the theory. If the $\Delta m = 4$ absorption were proportional to the square of the concentration, the $\Delta m = 4$ absorption per pair of impurities would have to be about a thousand times greater than the $\Delta m = 2$ absorption per impurity at $T = 4$ K!

The proposed model also predicts resonant absorption at $\Delta m = 3$ with a magnitude similar to that of $\Delta m = 4$. Although such a line should presumably be obscured by random strain broadening,[7] it is possible that such strains are partially field induced. If this is the case the $\Delta m = 3$ line would be observable at low frequencies and magnetic fields. A straightforward extension of this theory also predicts higher resonances at $\Delta m = 5, 6$ which are reduced in magnitude by the ratio $(G^2/Mv^2kT)^2$ over the $\Delta m = 3, 4$ resonances.

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Evaporation from Superfluid Helium*

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We calculate the angular and spectral distribution of atoms evaporating from superfluid helium using a kinetic-theory description of the liquid quasiparticles. Phase-space analysis indicates that the evaporation spectrum exhibits information about the single-particle properties of the liquid, but in particular does not manifest the quasiparticle density of states. The latter conclusion is in agreement with recent experimental results.

Several attempts have recently been made to determine theoretically the spectrum of atoms evaporating from superfluid He.[1-5] These efforts were stimulated in part by measurements of King and Johnston,[6] but in addition by the realization that evaporation probes microscopic properties of the liquid related to the single-particle Green's function. The present work indicates that while the kinematics of the evaporation process renders inaccessible the quasiparticle density of states, it does disclose other information about the single-particle spectral function of the bulk liquid.

We shall represent the elementary excitation system of liquid He by a gas of noninteracting quasiparticles. By considering the flux of quasiparticles incident upon the interface from below, we determine the rate of evaporation of atoms from the surface in terms of a phenomenological parameter which must be calculated from a microscopic theory. The frequency of the reverse process, in which a vapor atom condenses and excites a quasiparticle state of the liquid, is related by considerations of detailed balance,[1] which we discuss below. Following Anderson,[2] we neglect both inelastic processes and those processes involving more than one atom or quasiparticle.

The rate of arrival at unit surface area from solid angle $d\Omega$ of quasiparticles having energy[7] ω in an interval $d\omega$ is

$$\frac{d^2\dot{N}_{QP}}{d\omega\, d\Omega}\, d\Omega\, d\omega = f(\omega)\eta(\omega)\frac{d\omega}{dq}\cos\theta\, d\omega\, d\Omega. \qquad (1)$$

Here θ is the angle of incidence with respect to the surface normal, $f(\omega)$ is the Bose distribution function at temperature $T = (k_B\beta)^{-1}$,

$$f(\omega) = (e^{\beta\omega} - 1)^{-1}, \qquad (2)$$

and $\omega(q)$ is the energy of a quasiparticle with momentum $\vec{q}$. $\eta(\omega)$ is the quasiparticle density of

states per unit solid angle in the liquid,

$$\eta(\omega) = q^2(8\pi^3|d\omega/dq|)^{-1}. \quad (3)$$

The momentum $\vec{p}$ and energy E of an atom emitted from the liquid surface are uniquely determined by $\vec{q}$ and ω of the incident quasiparticle. Because of translational invariance parallel to the liquid surface, parallel momentum is conserved in the transfer process. Part of the energy ω must supply the binding energy $|\mu|/k_B = 7.17$ K required to liberate the atom, so that

$$E = p^2/2m = \omega - |\mu|, \quad (4a)$$

$$p_\parallel = q_\parallel. \quad (4b)$$

We note that the region of anomalous dispersion (q just below the roton minimum), for which the group velocity is opposite to $\vec{q}$, does not contribute to the evaporation process. When such a quasiparticle arrives at the surface, its momentum will be directed toward the liquid. Since the net surface force acts also in this direction, no final state of He atom moving outward toward the vapor is possible. We therefore ignore this branch of the excitation spectrum. One can show by a similar argument that phonons having momentum $q > q_1 \equiv m[c - (c^2 - 2|\mu|/m)^{1/2}]$ cannot contribute to evaporation because any resulting atom would have $p > q$.

The kinematic considerations above yield a relation between a given range of solid angle $d\Omega$ for incident quasiparticles and the corresponding range $d\Omega'$ at θ' for the resulting emitted atoms:

$$d\Omega/d\Omega' = p^2\cos\theta'/q^2\cos\theta. \quad (5)$$

We now introduce a phenomenological parameter $t_{\vec{q}\vec{p}}$ as the fraction of quasiparticles incident upon the surface which transfer to states of an evaporating atom having the appropriate momentum $\vec{p}$. If Eqs. (4) are satisfied, $t_{\vec{q}\vec{p}}$ should be of order unity. This is confirmed indirectly by experiments[8] which determine an average value of the condensation coefficient to be 0.7 to 0.9, although the possibility of inelastic processes makes this an upper limit. Combining (1)–(5), we obtain for the distribution of evaporating atoms

$$\frac{d^2\dot{N}_{ev}}{dE\,d\Omega'}(E,\theta') = \left(\frac{d^2\dot{N}_{QP}}{d\omega\,d\Omega}\, t_{\vec{q}\vec{p}}\,\frac{d\Omega}{d\Omega'}\right)_{\omega = E + |\mu|} = e^{-\beta(E+|\mu|)}\, mE\cos\theta'\, t_{\vec{q}\vec{p}}/4\pi^3, \quad (6)$$

where we have used $f(\omega) \approx \exp(-\beta\omega)$. A gap will appear in the spectrum between energies $(q_1 c - |\mu|)/k_B = 1.36$ K and $(\Delta - |\mu|)/k_B = 1.5$ K, corresponding to the high-energy-phonon limit mentioned above and the roton minimum (Δ) threshold, respectively.

We find, therefore, that proper treatment of phase space reveals an effective cancelation of the bulk quasiparticle density of states [Eqs. (4)]. This occurs because Eqs. (4) allow only those quasiparticles incident from within a cone $\theta < \theta_{max}$, centered about the surface normal, to contribute to evaporation. θ_{max} is determined from

$$\sin^2\theta_{max} = 2m(\omega - |\mu|)/q^2, \quad (7)$$

which gives 15° near the roton minimum, for example. Total internal reflection occurs for quasiparticles having $\theta > \theta_{max}$.

We may now rewrite Eq. (6) in terms of the saturated vapor pressure P and the angular distribution of atoms *incident* on the surface from the vapor side,

$$\frac{d^2\dot{N}_{inc}}{dE\,d\Omega'}(E,\theta') = P\beta^{+5/2}(2m\pi^3)^{-1/2}E\cos\theta' e^{-\beta E}, \quad (8)$$

$$P = e^{-\beta|\mu|}(m/2\pi)^{3/2}\beta^{-5/2}, \quad (9)$$

giving

$$\frac{d^2\dot{N}_{ev}}{dE\,d\Omega}(E,\theta') = \frac{d^2\dot{N}_{inc}}{dE\,d\Omega'}(E,\theta')\, t_{\vec{q}\vec{p}}. \quad (10)$$

Since by time-reversal invariance the coefficient $t_{\vec{p}\vec{q}}'$ for the reverse process $(\vec{p}, E \to \vec{q}, \omega)$ must equal $t_{\vec{q}\vec{p}}$, the right-hand side of Eq. (10) also equals the number of atoms impinging on the surface which "condense" into a quasiparticle state of the liquid:

$$\frac{d^2\dot{N}_{con}}{dE\,d\Omega'}(E,\theta') = \frac{d^2\dot{N}_{ev}}{de\,d\Omega'}(E,\theta'), \quad (11)$$

which is an explicit demonstration of the detailed-balance requirement of the surface.[9]

The fundamental result of the present work, Eq. (10), is remarkable in that the anomalous density of states of the liquid is not in evidence. The evaporation spectrum is like that of a free-particle "liquid," apart from possible variation in $t_{\vec{q}\vec{p}}$. In Fig. 1, the predicted spectrum is shown for the case of an assumed constant value of $t_{\vec{q}\vec{p}} = 0.5$. King, McWane, and Tinker[10] have recently repeated the evaporation measurement and obtained an approximately Maxwellian curve

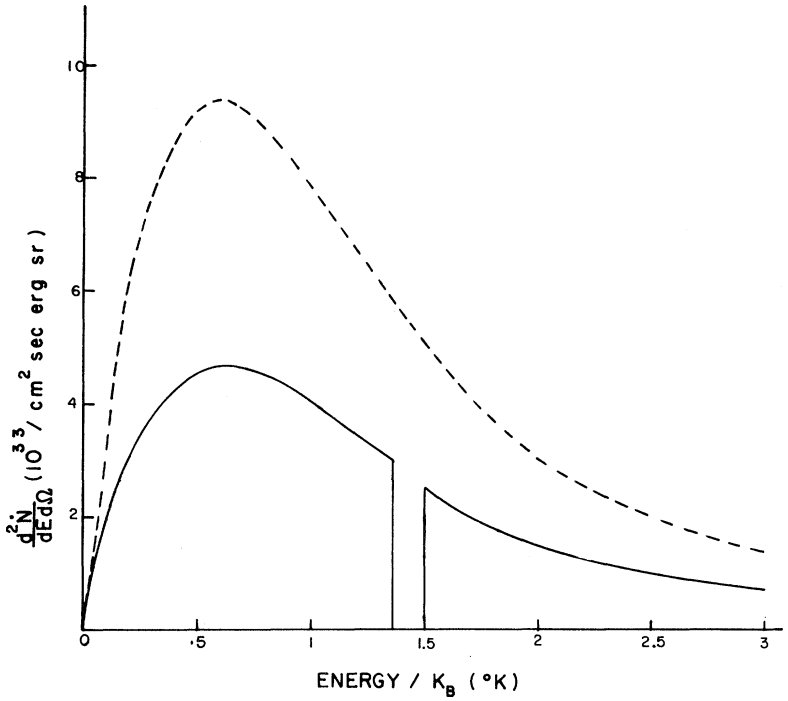

FIG. 1. Predicted evaporation spectrum for $\theta = 0$, assuming a constant transmission coefficient $t_{\vec{q}\vec{p}} = 0.5$ (solid curve). For comparison, the spectrum of atoms incident on the surface from the vapor is also shown (dashed curve).

corresponding to the experimental temperature $T = 0.6$ K. This tends to confirm the result (10) with an essentially constant transmission coefficient. However, no evidence appears in the data for the gap predicted above. This may be due to inelastic processes, which would preferentially smear the high-energy tail of the distribution, or perhaps to insufficient experimental resolution.

While this "kinetic theory" approach facilitates calculation of the E, θ dependence of evaporation, one can formulate this treatment equivalently in terms of the tunneling, or transfer, Hamiltonian method[11] first used for this problem by Widom.[1] This allows one to see the assumptions and limitations of the model. The Hamiltonian H of the complete system is divided into terms appropriate to liquid (H_l) and vapor (H_v) subsystems, plus a term which weakly couples the two via a transfer of atoms:

$$H = H_v + H_l + \sum_{\vec{q},\vec{p}} (T_{\vec{q}\vec{p}} b_{\vec{q}}^{\dagger} c_{\vec{p}} + T_{\vec{q}\vec{p}}^{*} c_{\vec{p}}^{\dagger} b_{\vec{q}}), \qquad (12)$$

where the $c_{\vec{p}}$ ($b_{\vec{p}}$) and $c_{\vec{p}}^{\dagger}$ ($b_{\vec{p}}^{\dagger}$) annihilate and create atoms in the vapor (liquid). The transfer rate is then proportional to the single-particle spectral weight functions of the subsystems, which we take to be[4,12]

$$A_v(\vec{p}, E) = 2\pi\delta(E - p^2/2m - |\mu|), \qquad (13a)$$

$$A_l(\vec{q}, \omega > 0) = 2\pi Z_q \delta(\omega - \omega(q)), \qquad (13b)$$

for vapor and liquid, respectively. These functions enter because the transfer process can occur only when the excitation is in the bare-particle configuration.[13] As pointed out by Griffin,[4] the strength Z_q of the quasiparticle pole may depend on the T-dependent Bose condensate fraction n_0, so evaporation may indirectly determine this elusive quantity. The total evaporation rate which results is

$$\dot{N}_{ev} = \sum_{\vec{q},\theta<\pi/2} W_{\vec{q}} Z_q \frac{d\omega}{dq} f(\omega) \cos\theta \delta_{p_{\parallel},q_{\parallel}}, \qquad (14)$$

where $W_{\vec{q}}$ is a bare-atom transfer coefficient[14] present in the term $|T_{\vec{q}\vec{p}}|^2$ in the transfer rate. Thus we identify the parameter $t_{\vec{q}\vec{p}}$ as the product $W_{\vec{q}} Z_q$. $W_{\vec{q}}$ is very small ($\approx 10^{-8}$) for electron tunneling problems in solids because of exponential decay of wave functions in the classically forbidden barrier region. No such region is present in evaporation so that we expect $t_{\vec{q}\vec{p}}$ to be

roughly of order unity. We have, however, neglected local self-energy effects by using for the spectral function its value in the bulk liquid instead of that near the surface.

The problem remains to calculate t^{++}_{qp} from microscopic theory. Preliminary results for this function, which have been obtained[15] from the Bogoliubov theory of superfluidity, indicate that t increases monotonically from zero for evaporation energy $E=0$ to a value of order unity for $E/k_B \approx 1$ K. However, the Bogoliubov approximation with δ-function pseudopotential does not give a roton minimum, so that a more realistic treatment is required for the evaporation problem.

It would be valuable to obtain both absolute and angular distribution measurements of the evaporating current. An equally crucial and difficult measurement is that of scattering by the liquid surface of a beam of He atoms. This experiment can yield detailed information about both elastic and inelastic processes occurring at the He surface.

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Numerical Simulation on Plasma Diffusion in Three Dimensions*

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Three-dimensional plasma numerical simulation on cross-field diffusion has been performed for cases with closed and nonclosed field lines of force. For the former case, the diffusion essentially takes the form found for two dimensions showing the three regions of diffusion, while for the latter, the diffusion follows the classical theory to stronger fields. However, the diffusion is always enhanced above the classical level when the magnetic field exceeds a critical value.

Three-dimensional (3D) computer experiments on plasma diffusion have been performed extending the previous 2D and $2\frac{1}{2}$D calculations.[1,2] The particle model used was an electrostatic dipole code[3] with an external magnetic field $\vec{B}$. The simulation was carried out in a cubic box of dimensions L^3 with periodic boundary conditions as sketched in Fig. 1. θ and ψ are the angles between the coordinate and the external magnetic field, and they are chosen at the beginning of calculations. Particles were initially distributed uniformly in the box with a Maxwell velocity distribution. All of the results reported here include the full dynamics of the particles. The diffusion coefficient is measured from the displacement of the guiding centers of a set of test particles $[D_{i,e}=(\Delta r_g)_{i,e}{}^2 t]$.

We find that the cross-field diffusion is always enhanced above the classical level when the magnetic field exceeds a critical value. When the magnetic field is parallel to one of the coordinate axes, say, the z axis, then this corresponds to a system with closed field lines, and enhanced diffusion due to plasma convections associated