# Dynamical Parton Model for Hadrons

E. P. Tryon
*Department of Physics and Astronomy, Hunter College of the City University of New York, New York, New York 10021*
(Received 6 March 1972)

Assuming that hadrons are bound states of large numbers of partons and antipartons, we argue that the hadronic medium is characterized by a tension which is *independent* of the amount by which it has been stretched. The dynamically stable configurations of such a medium are spinning loops, where the tension is balanced against the centrifugal force. The corresponding Regge trajectories are in excellent agreement with experiment, but rise only like $s^{2/3}$ for large $s$.

Several authors have proposed models wherein hadrons are composite particles consisting of large numbers of partons ($q$) and antipartons ($\bar{q}$) bound together in a deep potential.[1] In order for such a system to be stable against $q\bar{q}$ annihilations, it is necessary that such annihilations be inhibited by some mechanism. The most appealing explanation is that the depth of the binding potential is strongly dependent on the number of partons present. If this were true, then a system with many $q\bar{q}$ pairs could be stable for the simple reason that $q\bar{q}$ annihilations would reduce the depth of the potential so much that the final state would require a larger energy than is available from the initial state.

If we assume that the preceding explanation for stability against $q\bar{q}$ annihilations is correct, then some sort of saturation of the binding forces must also be operative, or else one could obtain a state of negative energy simply by adding a sufficiently large number of $q\bar{q}$ pairs. Therefore, the picture which emerges is that each hadronic bound state or resonance contains some optimal number of $q\bar{q}$ pairs specific to that state, and these $q\bar{q}$ pairs serve to minimize the total energy at some positive value, subject to whatever constraints may be implied by the other quantum numbers of the state.

If one were to "stretch" such a hadron, it would respond by using the work done on it to create enough new $q\bar{q}$ pairs to fill out the new enlarged volume with the energetically optimal density of partons. Therefore, the tension required to stretch the hadronic medium should be *independent* of the amount by which it has already been stretched,[2] provided that the system is larger than the distance over which the saturation of forces takes place. The value of the tension would depend on the parton rest mass, on the strength and range of the forces between partons, and on details of the saturation mechanism.

For any given set of internal quantum numbers different from those of the vacuum, there may exist a ground state with zero orbital angular momentum, whose collapse by $q\bar{q}$ annihilation is inhibited by the conservation laws specific to strong interactions, as well as by the energy considerations mentioned earlier. For every such ground state, there should exist a series of rotationally excited states, wherein the tension is balanced against centrifugal forces. Assuming the hadronic medium to be characterized by a tension which is independent of the amount by which it has been stretched, the stable rotating configurations will be spinning loops.[3] The diameters of the $q\bar{q}$ "chains" out of which the loops are made will be roughly the distance over which the saturation of forces takes place, and will thus be independent of the radii of the loops. Therefore, the linear tension along the length of each chain will be independent of the radius of the loop.

In this paper, we consider an idealized model of such loops, and derive the relation between angular momentum and energy. The corresponding Regge trajectories are in excellent agreement with experiment, but rise only like $s^{2/3}$ for large $s$. The analysis proceeds as follows.

Let us consider a circular loop of radius $r$, rotating with tangential velocity $v$. We shall imagine the loop to be divided into short segments which have length $dl$ in the loop's center-of-mass frame, and which have length

$$dl_0 = \gamma\, dl \tag{1}$$

in the instantaneous rest frame of each segment, where

$$\gamma \equiv (1 - v^2/c^2)^{-1/2}.$$

We shall assume that in the instantaneous rest frame of each segment, the chain of which the loop is made is characterized by a linear tension $T_0$ which is independent of the amount by which it has been stretched. Assuming the loop to have a rest energy $E_0$ even when of null radius and null tangential velocity (this corresponds roughly to the ground state), it follows that the total energy of the rotating loop is given by

$$E=\gamma(E_0+T_0 l_0), \tag{2}$$

where

$$l_0 \equiv \int dl_0 = \gamma 2\pi r. \tag{3}$$

In the instantaneous rest frame of $dl_0$, the segment undergoes an acceleration $a_0$ given by

$$a_0=\gamma^2 v^2/r. \tag{4}$$

This requires a force

$$dF_0=(dm_0)a_0, \tag{5}$$

where

$$dm_0=c^{-2}(E_0+T_0l_0)\,dl_0/l_0, \tag{6}$$

assuming that $E_0$ is distributed uniformly around the loop. In the rest frame of $dl_0$, the radius of curvature $R_0$ of $dl_0$ is given by

$$R_0=\gamma^{-2}r. \tag{7}$$

Assuming that the tension provides the entire centripetal force, we have

$$dF_0=(T_0/R_0)\,dl_0. \tag{8}$$

We have now written down all the relevant dynamical equations, and it remains only to examine their consequences. (Our present treatment is entirely classical—the quantization is left to future work.)

The first consequence of Eqs. (4)–(8) which we shall note is that

$$v/c=[T_0l_0/(E_0+T_0l_0)]^{1/2}, \tag{9}$$

so that

$$\gamma=(1+T_0l_0/E_0)^{1/2}. \tag{10}$$

Observe that we must have $E_0 \geq 0$ in order to avoid superluminal velocities.

Denoting the angular momentum of the loop by $L$, we have

$$L=\gamma m_0 v r, \tag{11}$$

where

$$m_0 \equiv \int dm_0 = c^{-2}(E_0+T_0l_0). \tag{12}$$

Using Eqs. (3) and (9)–(12), we can write $L$ as

$$L=(T_0l_0^2/2\pi c)(1+E_0/T_0l_0)^{1/2}. \tag{13}$$

The center-of-mass energy $E$ is given by Eq. (2), which is equivalent to $E=\gamma m_0c^2$. Using Eqs. (2) and (10), we obtain

$$E=\gamma^3E_0=(1+T_0l_0/E_0)^{3/2}E_0. \tag{14}$$

Expressing $L$ in terms of $\tilde{s} \equiv (E/E_0)^2$, it follows from Eqs. (13) and (14) that

$$L=\frac{E_0^2}{2\pi c T_0}(\tilde{s}^{1/3}-1)^2[1-\tilde{s}^{-1/3}]^{-1/2}. \tag{15}$$

Thus $L$ grows like $\tilde{s}^{2/3}$ in the limit of large $\tilde{s}$.

In addition to the "orbital" angular momentum $L$ carried by the loop, each parton may have intrinsic spin. However, we know from experiment that low-mass hadrons generally have low spin. Therefore, in any model where hadrons consist of large numbers of partons, the interaction between partons must be such as to favor small values of net spin.[4] Accordingly, we shall assume that the net intrinsic spin of each $q\bar{q}$ pair is zero. We shall denote the net spin of the nonrotating state with energy $E_0$ by $S_0$, and we shall assume that $S_0$ is small for those trajectories which have low-mass states on them.

In this paper, we shall restrict attention to leading Regge trajectories, where the total angular momentum $J$ is a maximum for any given energy. For this special case, $J$ is simply the algebraic sum of $L$ and $S_0$:

$$J=L+S_0. \tag{16}$$

To make a simple comparison of our model with experiment, let us consider the Regge trajectory which is known over the widest range of energies. This is the $\Delta(1236)$ trajectory, upon which five resonances have been observed (assuming that the $I=\frac{3}{2}$, positive-parity states at 2.85 and 3.23 GeV have the appropriate spins).

If we take the quark model as a guide, then partons are spin-$\frac{1}{2}$ objects, and the smallest value possible for $S_0(\Delta)$ is $\frac{1}{2}\hbar$. Assuming that the interaction selects this minimum value for $S_0$,[5] we have

$$S_0(\Delta)=\tfrac{1}{2}\hbar. \tag{17a}$$

For the remainder of our work, we shall use units wherein $\hbar=c=1$.

The two remaining parameters upon which the $\Delta$ trajectory depends are $T_0$ and $E_0$. If we determine these two parameters by making a least-squares fit to the five known points on the tra-

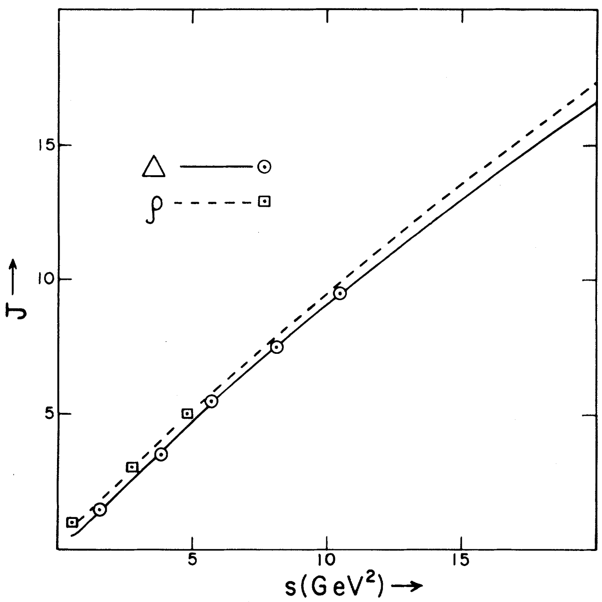


FIG. 1. Regge trajectories of the model fitted to the five experimental points on the $\Delta(1236)$ trajectory and the three experimental points on the $\rho$ trajectory.

jectory, we obtain

$$T_0(\Delta) = 0.0341 \text{ GeV}^2, \tag{17b}$$

$$E_0(\Delta) = 0.687 \text{ GeV}. \tag{17c}$$

In Fig. 1, we display the corresponding $J$ as a function of $s \equiv E^2$, together with the five experimental points. Observe that the Regge trajectories of our model have nonzero curvature, and that our model is in even better agreement with the five experimental points on the $\Delta$ trajectory than would be any straight line which could be drawn.

We have also indicated the three known points on the $\rho$ trajectory in Fig. 1 [including the $T(2200)$ resonance], together with our best fit to them. We have again followed the quark model in assuming that $S_0(\rho)$ is an integer, and we have used the smallest value which enables us to fit the data, namely,

$$S_0(\rho) = 1. \tag{18a}$$

The optimal values for $T_0$ and $E_0$ are

$$T_0(\rho) = 0.0343 \text{ GeV}^2, \tag{18b}$$

$$E_0(\rho) = 0.747 \text{ GeV}. \tag{18c}$$

Again our model is in excellent agreement with the data.

For all states with $L > 0$, our model predicts the radius of the loop, and also the tangential velocity with which it rotates. For example, we find for the $\Delta$ trajectory that $L = 1$ when $l_0 = 10.4$ GeV$^{-1}$, which together with Eqs. (3), (9), (10), and (17a)–(17c) implies that

$$r(\Delta(1236)) = 0.27 \text{ F}, \tag{19a}$$

$$v(\Delta(1236)) = 0.58c. \tag{19b}$$

The most easily tested prediction of our model is that Regge trajectories have a definite nonzero curvature. We have already noted that there is a hint of such curvature in the five known points on the $\Delta$ trajectory. It will be interesting to see whether this curvature is confirmed by the position of the $J = 23/2$ state, which our model predicts will have a mass of 3.61 GeV. By contrast, the linear trajectory fitted in a least-squares sense to the lowest three points on the $\Delta$ trajectory is given by $\alpha(s) = -0.05 + 0.958s$ (with $s$ in GeV$^2$), which implies a mass of 3.47 GeV for the $J = 23/2$ state.

In closing, we wish to emphasize that if the number of partons in a hadron is large, then the energy associated with each parton must be small. It then follows that $q\bar{q}$ pairs are easily created and annihilated within a hadron, so that these processes should play an important role in the dynamics. The primary virtue of our model is that it deals with the possibility of $q\bar{q}$ creation and annihilation in a semirealistic way. This is in marked contrast with other models,[1] where the number of constituent partons is held fixed and independent of the degree of excitation of the system.[6]

Like all parton models, our model raises almost as many questions as it answers. Its ultimate success or failure will depend on the extent to which it can be fully quantized, and of course on the extent to which its predictions are borne out by experiment.

It is a pleasure to acknowledge stimulating discussions with Dr. L. Susskind and Dr. J. T. Manassah. The author also wishes to thank Dr. G. Feinberg for a critical reading of the manuscript.

[1]Cf. Y. Aharonov, A. Casher, and L. Susskind, Phys. Lett. 35B, 512 (1971); also, J. T. Manassah and S. Matsuda, Phys. Rev. D 4, 3062 (1971). The partons may or may not be quarks.

[2]There would also be a surface tension, but this does not affect the calculation to be carried out in the present paper.

[3]A spinning disk is not stable, because only the outer-

most differential ring receives a net centripetal force from the tension.

[4]The author wishes to thank Dr. L. Susskind and Dr. J. T. Manassah for pointing this out.

[5]This assumption is not critical. The trajectory function is fairly insensitive to the value of $S_0$.

[6]The models with infinitely many constituent partons are defined as the large-$N$ limits of models with a fixed number $N$ of partons. If the interactions among the $N$ constituent partons are assumed to be due to virtual-parton exchange, then the total number of partons in a hadron should actually *decrease* in these models when a hadron is stretched, because multiparticle exchange should occur less frequently when distances are increased between the $N$ partons. The author thanks Dr. J. T. Manassah for a discussion of this point.

# ERRATUM

LOCALIZED-TO-ITINERANT ELECTRON TRANSITIONS IN RARE-EARTH COBALTATES. V. G. Bhide, D. S. Rajoria, Y. S. Reddy, G. Rama Rao, G. V. Subba Rao, and C. N. R. Rao [Phys. Rev. Lett. 28, 1133 (1972)].

Equation (1) should read as follows:

$$\mathrm{Co}^{3+}/\mathrm{Co}^{\mathrm{III}} = (N^2\mu^2/3R\chi_g T)^{-1} - 1.$$