(1969).
[10]J. W. Beams, Phys. Today 24, 34 (1971).
[11]H. Bondi, Rev. Mod. Phys. 29, 423 (1957).
[12]C. L. Lawson and R. J. Hanson, *Solving Least Squares Problems* (Prentice-Hall, Englewood Cliffs, to be published).

Origin of the Cosmic Microwave Background Radiation in a Chaotic Universe

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If the early universe possessed the maximum degree of irregularity compatible with its present large-scale uniformity and isotropy, dissipation of energy at redshifts $z \gtrsim 10^4$ would have generated a thermal microwave background with present temperature $\sim 3°K$.

The (apparently thermal) cosmic microwave background radiation is commonly interpreted as a relic of early pregalactic epochs when it was in equilibrium with hot opaque primeval plasma. In the widely accepted version of the "hot big bang" originally proposed by Gamow,[1] the radiation is assumed to have existed *ab initio* in an essentially homogeneous universe. The remarkable isotropy of the background temperature indicates that, at least back to the red shift when the microwave photons were last scattered, the gross structure of the universe was indeed homogeneous and isotropic. This supports Gamow's picture (and also raises severe problems for those theories which ascribe the origin of the microwave background to sources at modest red shifts).[2] But there is no *firm* evidence that, on scales up to the size of clusters of galaxies, the universe was ever any smoother than it is today. This Letter outlines some arguments which indicate how the microwave background could have been generated at red shifts $z \gtrsim 10^4$ by dissipation of energy associated with "primordial chaos."

The mass-energy density of 2.7°K blackbody radiation is $\rho_\gamma(t_0) \simeq 3\times10^{-34}$ g cm^{-3}, and we take this as an estimate of the energy density of the microwave background, even though its exact spectrum is unknown.[3] This compares with a matter density at the present epoch t_0 of $\rho_m(t_0) = 2\times10^{-29}\Omega$ g cm^{-3} (where Ω is the usual "density parameter," and a Hubble constant $H_0 \simeq 100$ km sec^{-1} Mpc^{-1} is assumed). If all this radiation were created at red shifts $\gtrsim z_{cr}$ and had expanded adiabatically since the corresponding epoch, then for $z < z_{cr}$

$$\rho_\gamma/\rho_m \simeq 1.5\times10^{-5}(1+z)\Omega^{-1}. \tag{1}$$

In the conventional "hot big bang" cosmology, z_{cr} is taken to be essentially infinite, the origin of the radiation being ascribed to the high entropy of the initial conditions, or at least relegated to exotic processes occurring at $z \gtrsim 10^{10}$. However, all that can strictly be deduced from its thermal spectrum is that the bulk of the microwave background was generated at an epoch when the matter was capable of thermalizing it. This obviously implies $z_{cr} \gtrsim 1500$, since at red shifts smaller than this, hydrogen and helium in equilibrium with the radiation would be primarily neutral, and hence essentially thermally decoupled. In fact, the condition $z_{cr} \gtrsim 1500$ is not sufficient: It is *also* necessary that the plasma, at temperature $2.7(1+z_{cr})$°K, should be able, within the time available, to generate enough photons to produce a radiation field with the full blackbody intensity. Equivalently, the mean free path of a photon with $h\nu \simeq 2.7k(1+z_{cr})$ for absorption (not merely for electron scattering) must be $\lesssim ct$, where the expansion time scale for a standard Friedmann model is $t \simeq 2\times10^{17}(1+z_{cr})^{-3/2}\Omega^{-1/2}$ sec (for $\rho_\gamma \lesssim \rho_m$, and shorter if the expansion is radiation dominated). One can easily show that this requires $z_{cr} \gtrsim z_{th}$, the "thermalization red shift" z_{th} being given by

$$(1+z_{th}) \simeq 4\times10^5\Omega^{-3/2}f^{-1}, \tag{2}$$

where f (≥ 1) is a "clumpiness factor," $\langle\rho_m^2\rangle/\langle\rho_m\rangle^2$.

Clearly the thermalization condition requires z_{cr} to be so large that [from (1)] the generation of the microwave background must have involved an energy release per unit volume of order $\rho_m c^2$. The only plausible source is therefore the gravitational or kinetic energy associated with *large-amplitude* primordial irregularities. The existence of galaxies and clusters of galaxies obviously implies that the early universe cannot have been *completely* homogeneous. It is found that density fluctuations of $\gtrsim 1\%$ or peculiar velocities $\gtrsim 0.01c$ must have existed in the fireball phase on the scale of protogalaxies and protoclusters. Dissipation associated with perturbations of the minimum required amplitude of $\sim 1\%$ can at most generate a small fractional increment in the pre-existing thermal radiation field (though several authors[4] have discussed this effect, and the spectral distortions resulting from energy injection at $z \lesssim 10^4$).

However, it can be argued[5] that, since the mass within the particle horizon of a Friedmann model goes to 0 as $t \to 0$, the assumed overall homogeneity involves the unpalatable assumption that different parts of the universe started to expand at the same time, and with the same curvature and entropy, even though there was then no causal connection which could have synchronized them. Thus, it may be the universe's present overall *isotropy*, *not* the small-scale inhomogeneity, which poses the major mystery. On this point of view, it is perhaps more natural to postulate the *maximum* degree of primordial chaos compatible with the observed large-scale uniformity. In particular, it is interesting to explore the possibility that the fluctuations on scales up to (say) protoclusters may already be of order unity when they come within the particle horizon. I have shown elsewhere[6] that this hypothesis is compatible with present observations —e.g., the limits on the background isotropy.

If the early universe approximates to a Friedmann model, then the mass within a particle horizon is $M_H(t) \propto t$. Irregularities on a mass scale M cannot dissipate until $M_H \gtrsim M$. Thus, as the expansion proceeds and the mass encompassed within a horizon increases, dissipation of gravitational or kinetic energy associated with progressively larger scales can provide a continuous heat input, at a rate (say) $\epsilon\rho c^2/t$ per unit volume [i.e., $\epsilon(t)$ measures the fraction of mass energy dissipated in an expansion time scale t]. If the fluctuations on a given scale M are already large when $M_H \simeq M$, then nonlinear dissipation will promptly ensue, yielding a value of ϵ of order unity. We leave the physical details of the dissipative processes as an open question. They may involve either damping of random motions, or collapse of a fraction of the material to black holes.

Consider, for illustrative purposes, a simple case when fluctuations on every scale M between $M_{\rm min}$ and $M_{\rm max}$ are of *comparable amplitudes* when they first come within the horizon. ϵ would then be roughly constant at all times such that $M_{\rm min} \lesssim M_H(t) \lesssim M_{\rm max}$. If the dissipated energy went primarily into radiation with $\gamma = \frac{4}{3}$, then, when $M_{\rm min} \ll M_H(t) \lesssim M_{\rm max}$, we would have

$$\rho_\gamma/\rho_m \simeq 2^{3/2}\epsilon. \tag{3}$$

At sufficiently early times, the plasma would be dense enough to thermalize the radiation at a temperature T, and

$$T \propto \rho_m^{1/4} \propto t^{-1/2}. \tag{4}$$

For *all* the microwave radiation to have been produced in this way, (1) and (3) imply that the energy input must have continued until the epoch corresponding to

$$1+z \simeq 2\times10^5\epsilon\Omega. \tag{5}$$

Furthermore, it must still be possible to thermalize the energy at this red shift, which entails [from (2)] that

$$f \gtrsim 2\Omega^{-5/2}\epsilon^{-1}. \tag{6}$$

In other words, if $\Omega \lesssim 1$ and $\epsilon \lesssim 1$, some clumpiness is essential in order to ensure adequate thermalization. (And f would have to be still larger if some extra form of energy density—e.g., gravitational waves—reduced the expansion time scale.) But if $\epsilon \simeq 1$ the matter distribution would certainly *not* be smooth. For instance, if the dissipation involved strong shock waves, each region being shocked (on average) once per expansion time scale, then one might expect $f \simeq \log(M_H/M_{\rm diff})$, where $M_{\rm diff}$ is the largest scale on which particle diffusion can smooth out fluctuations. We might have f as large as ~ 100 at the relevant epochs. (Note that the Jeans mass would be $\sim \epsilon^{3/2}M_H$, so the density inhomogeneities making the main contribution to f would be gravitationally stable.)

Thus the observed microwave background could have been generated if dissipation with amplitude $\epsilon \simeq 0.1-1$ persisted until the epoch corresponding to $z \simeq 2\times10^4\Omega - 2\times10^5\Omega$. A comparable energy input at later epochs, when (2) no longer holds, could not be thermalized and would indeed distort (via repeated Compton scattering) the thermal spectrum of radiation already present. Therefore, we must postulate that the primordial chaos is "truncated" above a scale $M_{\rm max}$, and that the dissipative heat input drops drastically at later times. $M_{\rm max}$ must be comparable to the value of M_H at the red shift given by (5), which is

$$2\times10^{17}\Omega^{1/2}\epsilon^{-3/2}M_\odot. \tag{7}$$

It seems significant that, for ϵ and Ω in the range 0.1–1, this is comparable to the largest known density inhomogeneities in the universe (clusters or superclusters of galaxies), and also with the largest scale on which fluctuations with amplitude ~ 1 would be compatible with the microwave background isotropy. Thus, we have *prima facie independent* reasons for requiring that the universe becomes inherently more homogeneous above this particular scale.

The preceding argument should at least have indicated that a naive "chaotic cosmology" can plausibly account for the presence of thermal background radiation with $T \simeq 2.7°$K. In the standard "hot big bang" model, it is entirely coincidental that the present ratio ρ_γ/ρ_m is such that (at least within 1 or 2 orders of magnitude) $\rho_\gamma = \rho_m$ at the epoch when matter and radiation decouple thermally; and that this is, in addition, the epoch when the particle horizon first became extensive enough to encompass the largest known scale of irregularity in the universe. These circumstances are, however, *straightforward consequences* of the theory proposed here, provided only that the "natural" value of ϵ is of order unity ("maximal chaos").

In this picture, the temperature could still have been arbitrarily high at very early times if $M_{\min}$ were small enough [see (4)], though the entropy per baryon would then be less than in the canonical picture. The simplifying assumption of constant ϵ is not essential to any part of our argument—unless ϵ varied with t more steeply than $\propto t^{-2/3}$, it would still be the largest scales ($M \simeq M_{\max}$), dissipating at the latest times, which make the dominant contribution. Of course, the assumption that the mean overall expansion rate is the same as for a homogeneous Friedmann model, even when $\epsilon \simeq 1$, is plainly a crude approximation at best.

The present background temperature of 2.7°K corresponds, as we have seen, to a reasonable value of ϵ and a reasonable lower limit on the factor f. As a corollary, the assumption of maximal chaos *determines* the present temperature of background radiation that could have been generated and thermalized prior to a red shift z, in terms of ϵ and Ω, as

$$T \simeq 5(\epsilon\Omega)^{1/4}[(1+z)/10^4]^{-1/4}. \tag{8}$$

The smallest red shift at which thermalization is possible is in general $\lesssim 10^5$; but, however large f may be, it cannot be less than ~ 1500. Thus, the last factor in (8) does not differ from unity even by a factor 2. If the other factor is also ~ 1, T is pinned down to much better than an order of magnitude. The prospect of being able to "predict" the background temperature in this way—in contrast to the situation in the canonical "hot big bang" model, where the entropy per baryon is a completely free parameter, unconstrained even in order of magnitude—seems sufficiently interesting to justify further investigations of dissipative processes in "chaotic" universes, along the lines outlined in the present Letter.

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[1]G. Gamow, Rev. Mod. Phys. 21, 367 (1949).

[2]Arguments pertaining to various interpretations of the microwave background are reviewed by M. J. Rees, in *Highlights of Astronomy*, edited by C. de Jager (Reidel, Dordrecht, 1971), p. 757.

[3]A. G. Blair, J. G. Berry, F. Eduskuty, R. D. Heibert, J. P. Shipley, and K. D. Williamson, Jr., Phys. Rev. Lett. 27, 1154 (1971); A. A. Penzias, K. B. Jefferts. amd R. W. Wilson, Phys. Rev. Lett. 28, 772 (1972), and earlier references cited therein.

[4]R. A. Sunyaev and Y. B. Zeldovich, Astrophys. Space Sci. 4, 301 (1969), and 7, 3 (1970); T. Matsuda, H. Sato, and H. Takeda, Progr. Theor. Phys. 46, 416 (1971).

[5]W. H. McCrea, Science 160, 1295 (1968); C. W. Misner, Astrophys. J. 151, 431 (1968), and Phys. Rev. Lett. 22, 1071 (1969).

[6]M. J. Rees, Mon. Not. Roy. Astron. Soc. 154, 187 (1971).