Mercury's Perihelion Advance: Determination by Radar

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Measurements of echo delays of radar signals transmitted from Earth to Mercury have yielded an accurate value for the advance of the latter's perihelion position. Given that the Sun's gravitational quadrupole moment is negligible, the result in terms of the Eddington-Robertson parameters is $(2+2\gamma-\beta)/3 \simeq 1.005 \pm 0.007$, where $\gamma=\beta=1$ in general relativity, and where 0.007 represents the statistical standard error. Inclusion of the probable contribution of systematic errors raises the uncertainty to about 0.02.

Interplanetary radar observations, although essentially uncoupled from the "fixed" stars, are nonetheless very sensitive to changes in orbital perihelion positions. For all presently practical purposes, the Sun and planets form a closed dynamical system; the perihelion position of one planet can be determined relative to that of another from the radar measurements of echo delay. Because of the large eccentricity and nearness to the Sun of Mercury's orbit, its non-Newtonian perihelion advance is the easiest to estimate accurately. That radar could provide data for a significant test of the related prediction of general relativity has long been recognized.[1] Here we report the results of an analysis of five years of radar observations of Mercury and Venus as they relate to this well-known test of Einstein's theory.[2] These data were obtained primarily between 1966 and 1971 at the Haystack (Massachusetts) and Arecibo (Puerto Rico) Observatories. All told, there are about 150 Arecibo-Mercury and 200 Haystack-Mercury time-delay measurements; the former were obtained at a radar frequency of 430 MHz and the latter at 7840 MHz. The individual measurement errors are mostly between 5 and 20 μsec. The number of Earth-Venus observations[3] exceeds 500 with many of the recent Haystack time-delay measurements having uncertainties of only 1 μsec. Doppler-shift measurements from Haystack's Mercury and Venus observations were included but have little effect on the results. No optical data at all were included.

In our analysis we considered all potentially important gravitational interactions within the framework of general relativity. As expected, it proved more than sufficient to use for the planets the equations of motion that follow from the Schwarzschild metric for the Sun, supplemented by the Newtonian interplanetary perturbations. To estimate the non-Newtonian perihelion advance explicitly, we parametrized the equations of motion, expressed in harmonic coordinates, such that all non-Newtonian terms were multiplied by the *ad hoc* parameter λ_p. This parametrization does not test the contributions of the individual relativistic terms, but just their cumulative effect. This limitation is of no practical consequence since the existing radar data are very insensitive to all of the predicted relativistic effects on planetary motion save for the non-Newtonian perihelion advance. An estimate of λ_p under these circumstances is therefore equivalent to an estimate of $(2+2\gamma-\beta)/3$, where γ and β are the Eddington-Robertson parameters.[4]

Although the non-Newtonian advance of Mercury's perihelion position is by far the largest relativistic effect, radar observations of Venus play an important role in our estimate of λ_p. The reason is simple. All observations are made from Earth and hence its orbital elements must be determined along with Mercury's when estimating λ_p. Primarily because of the closer approaches of Venus to Earth, much of the corresponding radar echo-delay data are more accurate than the Earth-Mercury data (see above) and serve to determine a more precise orbit for Earth than would be possible with the latter data alone.[5]

In addition to λ_p, therefore, we had to estimate from these data the four "in-plane" orbital parameters[6] for each of the three innermost planets; the light-second equivalent of the astronomical unit (a consequence of our choice of units[6]); the mass of Mercury; the mean equatorial radii of Mercury and Venus; a plasma parameter to account for the interplanetary medium; and two "bias" parameters to disclose possible systematic differences between the Haystack and Arecibo observations of Mercury and Venus.[6] The other orbital elements and masses of the inner planets, as well as the orbits and masses of the Moon and outer planets, are known sufficiently well from other observations so that the uncertainties are too small to affect significantly our estimate for λ_p. A similar comment applies *a fortiori* to the rotation of Earth about its center of mass. The solar corona is of no concern since the data that determine λ_p were obtained mostly near inferior conjunctions where the radio signals do not penetrate inside Mercury's orbit. We also made the assumption that the standardly defined dimensionless parameter J_2, describing the Sun's gravitational quadrupole moment, is zero. The data do not allow a useful result to be obtained from a simultaneous estimate of both λ_p and J_2.

The only other possibly significant source of systematic error is the variation of topography over the equatorial regions of the target planets. Venus's topography presents perhaps the greatest difficulty because of the apparently strong coupling between its spin and the relative orbital motions of Earth and Venus. Nonetheless, significant progress has been made and agreement is reasonable between the results for surface-height variations obtained from echo-delay data and those obtained from absorption effects of the Venus atmosphere on Haystack's X-band radar signal.[7] For the main target, Mercury, the surface-height variations are relatively inconspicuous. None have yet been detected reliably from the echo-delay measurements to the subradar point. But about one third of the equatorial circumference has been mapped by another technique which discloses occasional small (<2 km) peaks and valleys.[8] Aside from these apparently minor variations, there is another reason why Mercury's topography should not affect seriously our estimate of λ_p: The orbital periods of Earth and Mercury are incommensurable and measurements at many different parts of Mercury's orbit have each been made from different parts of Earth's orbit. Thus, although Mercury's spin is coupled to its orbital motion, the effects of topography on the estimate of the orbit will tend to average out over the more than five-year span of the radar data.

Our weighted-least-squares solution for the twenty parameters described above yields

$$(2+2\gamma-\beta)/3 \simeq \lambda_p = 1.006 \pm 0.006,$$

where 0.006 represents the formal standard error and is based on the error assigned to each of the echo delays from a consideration of the signal-to-noise ratio and the relevant systematic errors that might be introduced by the measurement apparatus. Because the lack of a representation of planetary topography is the most serious deficiency in the twenty-parameter theoretical model, we added two-dimensional Fourier series to represent separately the surface-height variations in the equatorial regions of each planet. We considered different numbers of terms in the Fourier series to test more thoroughly the sensitivity of our estimate for λ_p to the surface-height variations. The number of extra parameters involved ranged from 40 to 122 for each planet; the total number in a given solution reached a high of nearly 250. The result for λ_p in no case differed from unity by more than about 0.01. The solution with a typical representation of topography (eighty extra parameters) yielded

$$(2+2\gamma-\beta)/3 \simeq \lambda_p = 1.004 \pm 0.007.$$

The post-fit residuals for the Mercury echo-delay observations for this solution, displayed in Fig. 1 as a function of the longitude of the subradar point,[9] have a weighted-rms value near unity as expected.

The estimates obtained for the other parameters from each of the solutions were all of a nonsurprising nature, with one exception. The bias between the Arecibo and Haystack Mercury observations was consistently in the range from 7 to 10 μsec, with the delays measured at Arecibo being the larger. (The corresponding bias for

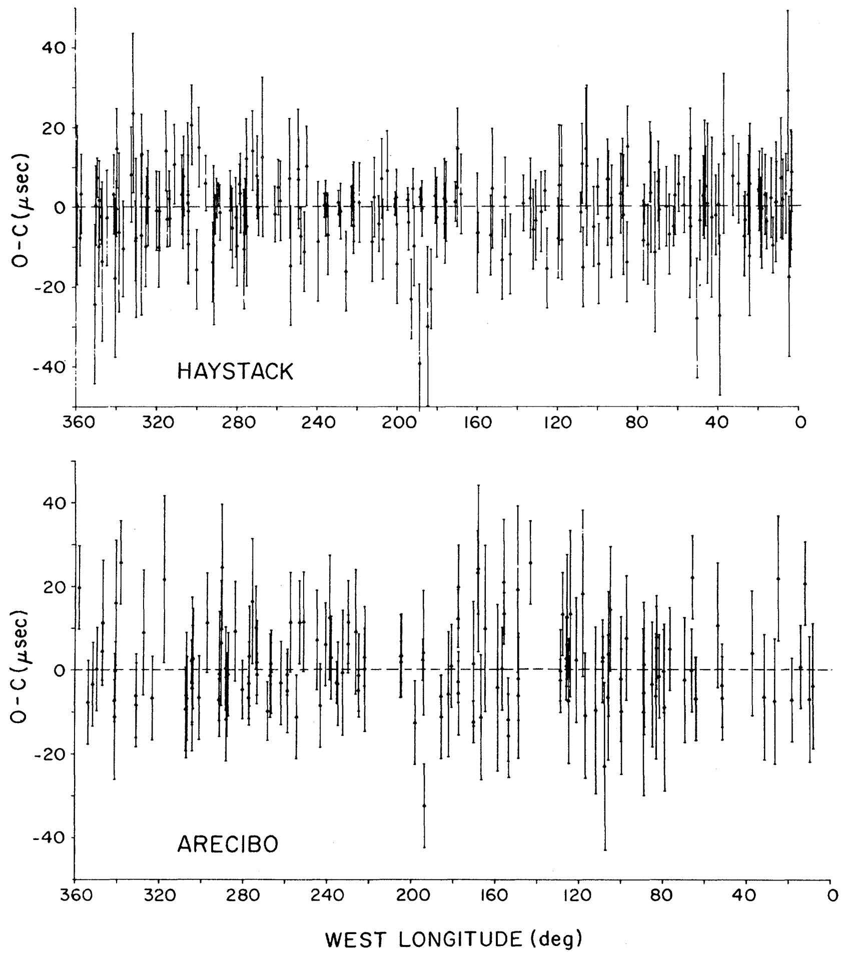

FIG. 1. Post-fit residuals for Earth-Mercury time-delay measurements made at the Haystack and Arecibo Observatories. The residuals are displayed as a function of the longitude of the subradar point on Mercury; low-frequency longitudinal variations in surface heights were removed by means of a Fourier-series parametrization (see text).

the Venus observations was invariably 1 μsec or less.) The possible cause of this discrepancy is still under investigation; it may be related to the radar scattering law since Mercury's surface seems in general to be far coarser on the scale of a wavelength then does Venus's surface. To investigate the possible effect of such a bias on our result for λ_p, we made additional least-squares solutions by alternately suppressing the Haystack and Arecibo data. In some cases the surface-height parameters were also omitted. The results for λ_p were each consistent with a 1.00 value within the somewhat larger bounds of 0.02 and 0.04, for Haystack and Arecibo data, respectively.

In consideration of all of the above, our best judgement is that the radar data alone yield a value for λ_p that does not differ significantly from unity; our estimate of the standard error is about 0.02, mainly attributable to an allowance for the probable contributions of systematic errors.[10] Combining the above with our result for γ[6] yields

$$\gamma \simeq 1.0 \pm 0.1, \quad \beta \simeq 1.1 \mp 0.2$$

for the Eddington-Robertson parameters. Our analysis, as mentioned above, is predicated on the assumption that the solar gravitational quadrupole moment is zero. The result for λ_p can therefore be considered a confirmation of general relativity only insofar as the contribution of J_2 to the orbit of Mercury is negligible.[11] Approximately five years of additional radar observations of the inner planets are required in order to separate usefully the contributions to their orbits of λ_p and J_2. A covariance analysis based on expected improvement in measurement accuracy and in the modeling of planetary topography indicates that the uncertainty of J_2 would be reduced to about 3×10^{-6} and that of λ_p to about 0.3%.

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[1]I. I. Shapiro, Phys. Rev. Lett. 13, 789 (1964), and Massachusetts Institute of Technology Lincoln Laboratory Technical Report No. 368, DDC No. 614232, 1964 (unpublished).

[2]The radar observations of Mars, at tne present state of analysis, cannot contribute usefully to this test.

[3]These include observations from the Millstone Hill radar of the Massachusetts Institute of Technology Lincoln Laboratory and from the Goldstone radars of the Jet Propulsion Laboratory. The latter were kindly sent to us by R. M. Goldstein, J. H. Lieske, and W. G. Melbourne.

[4]See, for example, H. P. Robertson, in *Space Age Astronomy,* edited by A. J. Deutsch and W. E. Klemperer (Academic, New York, 1962), p. 228.

[5]The existing radar observations of Venus do not allow the advance of its orbital perihelion, or Earth's, to be determined with an accuracy useful for testing general relativity.

[6]I. I. Shapiro, M. E. Ash, R. P. Ingalls, W. B. Smith, D. B. Campbell, R. B. Dyce, R. F. Jurgens, and G. H. Pettengill, Phys. Rev. Lett. 26, 1132 (1971). [Note that the result of the first radar time-delay test of general relativity $(1+\gamma)/2\simeq 0.9\pm0.2$ was misprinted as 0.09 ± 0.2 in this reference.]

[7]D. B. Campbell, R. B. Dyce, R. P. Ingalls, G. H. Pettengill, and I. I. Shapiro, Science 175, 514 (1972); A. E. E. Rogers, R. P. Ingalls, and L. P. Rainville, Astron. J. 77, 100 (1972).

[8]R. P. Ingalls and L. P. Rainville, Astron. J. 77, 185 (1972).

[9]We use the definition of longitude adopted in *Proceedings of the Fourteenth General Assembly of the International Astronomical Union,* edited by C. De Jager and A. Jappel (D. Reidel, Dordrecht, Holland, 1971), p. 28. In this system, the Sun was above the zero meridian at the time of Mercury's first perihelion passage in 1950.

[10]Prior determinations of the perihelion advance of Mercury's orbit, based solely on optical observations, yielded the equivalent of $\lambda_p\simeq 1.00\pm0.01$ [G. M. Clemence, Astron. Papers Amer. Ephemeris Nautical Almanac 11, Part 1 (1943); see also R. L. Duncombe, *ibid.* 16, Part 1 (1959)].

[11]By the same token, given that $\lambda_p\equiv1$, our data show that $J_2 < 5\times10^{-6}$.

Measurement of $|\eta_{00}|/|\eta_{+-}|$†

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We have measured the quantity $|\eta_{00}|/|\eta_{+-}|$ which is defined in terms of K_L and K_S decay rates by the ratio $[\Gamma(K_L\to\pi^0\pi^0)\Gamma(K_S\to\pi^+\pi^-)/\Gamma(K_L\to\pi^+\pi^-)\Gamma(K_S\to\pi^0\pi^0)]^{1/2}$. We find $|\eta_{00}|/|\eta_{+-}|=1.03\pm0.07$.

In the study of CP nonconservation in K_L decay[1] the crucial question has become the following: Does a single parameter in the mass matrix account for all the CP-nonconservation phenomena observed? If the ratio $|\eta_{00}|/|\eta_{+-}|^2$ were found to be different from unity, such a single parameter could not describe the CP-invariance violation. From an experimental point view, the principal difficulty in the determination of this ratio has been the measurement of the decay rate $\Gamma(K_L \to \pi^0\pi^0)$ with sufficient precision.

We have measured the ratio $|\eta_{00}|/|\eta_{+-}|$ using a K_L beam of mean momentum 6 GeV/c produced by the alternating gradient synchrotron at Brook-