to the backflow.[7] (Using the viscosity coefficients of MBBA,[8] one gets $A_1 = 10^{-3}$, $A_3 \sim 0.75$.) The results of the coupled equations (10) and (11) are given in Fig. 1 for geometry 3. The factor $X = kd/2$ (curve a) gives the wave vector of the distortion compared with the static value $k = \pi/d$. The fastest increase (case A) and the slowest decay (case B) of θ_M, which are seen in the experiments, are both given by the smallest X solution [see Eq. (10)] which corresponds to the weakest distortion. In geometry 2, where there is no backflow $(A_2 = \lambda_2 = 0)$, the time constant $s_0^{-1}(h)$ can be calculated directly from (6);

$$s_0(h) = \chi_a H_c^2 (h^2 - 1)/\gamma_1. \tag{12}$$

The variation of $s(h)$ is plotted in terms of the ratio (curve b)

$$s_0(h)/s(h) = \gamma_1^*(h)/\gamma_1. \tag{13}$$

The variation of the effective viscosity $\gamma_1^*(h)$ should not be observable in our experiments where $0 < h < 2$. We thus expect, using (12) and (13),

$$\frac{s(h)}{|s(h=0)|} = \frac{\chi_a(h^2-1)H_c^2}{\gamma_1^*} \frac{\gamma_1^*}{\chi_a H_c^2} = h^2 - 1,$$

as was accurately obtained in the experiments, shown in Fig. 3.

Finally, we determine an absolute value of γ_1^*

from the exponential variation of N versus t when a field is suppressed (see inset of Fig. 3). Using $\chi_a \simeq 1.2 \times 10^{-7}$ and the correction $\gamma_1 = 1.16\gamma_1^*$ (see Fig. 1, curve b), we get $\gamma_1 = 1.95$ at 16°C and $\gamma_1 = 1.25$ at 22°C, in agreement with other temperature-dependent results.[9] More detailed results and data for geometry 1 will be given elsewhere.[2]

It is a pleasure to acknowledge critical discussions with P. G. de Gennes and L. Léger.

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[6]In case A, $\theta_M^2(0)$ is the mean square value of the thermal fluctuations which induces a transition from a maximum of F for $\theta = 0$ to the equilibrium state. The equipartition theorem leads to $\theta_M^2(0) \sim a/d$, where a is a molecular dimension. Other contributions come from residual misalignment of H or in the film (Ref. 2).

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Solid Surface Shape and the Alignment of an Adjacent Nematic Liquid Crystal

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We show that elastic strain energy may account for the tendency of some nematic liquid crystals to lie parallel to the direction of rubbing on a solid surface that has been slightly deformed by rubbing, or perpendicular to a surface that is slightly rough in two dimensions.

It is well known that the directors of many nematic liquid crystals tend to assume an orientation parallel to the direction in which an adjacent solid surface has previously been rubbed. Such rubbing may be done on glass with fresh dry lens tissue or cloth, or with a polishing lap charged with jeweler's rouge or other polishing material. Dreyer[1] reported that parallel alignment sometimes occurs even in replicas of rubbed surfaces. This observation suggests that one mechanism for such alignment is based primarily on geomet-

rical factors rather than detailed molecular forces. A simple explanation of orientation parallel to the rubbing direction can be made on the basis of the additional elastic energy that would occur in a nematic liquid crystal due to distortion near a gratinglike wavy surface if the liquid crystal molecules were forced to lie against the surface with directors lying across, rather than parallel to, the grooves and ridges. If the surface is rough in both dimensions, such energy considerations would explain a tendency for the molecules

to line up normal to the surface even when they would lie flat on a perfectly smooth surface of the same material at the same temperature.

A different mechanism of surface alignment is involved when a monolayer of long molecules that are wetted by a nematic are bonded in some particular orientation to a surface. This may occur intentionally or accidentally when a surface is rubbed with an impregnated cloth or paper. However, the elastic-energy mechanism is likely to be dominant in many cases. At the end of this paper we describe experiments suggesting the effects of both mechanisms.

First, let us consider the effect of a gratinglike deformation of surface shape caused by rubbing. Suppose that both ends of each liquid crystal molecule have affinity for the material on the surface so that they tend to lie flat against the surface rather than perpendicular to it if it is smooth. Suppose that rubbing the surface scribes small grooves or deposits small threads on the surface giving a shape that can be approximately described by a sinusoidal wave

$$z \approx A \sin qx. \tag{1}$$

The principle of the energy dependence on surface shape and molecular orientation can be illustrated with the following over-simplified model.

Consider a nematic liquid crystal that obeys the Oseen[2]-Frank[3] elastic continuum theory. For simplicity, suppose that the bend and splay elastic constants, k_{11} and k_{33}, are alike. Let $\varphi(x, z)$ be the azimuth of molecules if they lie in the x, z plane normal to the grooves and ridges on the surface. The minimum energy for such a material with directors oriented in this plane would occur if the azimuth φ obeys the equation[2,3]

$$\partial^2 \varphi / \partial x^2 + \partial^2 \varphi / \partial z^2 = 0. \tag{2}$$

A solution to this equation that would match a gratinglike boundary is

$$\varphi(x, z) = Aq \cos(qx) e^{-qz}. \tag{3}$$

Notice that the surface defined by Eq. (1) coincides quite closely with the directors in Eq. (3) if the dimensionless product Aq is not large. Since the shape of the supposed gratinglike surface is somewhat arbitrary anyway, it can be altered slightly in the reader's imagination to fit the directors in Eq. (3) exactly.

The energy density due to elastic strain in this simplified nematic liquid crystal model is

$$u_\perp = \frac{k_{11}}{2} \left[\left(\frac{\partial \varphi}{\partial x} \right)^2 + \left(\frac{\partial \varphi}{\partial z} \right)^2 \right]$$

$$= \frac{k_{11}}{2} (Aq)^2 q^2 \exp(-2qz); \tag{4}$$

and the total energy per unit area, which is largely confined within a distance $(2q)^{-1}$ of the surface, is

$$\rho_\perp = \int_0^\infty u(z) dz = \tfrac{1}{4} k_{11}(Aq)^2 q. \tag{5}$$

If the molecules lie parallel to the ridges, there is no strain energy near the surface. u and ρ represent extra elastic energy that can be removed by reorientation of the molecule to a position parallel to the ridges. The problem of the energy at intermediate orientations requires solution of very complicated equations that involve the twist elastic constant[3] k_{22} as well. If we make the further assumptions that all three elastic constants are alike and that solutions to the equations involve negligibly small variation of the angle θ between the director and the y axis as the azimuth φ varies, then it is possible to show that

$$u(z, \theta) = u_\perp(z) \sin^2\theta, \tag{6}$$

$$\rho(\theta) = \rho_\perp \sin^2\theta. \tag{7}$$

However, the detailed behavior of the energy as the molecules go through a hypothetical reorientation from $\theta = 90°$ to the parallel orientation, $\theta = 0°$ or $180°$, is not very important to the present argument.

A very good optically polished glass or fused quartz surface has roughness amplitude of about 10 Å,[4] but the autocorrelation length of the roughness is on the order of several hundred angstroms. We suppose that rubbing with certain materials does something to make more closely spaced, parallel roughness but does not greatly increase the amplitude. There is no visible deterioration of the optical finish when polished glass or fused quartz is rubbed in such a way as to cause it to align a nematic sample. Electron micrographs of replicas of rubbed glass surfaces show a wide variety of forms. As an illustration we shall suppose that rubbing the surface results in approximately sinusoidal waves of length $2\pi/q = 200$ Å and that the amplitude is 10 Å. Then the shape factor Aq is about 0.3. Using $k_{11} = 10^{-6}$ (cgs units) [a typical value, approximately correct for para-azoxydianisole (PAA)[3]], we get $\rho = 0.08$ erg/cm² more energy for alignment across than for alignment parallel to the waves, and the additional energy density at $z = 0$ is $u = 5 \times 10^5$ erg/cm³.

In order to make such an energy density differ-
ence meaningful we compute the electric and mag-
netic fields that would be necessary to produce
polarization energy density of equal magnitude.
These would be approximately the minimum fields
necessary to hold the molecular alignment at the
surface across rather than parallel to the ridges
on the surface. The reduction in the minimum
fields that might result from torque transmitted
by elastic forces from a region of the liquid crys-
tal beyond a distance $(2q)^{-1}$ from the surface
would be slight.

Maier and Meier[5] give the following low-fre-
quency dielectric tensor components for PAA:
$\epsilon_1 \approx 5.6$, $\epsilon_2 = 5.9$. Setting

$$u = 5 \times 10^5 \text{ erg/cm}^3 = E^2(\epsilon_2 - \epsilon_1)/8\pi,$$

we get $E = 2.0 \times 10^4$ statV/cm or 6×10^6 V/cm.
Massen, Poulis, and Spence[6] measured the mag-
netic anisotropy of PAA at 100°C in a strong mag-
netic field and obtained a value of about 2.4×10^{-7}
for $\mu_2 - \mu_1$. Setting

$$u = H^2(\mu_2 - \mu_1)/8\pi,$$

we get $H = 7 \times 10^6$ Oe. The elastic distortion ener-
gy would be almost impossible to overcome with
magnetic or even electric fields for a surface as
rough as the one imagined in the example.

If the surface is equally rough in both the x and
y directions, the way for the molecules to avoid
elastic strain energy near the surface is to stand
on end. Whether or not this will occur on a rough
surface will depend on the degree of surface
roughness and on the difference in energy for a
surface layer of molecules lying parallel or per-
pendicular to a perfectly smooth surface of the
same material at any given temperature. Certain
nematic liquid crystals show the vertical (homeo-
tropic) orientation at high temperatures and par-
allel orientation at lower temperatures. The
temperature of this change in orientation should
be slightly lower for rougher surfaces if the elas-
tic energy at the surface is an important factor.

We have observed what we believe to be sur-
face-shape-induced alignment of PAA. In one ex-
periment, glass microscope slides were rubbed
in one direction several times over a pitch lap
charged with fresh cerium oxide slurry. They
were then washed with soap, soaked for about
two minutes in chromic and concentrated sulfuric
acid glass cleaner, rinsed with running water
and then distilled water, and finally dried and
baked for $\frac{1}{2}$ h at 500°C in air. When PAA was
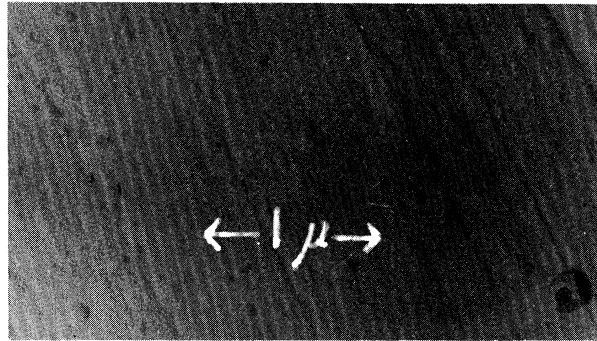heated to its nematic phase and allowed to flow

FIG. 1. Electron micrograph of a gold-shadowed car-
bon replica of a microscope slide rubbed with nominal-
ly 1-μm diamond paste on leather and cleaned as de-
scribed.

between two such slides separated by a few mi-
crometers, the extraordinary optical axis of the
PAA oriented parallel to the direction of rubbing
at temperatures up to about 134°C. It then as-
sumed the vertical orientation up to the isotropic
transition at 136°C. These changes were revers-
ible. We also rubbed microscope slides with
hard leather charged with 1-μm diamond paste
and cleaned them in the same way. These sub-
strates also oriented PAA in the direction of rub-
bing and no range of temperatures with vertical
orientation was observed.

Figure 1 is an electron micrograph of a micro-
scope slide rubbed with diamond paste and cleaned
as described. A slide rubbed with cerium oxide
showed fewer and much less pronounced grooves.
Rubbing with diamond paste gave more reproduc-
ible results than with cerium oxide.

Substrates rubbed with either kind of lap changed
their aligning effect if PAA was cooled to the
crystalline phase against the surface and then
heated again to the nematic phase. Thereafter
the nematic phase tended to align in a direction
between the former direction of the molecules in
the crystallites and the rubbing direction. The
effect persisted even after the PAA was heated
to a little above the isotropic transition and cooled
again. After heating to about 200°C and cooling,
the alignment was more nearly parallel to the
direction of rubbing. These observations suggest
that when the PAA crystallized, a layer of PAA
molecules or of some impurity became weakly
bonded to the surface in a direction determined
by the crystal axes and that this layer had an ori-
enting effect comparable in strength to the sur-
face-shape effect.

I wish to acknowledge helpful discussions with
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Taylor.

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Enhanced Scattering Inherent to "Loss-Cone" Particle Distributions*

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Particle scattering due to a Rosenbluth-Post convective loss-cone instability is calculated using a slab model. This collective contribution to the scattering rate is found to be the classical scattering rate increased by a factor $(T_i/T_e)^{3/2}(m_e/m_i)^{1/2}(\ln\Lambda)^{-1}\times$(energy amplification of the convective instability in the finite plasma).

Because of their loss-cone distribution functions, plasmas confined in open-ended configurations have been shown by Rosenbluth and Post (RP)[1,2] to be inherently subject to an instability which convects along the magnetic field $\vec{B}$ with wavelengths perpendicular to $\vec{B}$ short compared to the ion gyroradius. They obtained a stability criterion based on the limit of ten e foldings of the instability in the length of the plasma. Current mirror experiments satisfy this criterion.

In such mirror experiments as 2X[3] and 2X-II,[4] under the most favorable conditions, the decaying plasma is apparently quiescent. However, the density decay rate is always a few times the classical one and is relatively insensitive to ion temperature. A possible cause of this anomalous loss is that, although the machines satisfy the RP ten-e-folding criterion, there may exist a level of fluctuation over the classical value, the source of which is the two-particle scattering process amplified by the appropriate convective growth. Such fluctuations would be expected to be more effective at scattering ions than ion-cyclotron instabilities of the same amplitude because of their shorter wavelength and the occurrence of ion-wave resonance. It is our purpose to describe the fluctuation level and the associated particle scattering rates for this process.

Such a calculation of the fluctuation level and scattering due to a convective instability in a finite plasma differs in principle from quasilinear or other nonlinear theories for absolute (standing wave) instabilities (or convective instabilities in infinite plasmas). In the latter case, growth is stopped by ion scattering modifying the distribution function. In a finite plasma which is only convectively unstable the fluctuation level is automatically limited,[5] and one can envisage (with a source of ions) a steady state with enhanced scattering into the loss cone. The familiar expansion of kinetic theory holds with the small parameter (number of particles in a Debye sphere)$^{-1}$ increased by the energy amplification of a wave in crossing the plasma.

The fluctuation level of an unstable drift-type wave convecting across the magnetic field has been obtained by Kent and Taylor,[6] although they do not obtain the accompanying particle scattering. Because the wave we are interested in has a parallel group velocity greater than that of any particle (which is to say that the wave is convective in the frame of all particles), our result is similar to theirs with alteration in the direction of propagation. The method is based upon constructing a two-particle correlation function by a superposition of uncorrelated but dressed test
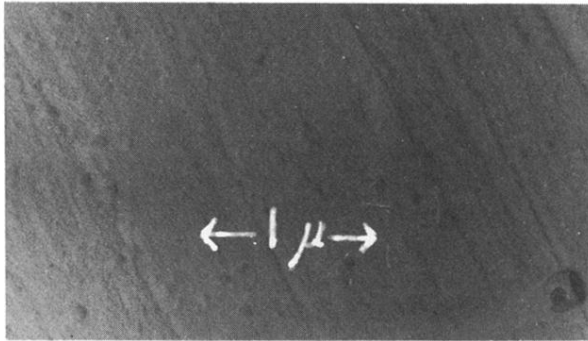
FIG. 1. Electron micrograph of a gold-shadowed carbon replica of a microscope slide rubbed with nominally 1-μm diamond paste on leather and cleaned as described.