Unified Weak and Electromagnetic Interactions without Neutral Currents*

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We construct unified models of weak and electromagnetic interactions using just three gauge fields which correspond to the intermediate vector boson and the photon. These models may be renormalizable, and do not predict processes involving neutral lepton currents.

Recent work[1,2] indicates the possibility of constructing a renormalizable theory of weak and electromagnetic interactions. The published models[1,3,4] require the existence of a neutral massive intermediate vector boson coupled to leptons and to a strangeness-conserving neutral hadron current: They predict the occurrence—with typical weak cross sections—of a number of (as yet) unobserved processes.[5] In this note we show how it is possible to synthesize weak and electromagnetic interactions without the appearance of any neutral current besides the electromagnetic current.

First, we sketch the format of existing models. The Lagrangian is exactly invariant under a local gauge group $\mathcal{G}$, and involves a number of massless self-coupled spin-1 gauge fields equal to the number of generators of $\mathcal{G}$. These are invariantly coupled to a multiplet of fermions (including leptons and hadrons) and to a multiplet of spinless mesons φ. The charges associated with the observed weak interactions Q_W and the electromagnetic charge Q_E generate an important subgroup $\mathcal{G}'$ of the gauge group, which in published theories is isomorphic to SO(3)⊗U(1). The SO(3) generators are given by Q_W, $Q_{\overline{W}}$, and Q_3, where $[Q_W, Q_{\overline{W}}]=Q_3$; $[Q_W,Q_3]=Q_W$; $[Q_3,Q_{\overline{W}}]=Q_{\overline{W}}$; and the remaining generator satisfies

$$[Q_3, Q_4]=[Q_W, Q_4]=[Q_{\overline{W}}, Q_4]=0,$$

with

$$Q_E=Q_3+Q_4.$$

Thus the electrical charge is given as a mixture of singlet and triplet under the rotation group generated by Q_W. In some models $\mathcal{G}'=\mathcal{G}$ and there are just four gauge fields[1,4]; in other models there are more than four fields.[3,6]

The gauge symmetry is spontaneously broken: The scalar mesons develop nonzero vacuum expectation values. Instead of the appearance of massless Goldstone bosons, the Higgs mechanism[7] causes certain of the massless gauge fields to acquire mass: Only those gauge fields whose generators annihilate $\langle\varphi\rangle_0$ remain massless. With proper choice of the scalar-meson multiplet, it is only the photon that remains massless. There remains at least one "new" massive vector boson which mediates such phenomena as

$$\nu'+e\to\nu'+e, \quad \nu'+p\to\nu'+\cdots.$$

These processes have not been seen.[8] Can we modify this construction so as to avoid such predictions?

One way[9] is to choose a smaller gauge group; we must arrange matters so that the electrical charge and the weak charge generate a rotation group among themselves:

$$[Q_W, Q_{\overline{W}}]=Q_E; \quad [Q_W, Q_E]=Q_W; \quad [Q_E, Q_{\overline{W}}]=Q_{\overline{W}}.$$

The gauge group will then be isomorphic to SO(3) and the only gauge fields will be $W^\pm$ and the photon A.[10]

The weak currents, involving the electron and its neutrino, do not generate this algebra. The simplest way we have found to remedy this is to introduce new leptons: Together with the electron and its neutrino we associate a pair of unobserved heavy leptons X^0 and X^+. We define two triplets of electronic states:

$$\vec{\psi}_{eR}=\tfrac{1}{2}(1-\gamma_5)[X^+; X^0; e^-],$$

$$\vec{\psi}_{eL}=\tfrac{1}{2}(1+\gamma_5)[X^+; X^0\cos\beta+\nu\sin\beta; e^-],$$

which transform as vectors under the gauge group. The remaining states

$$s_L=\tfrac{1}{2}(1+\gamma_5)(X^0\sin\beta-\nu\cos\beta),$$

$$s_R=\tfrac{1}{2}(1-\gamma_5)\nu,$$

transform as singlets. The coupling of the vector bosons to these fermions is

$$e\vec{W}_\mu\cdot(\vec{\psi}_e\times\gamma_\mu\vec{\psi}_e),$$

where $W_3 = A$; and in a more compact notation

$$\vec{\psi}_e = \vec{\psi}_{eL} + \vec{\psi}_{eR} = [X^+; (X_R^0 + \cos\beta X_L^0 + \sin\beta \nu_L); e^-].$$

More explicitly, we have

$$eA_\mu(\bar{X}^+\gamma_\mu X^+ - \bar{e}^-\gamma_\mu e^-) + [\tfrac{1}{2}eW_\mu^+(\sin\beta\bar{e}^-\gamma_\mu(1+\gamma_5)\nu + \cdots) + \text{H.c.}],$$

where $\cdots$ indicates terms involving the weak interactions of unobserved leptons. For the dimensionless coupling strength of the observed weak interactions, we identify

$$g_W = \tfrac{1}{2}e \sin\beta,$$

and hence obtain $M_W = 53.0 \sin\beta$ GeV/c^2.[11]

For the muons, we must again introduce a pair of heavy unobserved muonic leptons, Y^0 and Y^+, and we construct another triplet

$$\vec{\psi}_\mu = [Y^+; (Y_R^0 + Y_L^0\cos\beta' + \nu_L'\sin\beta'; \mu^-]$$

and singlet

$$s_L' = (Y^0\sin\beta' - \nu'\cos\beta')_L, \quad s_R' = \nu_R',$$

which is coupled to the vector bosons according to

$$e\vec{W}_\lambda\cdot(\vec{\psi}_\mu \times \gamma_\lambda \vec{\psi}_\mu).$$

In this model, because it is "vectorlike,"[12] the leptons may have masses before the gauge symmetry is broken. Most generally, each of the lepton multiplets $\vec{\psi}_e$, $\vec{\psi}_\mu$, s, and s' may have its own invariant mass. In order to guarantee that the neutrinos remain massless, the masses of s' must be zero.

Scalar mesons are introduced in order to spontaneously break the gauge symmetry and to reproduce the observed lepton spectrum. This may be done most simply with a single real triplet of scalar mesons $\vec{\varphi}$, invariantly coupled to the vector bosons and to the leptons. The unique neutral member of the triplet develops a vacuum expectation value and survives as a physical particle; the charged members of the triplet disappear while the $W^\pm$ gets its mass.

The scalar-meson triplet is invariantly coupled to the leptons so as to independently conserve muon number and electron number,[13] and is not coupled to the unobserved neutrino states ν_R and ν_R'. Its couplings are

$$\vec{\varphi}\cdot[g_1\vec{\psi}_e\times\vec{\psi}_e + g_2\vec{\psi}_\mu\times\vec{\psi}_\mu + g_3(\vec{\psi}_e s_L + \text{H.c.}) + g_4(\vec{\psi}_\mu s_L' + \text{H.c.})].$$

When φ^0 develops a vacuum expectation value χ, the six nonvanishing lepton masses and the angles β and β' are determined in terms of the six parameters $g_i\chi$ and the initial mass of each triplet. Two relations among these quantities may be deduced:

$$2m(X^0)\cos\beta = m(X^+) + m(e^-),$$

$$2m(Y^0)\cos\beta' = m(Y^+) + m(\mu^-).$$

We shall choose $\beta = \beta'$ to incorporate muon-electron universality: Note that this gives a relation among the lepton masses.

The model of leptons we have constructed has the following properties: (1) No neutral currents but the electromagnetic current; no neutral gauge field but the photon. (2) The theory is "vectorlike"—it is free of Adler-Bell-Jackiw anomalies.[14] (3) Muon and electron numbers are independently and exactly conserved. (4) The neutrinos, if initially massless, do not develop mass. (5) The vector boson must be lighter than 53.0 GeV/c^2. (6) Four new unobserved heavy leptons have been introduced, as well as one parametric angle β. The strength of observed weak interactions, and hence the W mass, is determined by β. (7) Muon-electron universality is not automatic, but has been put in by hand. (8) Just one neutral scalar meson survives as an observable particle; its only couplings to the lepton system involve unobserved states.

The SO(3) scheme is readily extended to hadrons without the introduction of any scalar field besides $\vec{\varphi}$. A minimum of five quark fields is needed: a unitary triplet (p, n, λ) of ordinary integral-charge quarks of charges $(1, 0, 0)$ and two presumably heavier singlets (q^-, q^0) of charges $(-1, 0)$. The model is compatible with the existence of a massive vector gluon coupled to quark number, giving rise to strong interactions. These strong interactions are invariant under chiral U(5). The observed hierarchy of strong-interaction symmetries results from the quark mass spectrum. Mesons are constructed out of ordinary quark-antiquark pairs, while baryons involve a single q^0 together with one or more ordinary quark-antiquark pairs. Hadrons containing q^- have not yet been observed, presumably because $m(q^-) > m(q^0)$.

Under the SO(3) gauge group, the five quarks transform as a triplet $\vec{\psi}_H$ and two singlets. The gauge-invariant vector-boson coupling is, as for leptons, $e\vec{W}_\mu\cdot(\vec{\psi}_H\times\gamma_\mu\vec{\psi}_H)$.

The "observed" quark mass spectrum is produced by the interplay of gauge-invariant mass terms ($\vec{\psi}_H \cdot \vec{\psi}_H$, etc.) and terms proportional to χ arising from the SO(3)-invariant (but not space-reflection invariant) couplings of $\vec{\varphi}$ with the quarks. Much as for leptons, the parameters may be chosen such that the triplet is

$$\vec{\psi}_H = [p; q_R{}^0 + (q^0 \cos\beta + n \cos\theta \sin\beta + \lambda \sin\theta \sin\beta)_L; q^-],$$

and the singlets are λ_R, n_R and

$$(\lambda \cos\theta - n \sin\theta)_L,$$

$$(n \cos\theta \cos\beta + \lambda \sin\theta \cos\beta - q^0 \sin\beta)_L.$$

As for the lepton systems, the neutral current is the electromagnetic current, and the charged current is

$$\tfrac{1}{2} e \sin\beta \bar{p} \gamma_n (1+\gamma_5)(n \cos\theta + \lambda \sin\theta) + \cdots,$$

where again, terms corresponding to unobserved transitions to "charmed" hadron states have been omitted. Universality is maintained by choosing β the same as for the lepton currents.

One relationship emerges among the quark masses and the angles

$$2m(q^0) \cos\beta = m(q^-) + m(p).$$

To ensure that q^0 is lighter than q^- we must have $\cos\beta > \frac{1}{2}$, and hence $M_W < 45.6$ GeV/c^2. However, we shall obtain a much more stringent upper limit on M_W in this model.

Second-order weak interactions will yield finite results for $\Delta Y = 2$ processes (e.g., K_1-K_2 mass splitting) and for $\Delta Y = 1$ couplings to neutral lepton currents (e.g., $K_2{}^0 \to \mu\bar{\mu}$) of order $G(GM_W{}^2)$.[15] But limits on these processes suggest that $M_W < 4$ GeV/c^2.[16] Thus theories of this kind are acceptable only if β is chosen to be quite small ($\beta < 0.08$). The most striking property of the five-quark model is that the W must be light.

Bjørken has found[15] that with a larger number of quark fields it is possible to circumvent the appearance of $\Delta Y = 2$ and strangeness-changing neutral lepton currents to order $G(GM_W{}^2)$ by an artifice like that of Glashow, Iliopoulos, and Maiani.[17] For instance, we can introduce eight quarks with an SO(3) structure like the leptons: two triplets and two singlets. Again the triplet $\vec{\varphi}$ of scalar fields suffices to break the symmetry and account for the quark mass spectrum. In such models, the unwanted processes still appear, but to order $G(G\Delta^2)$ where Δ is a typical quark mass splitting, and no drastic upper limit on M_W may be deduced. There is considerable arbitrariness in such models, so that a detailed discussion seems premature until an appealing model is found.

Whichever hadron scheme is adopted, the resulting theory has certain attractive features. We have achieved a weak-electromagnetic synthesis involving only $W^\pm$ and A. There are no unobserved neutral currents; instead there are extra quark and lepton states. Only three gauge fields and three scalar mesons are needed. There are no triangle anomalies, either for the leptonic or hadronic parts of the currents. However, anomalies do appear in the strong axial-vector current whose divergence has the quantum numbers of the pion. Since the relevant quarks are just $p(Q=1)$ and $n(Q=0)$, the correct sign and magnitude of the amplitude for $\pi^0 \to 2\gamma$ is obtained.[18]

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[1]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967).

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[4]C. Bouchiat, J. Iliopoulos, and Ph. Meyer, to be published.

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[8]Only upper limits on these cross sections are available in the literature. For $\nu' p \to \nu' p$ see D. C. Cundy *et al.*, Phys. Lett. 31B, 478 (1970); for $\nu' e \to \nu' e$ see C. H. Albright, Phys. Rev. D 2, 1330 (1970).

[9]Another way is to arrange things so that the neutral current does not involve neutrinos, as in any model where the neutrino is the central member of a triplet under the relevant SU(2) gauge group.

[10]Such a possibility was first suggested by J. Schwinger [Ann. Phys. (New York) 2, 407 (1957)], and S. L. Glashow [Nucl. Phys. 10, 107 (1959)], and A. Salam and J. C. Ward [Nuovo Cimento 11, 568 (1959)].

[11]Cf. T. D. Lee, Phys. Rev. D 3, 801 (1971).

[12]Interactions are vectorlike if they may be written in terms of vector currents alone with a suitable definition of the fermion fields. Such interactions have no

triangle anomalies. See Ref. (6).

[13]The four electronic leptons have lepton number one; the four muonic leptons have muon number one.

[14]S. L. Adler, Phys. Rev. 177, 2426 (1969); J. S. Bell and R. Jackiw, Nuovo Cimento 60, 47 (1969). In the present context, these anomalies have been studied by D. Gross and R. Jackiw, to be published.

[15]J. D. Bjørken, private communication.

[16]E.g., F. E. Low, Comments Nucl. Particle Phys. 2, 33 (1968). He concludes that the cutoff in a divergent theory of weak interactions must be less than 4 GeV. In the present finite theory, the role of the cutoff is played by the W mass.

[17]S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[18]S. Adler, in *Lectures on Elementary Particles and Quantum Field Theory; 1970 Brandeis University Summer Institute in Theoretical Physics,* edited by S. Deser *et al.* (Massachusetts Institute of Technology Press, Cambridge, Mass., 1970), Vol. 1, pp. 1–164.

ERRATA

STRESSES PRODUCED BY A CONTINUOUS DISTRIBUTION OF MOVING DISLOCATIONS IN AN ISOTROPIC CONTINUUM. Sitiro Minagawa and Takao Nishida [Phys. Rev. Lett. 28, 353 (1972)].

Equation (4) should read as follows:

$$\psi_{ij} = 2\rho[\psi_{ij}' - (1-2\nu)^{-1}\delta_{ij}\psi_{kk}'],$$
$$2\rho\psi_{ij}' = \psi_{ij} - (2\nu)^{-1}\delta_{ij}\psi_{kk}.$$

PARTICLE CREATION IN ISOTROPIC COSMOLOGIES. Leonard Parker [Phys. Rev. Lett. 28, 705 (1972)].

In Eq. (3), dt^2 should be replaced by $-dt^2$. In the sixth line following Eq. (11), replace $(-t)^{-1/2}$ by $(-t)^{1/2}$. On page 708, the second line should read "$\cdots m^3$ and energy density of order $m^4\cdots$."

COSMIC-RAY PROTON AND HELIUM SPECTRA ABOVE 50 GeV. M. J. Ryan, J. F. Ormes, and V. K. Balasubrahmanyan [Phys. Rev. Lett. 28, 985 (1972)].

On page 987 the formulas for the proton and helium fluxes should read as follows:

$$dN_p/dE = (2.0 \pm 0.2)\times 10^4 E^{-2.75\pm 0.03} \text{ protons m}^{-2}\text{ sr}^{-1}\text{ sec}^{-1}\text{ GeV}^{-1},$$

and

$$dN_{\mathrm{He}}/dE = (8.6 \pm 1.4)\times 10^2 E^{-2.77\pm 0.05} \text{ He m}^{-2}\text{ sr}^{-1}\text{ sec}^{-1}\text{ (GeV/nucleon)}^{-1}.$$

We would like to thank Howard Verschell of the University of New York for bringing this error to our attention.

USE OF THE (^{7}Li, ^{7}Be) REACTION TO MEASURE THE MASS OF ^{26}Na. G. C. Ball, W. G. Davies, J. S. Forster, and J. C. Hardy [Phys. Rev. Lett. 28, 1069 (1972)].

The footnotes to Table I were omitted and should read as follows: [a]Wapstra and Gove (Ref. 8). [b]Calculated using Eq. (2) of Ref. 7 with $N-Z = 1$ for both even and odd values of N. The parameters N and Z are as defined in Ref. 7. [c]Calculated assuming ^{10}Li is unbound to neutron emission and ^{14}B is bound. [d]Present work.

MISSING-MASS SPECTRA PRODUCED BY 2-GeV PROTONS IN THE REACTION $p + d \to \mathrm{He}^3 + X^0$. H. Brody, E. Groves, R. Van Berg, W. Wales, B. Maglich, J. Norem, J. Oostens, and G. B. Cvijanovich [Phys. Rev. Lett. 28, 1215 (1972)]; and ANALYSIS OF $p + d \to \mathrm{He}^3 + X^0$ EXPERIMENTS. H. Brody [Phys. Rev. Lett. 28, 1217 (1972)].

Figure 1 on page 1216 should be interchanged with Fig. 3 of page 1219. (The figure captions are correct as they stand.)