Finite-Dispersion-Relation Restrictions on Interfering Meson Models

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Using the finite-dispersion-relation method for treating three-body decays, we find that bounds can be placed upon the coupling strengths of two interfering resonances. A special result is that two narrow interfering coherent (or incoherent) resonances for the A_2 meson fail to satisfy a finite-dispersion-relation, finite-energy sum rule for coupling strengths (widths) which follows from the $\eta \to \pi^+\pi^-\gamma$ decay rate. This appears to be the first evidence independent of mass distributions indicating that one of the tabulated descriptions of the A_2 is wrong.

When well-known mesons mix or interfere, the resulting distortions in mass distributions lead to further information about the particles involved. However, if only the distorted mass distribution is available, as has been the case with the A_2 meson, little reliably certain information on the two particles is obtainable; the present particle data tabulations therefore list several possible particle configurations for the A_2 meson.

In this paper we present arguments showing that it is highly unlikely that A_2 meson consists of two narrow interfering or noninterfering particles. The present work does not depend on A_2 mass distributions; it is based mainly on the concepts of finite-energy sum rules (FESR), duality, finite dispersion relations (FDR), and crossing to place parameter-free limits on products of A_2 (or A_2 constituents in interfering resonance models) coupling constants from the measured three-body decay rate for $\eta \to \pi^+\pi^-\gamma$. These limit requirements can be cast in the form of a sum rule, should one so wish. This sum rule can readily be satisfied by a broad A_2 meson, as in a single Breit-Wigner description or in models with broad and narrow mixed or interfering components, because considerable coupling (at least to the $\pi\rho, \pi\eta$ channels) is required. This result justifies the usual phenomenological treatment of the A_2 as a single object in, e.g., t-channel Regge-pole exchange.

High-energy scattering techniques and ideas incorporating duality have led to useful insight and to successful predictions for several low-energy decay-type processes. For example, Aviv and Nussinov[1] and Gounaris and Verganelakis[2] easily explained the branching ratios found experimentally for $\omega \to \pi\pi\gamma$ and $\eta \to \pi\gamma\gamma$, respectively, which were considerably larger than expected from conventional pole models such as the Gell-Mann–Sharp–Wagner model. Similarly, the results of octet dominance in S-wave hyperon decays have been obtained, neatly incorporating absence-of-exotics requirements with duality, by Nussinov and Rosner[3] and Kawarabayashi and Kitakado,[4] without use of current algebra.

The present work makes assumptions similar to those in the calculations mentioned above (Refs. 1–4). We write a dispersion relation in $\nu = \frac{1}{2}(s-u)$ for the scattering process $\eta\pi \to \gamma\pi$ obtained by crossing from $\eta = \pi^+\pi^-\gamma$. Duality is used to argue that the cut integral along Reν need only be taken to a finite ν value from where Regge behavior completes the amplitude description. FESR are used to evaluate the Regge residue, which is in remarkable agreement with independent SU(3) predictions.

The process of interest, depicted in Fig. 1 with four-momenta and polarization labeling the particle lines, is described by the amplitude

$$A = \epsilon^\mu \epsilon_{\mu\nu\lambda\rho} q_1{}^\nu q_2{}^\lambda k^\rho B(\nu, t), \tag{1}$$

where $s = (p - q_1)^2$, $t = (q_1 + q_2)^2$, $u = (p - q_2)^2$, and $\nu = \frac{1}{2}(s-u)$. Under s-u crossing ($\nu \to -\nu$), $B(\nu, t) = B(-\nu, t)$.

In the s and u channels, quantum numbers are such that of the low-lying resonances only the A_2 contributes:

$$B_{\mathrm{res}}(\nu, t) = -4 g_{A_2\pi\gamma} g_{A_2\eta\pi} \left[\frac{t - u - m_\pi{}^2(m_\eta{}^2 - m_\pi{}^2) m_{A_2}{}^{-2}}{s - m_{A_2}{}^2} + \frac{t - s - m_\pi{}^2(m_\eta{}^2 - m_\pi{}^2) m_{A_2}{}^{-2}}{u - m_{A_2}{}^2} \right]. \tag{2}$$

Since only the absorptive part of B_{res} enters in the following, $g_{A_2\pi\gamma}$ and $g_{A_2\eta\pi}$ can be taken as physical couplings. In the t channel ($\gamma\eta \to \pi^+\pi^-$), the odd-signature ρ Regge-pole exchange amplitude is

$$B_\rho(\nu, t) = \frac{\pi\beta}{\Gamma(\alpha_\rho(t)) \sin\pi\alpha_\rho(t)} \left[\nu^{\alpha_\rho(t) - 1} + (-\nu)^{\alpha_\rho(t) - 1} \right]. \tag{3}$$

Through use of the Cauchy theorem, a FDR[1] expresses $B(\nu, t)$ in terms of cut integrals along the Reν axis (where the absorptive part of B_{res} is the spectrum along the cut), and in terms of semicircles closing the contour at a finite value, $|\nu| = N$, which is large enough so that the Regge description is valid for $\nu \geq N$. As a result, $B(\nu, t)$ can be written in terms of low- and high-energy parts, $B \equiv B_L + B_H$, with the resonance contribution

$$B_L(\nu, t) = -4g_{A_2\pi\gamma}g_{A_2\eta\pi}\left[\frac{2t + m_{A_2}{}^2 - m_\eta{}^2 - 2m_\pi{}^2 - m_\pi{}^2(m_\eta{}^2 - m_\pi{}^2)m_{A_2}{}^{-2}}{\nu - (m_A{}^2 - \frac{1}{2}m_\eta{}^2 - m_\pi{}^2 + \frac{1}{2}t)} + (\nu \to -\nu)\right], \tag{4}$$

and the Regge contribution

$$B_H(\nu, t) = -\frac{2\beta}{\Gamma(\alpha_\rho(t))}\sum_{n=0}^{\infty}\left(\frac{\nu}{N}\right)^{2n}\frac{N^{\alpha_\rho(t)-2n-1}}{\alpha_\rho(t) - 2n - 1}. \tag{5}$$

The lowest-moment finite-energy sum rule,

$$\frac{\beta}{\Gamma(\alpha_\rho(t))}\frac{N^{\alpha_\rho(t)+1}}{\alpha_\rho(t)+1} = \frac{1}{\pi}\int_0^N d\nu\, \nu\, \mathrm{Im}B_{res}(\nu, t), \tag{6}$$

is used to determine β at $t = 0$ as[5]

$$\beta = g_{A_2\pi\gamma}g_{A_2\eta\pi}6\sqrt{\pi}\,N_0{}^{-3/2}(m_{A_2}{}^2 - \tfrac{1}{2}m_\eta{}^2 - m_\pi{}^2)[m_A{}^2 - m_\eta{}^2 - 2m_\pi{}^2 - m_\pi{}^2(m_\eta{}^2 - m_\pi{}^2)m_{A_2}{}^{-2}], \tag{7}$$

where $N = N_0 + \frac{1}{2}t$, $\alpha_\rho(t) = \frac{1}{2} + t/2m_\rho{}^2$, and $N_0 = \frac{5}{3}m_{A_2}{}^2 - \frac{1}{2}m_\eta{}^2 - m_\pi{}^2 \simeq 2.65$ GeV^2, which follows from the duality prescription used in Refs. 1 and 2 for defining N as the average of $m_{A_2}{}^2$ and the next recurrence on the A_2 trajectory. Equations (1)–(7) determine the amplitude for our Fig. 1 process completely in terms of the couplings $g_{A_2\pi\gamma}$ and $g_{A_2\eta\pi}$, which themselves follow from the partial widths for A_2 decay into $\pi\gamma$ and $\eta\pi$.

For the A_2 treated as a *single* Breit-Wigner resonance, with $m_{A_2} = 1300$ MeV, $\Gamma = 85$ MeV, and fractional decay[6] into $\rho\pi$ and $\eta\pi$ as 76% and 18%, respectively, we find $g_{A_2^\pm\rho\pi^\mp}{}^2/4\pi = 1.79$ GeV^{-4} and $g_{A_2^\pm\pi^\mp\eta}{}^2/4\pi = 0.614$ GeV^{-2}. From vector meson dominance, $g_{A_2^\pm\pi^\pm\gamma}{}^2/4\pi = 5.1\times10^{-3}$ GeV^{-4} follows, where a ρ-γ coupling $ef_\rho{}^{-1}$ with $f_\rho{}^2/4\pi = 2.56 \pm 0.22$ is used.[7] These coupling constants and Eqs. (1), (4), (5), and (7) lead to the theoretical prediction $\Gamma^{\mathrm{theor}}_{\eta\to\pi\pi\gamma} = 0.148$ keV, to be compared with the experimental value[6] $\Gamma^{\mathrm{expt}}_{\eta\to\pi\pi\gamma} = 0.123 \pm 0.032$ keV. The distribution of events as a function of photon momentum is, furthermore, also in excellent agreement with experiment.[8]

The A_2 or its constituents enter the FESR calculation in the narrow-resonance limit, which is justified in the present work since $s, u \leq (m_\eta - m_\pi)^2$ is much less than $m_0{}^2 = (1300)^2$ MeV^2. For two interfering[9] resonances in the A_2 region, the resonance amplitude can be written in this limit as

$$T_{\pi\gamma,\pi\eta}(s) \to \frac{F_1{}^{\pi\gamma}G_1{}^{\pi\eta} + F_2{}^{\pi\gamma}G_2{}^{\pi\eta}}{s - m_0{}^2 + i\epsilon}, \tag{8}$$

where $F_j{}^{\pi\gamma}$ and $G_j{}^{\pi\eta}$ denote the decay and production amplitudes of resonance j into or from the states $\pi\gamma$ and $\pi\eta$. The experimental result for $\Gamma_{\eta\to\pi\pi\gamma}$ implies restrictions on the residue $R \equiv F_1{}^{\pi\gamma}G_1{}^{\pi\eta} + F_2{}^{\pi\gamma}G_2{}^{\pi\eta}$ in Eq. (8),

$$10.5\ \mathrm{MeV}^2 \leq |F_1{}^{\pi\gamma}G_1{}^{\pi\eta} + F_2{}^{\pi\gamma}G_2{}^{\pi\eta}|^2 \leq 14.0\ \mathrm{MeV}^2, \tag{9}$$

where the normalization definitions are such that the single Breit-Wigner case (from values given above) yields $|F^{\pi\gamma}G^{\pi\eta}|^2 = 13.7$ MeV^2. For two nearby narrow coherent resonances, each of width $\Gamma = 25$ MeV (the so-called high-low model[6]),

$$|R_{\mathrm{high-low}}|^2 \leq 5\ \mathrm{MeV}^2, \tag{10}$$

well outside the limits in Eq. (9). Various explicit pictures for what happens in the high-low model will lead to considerable variation in $|R|^2$, but the upper limit of 5 MeV^2 is independent of the model.

Models ascribing different J^P values to the two narrow resonances are *more* strongly discriminated. For example, for the special case[10] of $J^{PC} = 1^{-+}$ and 2^{++} mesons making up the A_2, we

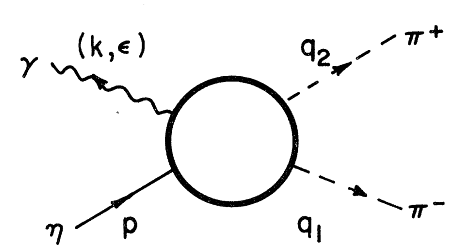

FIG. 1. Diagram for $\eta \to \pi^+\pi^-\gamma$ with four-momenta and polarization labeling particle lines.

find

$$|R_{1^{-+},2^{++}}|^2 \lesssim 2.5 \text{ MeV}^2. \qquad (11)$$

To summarize, in this paper the finite-dispersion-relation technique, though applied here for the purpose of getting information on the A_2 meson, has been shown to provide a useful tool with which to obtain information on resonance couplings. Other explorations with this method are therefore in progress.

In the present application, the single Breit-Wigner model for the A_2 satisfied Eq. (9) with a parameter-free prediction for the residue R (absolute value squared).[11] Models with mixed or interfering broad and narrow components can also satisfy Eq. (9), but the coupling of the broad component to $\pi\rho$ and $\pi\eta$ states must dominate. Two narrow resonances, whether interfering or not, do not satisfy condition (9); this strongly suggests that the results of analyses[12] using such narrow interfering resonances are not physically relevant and should not be included in particle data tables.

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[1]R. Aviv and S. Nussinov, Phys. Rev. D 2, 209 (1970). An enhancement factor of 5 over earlier predictions was found for $\omega \to \pi\pi\gamma$.

[2]G. J. Gounaris and A. Verganelakis, Nucl. Phys. B34, 418 (1971). An enhancement factor of 80 over earlier calculations was found for $\eta \to \pi\gamma\gamma$.

[3]S. Nussinov and J. L. Rosner, Phys. Rev. Lett. 23, 1264 (1969).

[4]K. Kawarabayashi and S. Kitakado, Phys. Rev. Lett. 23, 440 (1969).

[5]Using this result, Eq. (7), we find that $f_{\rho\eta\gamma}^2/4\pi = 0.115\alpha[G(m_\rho^2)]^2$, where $G(t)$ is a possible mild form factor expressing allowed t dependence in going from $t=0$ to the ρ pole. The only unreliability, perhaps 20% or less, comes from the usual uncertainties in FESR calculations. On the other hand, from SU(3) and the experimental decay rates for $\eta \to \gamma\gamma$ and $\pi^0 \to \gamma\gamma$, we find $f_{\rho\eta\gamma}^2/4\pi = (0.139 \pm 0.039)\alpha$ or $(0.015 \pm 0.026)\alpha$, depending on whether $\Gamma_{\pi^0 \to \gamma\gamma}$ is the average from the Particle Data Tables [A. Rittenberg *et al.*, Rev. Mod. Phys. 43, S1 (1971)] or the DESY value from G. Belletini *et al.*, Nuovo Cimento 66A, 243 (1969). The quark model value is $f_{\rho\eta\gamma}^2/4\pi = 0.075\alpha$.

[6]Rittenberg *et al.*, Ref. 5.

[7]This value is obtained from either the $\rho \to \pi\pi$ decay width by means of universality or the recent $\omega \to \rho$ interference experiment; see J. Lefrancois, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, 1971* (Laboratory of Nuclear Studies, Cornell Univ., Ithaca, N. Y., 1972), p. 51.

[8]A. M. Cnops *et al.*, Phys. Lett. 26B, 398 (1968).

[9]We follow the notation of K. E. Lassila and P. V. Ruuskanen, Phys. Rev. Lett. 19, 762 (1967), and in *Third Topical Conference on Resonant Particles, Athens, Ohio, 1967* (Ohio Univ. Press, Athens, Ohio, 1967).

[10]These particular values were predicted apparently by M. Gell-Mann and G. Zweig from a four-dimensional harmonic-oscillator model. See H. Harari, in *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September, 1968*, edited by J. Prentki and J. Steinberger (CERN Scientific Information Service, Geneva, Switzerland, 1970), pp. 197–198, for a summary of this work along with predictions.

[11]This result tends to enhance the credibility of the Northeastern University–State University of New York experiment, D. Bowen *et al.*, Phys. Rev. Lett. 26, 1663 (1971), whose results apparently disagreed with those of the CERN missing mass group, G. Chikovani *et al.*, Phys. Lett. 25B, 44 (1967).

[12]See, e.g., K. W. J. Barnham and G. Goldhaber, UCRL Report No. UCRL-20293, 1971 (unpublished), and in *Phenomenology in Particle Physics*, edited by C. B. Chiu, G. C. Fox, and A. J. G. Hay (California Institute of Technology Press, Pasadena, Calif., 1971), p. 307.