering.

For reasons mentioned below, we defer quantitative evaluation of (5) and determine only its scaling with plasma parameters by introducing dimensionless variables in the integrals. From (3) we note that

$$\mathrm{Im}k_0=\alpha(\omega_{pe}/\omega_{ce}a_i)(1+\omega_{pe}^2/\omega_{ce}^2)^{-1/2}, \tag{6}$$

where a_i is the Larmor radius and α $\{\lesssim\frac{1}{2}[yF(y)]_{\max\,\mathrm{neg}}$ in the notation of Ref. 2$\}$ is a constant which depends on the ion distribution function. For a collisional distribution α is about 0.1, but for peaked distributions it is much larger.[2] We note from (3) that, when $\omega=\vec{k}_\perp\cdot\vec{v}$, $\mathrm{Im}k_0$ is independent of $k_\perp$, so that the upper limit of the $k_\perp$ integration in (5) is set by the onset of electron Landau damping. Using (2), $\partial\epsilon/\partial k_\parallel=-2\omega_{pe}^2k_\parallel/\omega^2k_\perp^2$; and in (5) introducing the dimensionless variables $\tilde{k}_\perp=k_\perp k_{De}^{-1}(1+\omega_{pe}^2/$

$\omega_{ce}{}^2)^{1/2}$, $\tilde{\vec{v}}=\vec{v}/v_{Ti}$, and $\tilde{z}=z\,\mathrm{Im}k_0$, we may estimate for (4)

$$\left(\frac{d\ln f}{dt}\right)_{\text{ion-ion collective}}\sim\frac{1}{\tau_{sp}}\left(\frac{T_i}{T_e}\right)^{3/2}\left(\frac{m_e}{m_i}\right)^{1/2}\frac{1}{(1+\omega_{pe}{}^2/\omega_{ce}{}^2)^2}\,\frac{A}{\ln\Lambda},\tag{7}$$

where we have normalized to the classical ion-ion scattering (Spitzer) time $\tau_{sp}{}^{-1}\equiv4\pi ne^4\ln(\Lambda)/m_i{}^{1/2}T_i{}^{3/2}$; $\ln\Lambda$ is the Coulomb logarithm. Here A is a numerical factor, *including the exponential,* resulting from the integral in (5) in dimensionless variables. When there is significant growth, the result in (5) is z dependent, being sharply peaked near the axial ends of the plasma. Averaging over a plasma length L we find that, for this latter case, A scales as $\exp[-2\int_{-L/2}^{L/2}dz\times\mathrm{Im}k_0(z)]/\mathrm{Im}k_0L$.

Whether scattering of this type dominates or is dominated by classical scattering of time scale τ_{sp} depends on the magnitude of the numerical coefficent. In 2X and 2X-II the temperature ratio term offsets the mass ratio term by a factor of 3. Also, the plasma is about 60 gyroradii long and $2>\omega_{pe}/\omega_{ce}>0.3$, so that using simple midplane values we obtain an exponent of 3 to 5 in the amplification. Our result is of the order of magnitude as that observed in 2X and 2X-II and except for the convective growth part has the weak dependence on ion energy. A more quantitative comparison is limited by the model selected. We may expect the slab model to adequately represent an axisymmetric mirror within which a proper eikonal treatment of converging field lines would yield a $k_\perp$ varying as $B^{1/2}(z)$ along a field line. In a minimum-B well, $k_\perp$ dilates as the line elements in moving into the fans, and in fact changes by a factor of 5 in 2X-II. The effective length over which a wave avoids Landau damping would be correspondingly reduced. This effect is under investigation, and quantitative comparison with 2X-II awaits this result.

Note that enhanced scattering may occur in a stable (non-loss-cone) plasma with low electron temperature, wherein the enhancement $(T_i/T_e)^{3/2}\times(m_e/m_i)^{1/2}(\ln\Lambda)^{-1}$ still holds. This result may be obtained from the Balescu-Lenard collision integral[11] by allowing for a root of $\epsilon=0$ near the real $k_\parallel$ axis, or from (5) by integrating by parts in z', because then $\mathrm{Im}k_0>0$. In this case $A\sim1$ if the waves are damped, but $A\to\infty$ as the waves become marginally stable—indicating the importance of finite geometry effects. This enhanced scattering in a stable magnetized plasma can be shown to not change the parallel or perpendicular energy (to leading order in m_e/m_i). Our result is therefore not in contradiction to the results of Ichimaru and Rosenbluth,[12] who consider temperature-change rates in stable magnetized plasmas and find little change from the classical result.

In summary, we have described a new, irreversible and irreducible scattering mechanism for ions confined in magnetic mirror machines. The calculation describes the transition from classical ion scattering to the case when the RP convective instability[1,2] leads to rapid plasma loss.

More generally, we expect that a similar enhancement of scattering (or diffusion) will occur in other circumstances, wherein a convective instability is deemed benign because of limited amplification. Because such scattering is due to collective wave (convective microinstability) enhancement of a part of the fluctuation spectrum generated by the two-particle scattering process, it might be considered a "quasiclassical" scattering process.

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