when analyzed using (22), gave a result $3\frac{1}{2}$ standard deviations below that of the second (g-IV).[1] But $\omega_{\parallel}/a\omega_0 \approx 5.5$ in g-III, so the limit (20) is applicable. When we reanalyze the g-III data using (20), we find $a_{g\text{-III}} = (1\,159\,690 \pm 30) \times 10^{-9}$, which is about 1 standard deviation above the g-IV value.

For g-IV the limit (22) does not quite apply since $\omega_{\parallel}/a\omega_0 \approx 0.4$ is not negligibly small. When we reanalyze the g-IV data using the general expression (18), we find $a_{g\text{-IV}} = (1\,159\,656.7 \pm 3.5) \times 10^{-9}$, which is about one part per million below the previously reported value.[1] This is to be compared with the latest theoretical value,[8] $a_{\text{th}} = (1\,159\,655 \pm 2) \times 10^{-9}$.

Details will appear in a forthcoming publication. We want to thank A. Rich and J. Wesley for making their data available to us and for many helpful conversations.

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Chiral Symmetry Realization in the Quark Model*

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Chiral symmetry realizations of the quark-gluon model are considered in the absence of self-energy insertions for the gluon and photon, so the bare fermion mass vanishes. We exhibit the conditions which realize the chiral symmetry with Nambu-Goldstone bosons as bound states. A formal argument indicates that in this model electromagnetic corrections to the strong interactions are then finite to leading order and possibly all orders.

It has been appreciated for some time that chiral SU(3)⊗SU(3) may be an approximate symmetry of the strong-interaction Hamiltonian provided that the vacuum state is just SU(3) invariant. Such a realization of the exact symmetry requires an octet of Nambu-Goldstone bosons (NGB) corresponding to π, K, and η.

A popular model from the point of view of the algebraic simplicity of the structure of the currents and light-cone algebras is the quark-gluon model. This model is essentially a generalization of quantum electrodynamics (QED) to include a triplet of quark fermions and a massive neutral vector meson. In this model, however, the possibility of a Goldstone realization is obscure.

What we propose here is to investigate as a viable model for the strong interactions the quark-gluon model with emphasis on the Goldstone realizations of the symmetry. This undertaking is necessarily nonperturbative in the gluon coupling g_0 because all the hadrons and in particular the ground-state mesons must emerge as bound states of the quarks. The proposed approach is along the lines of investigations originally persued by Nambu and Jona-Lasinio.[1] However, their model was characterized by a strong cutoff dependence which is absent in this model. The model we propose is not new, but the emphasis on the solutions exhibiting collective phenomena is.

Two important features of this model emerge from this preliminary investigation. First, if the bare fermion mass vanishes, and we ignore self-energy insertions in the photon and gluon propagators, so that the physical mass is generated by the interaction, then radiative corrections to the strong interactions are damped. To leading order in the electromagnetic coupling the corrections are finite, and we conjecture that this is true to all orders. Secondly, we point out that the model possesses solutions with NGB.

The Lagrangian is $\mathcal{L}=\mathcal{L}_0+\mathcal{L}'$ with

$$\mathcal{L}_0=-i\bar{q}(x)\gamma\cdot\partial q(x)-\tfrac{1}{4}G_{\mu\nu}G^{\mu\nu}-\tfrac{1}{2}\mu_0{}^2W_\mu W_\mu - g_0W_\mu\bar{q}(x)\gamma_\mu q(x)-[\partial_\mu W_\mu(x)]^2/\lambda^2.$$

$G_{\mu\nu}=\partial_\mu W_\nu-\partial_\nu W_\mu$, λ is the gauge parameter, and $\mathcal{L}'$ is a term which breaks the formal U(3) $\otimes$U(3) symmetry of $\mathcal{L}_0$. Presently we will set $\mathcal{L}'=0$. That the gluon has a mass is a formal difference[2] between this model and QED; the essential feature is that it couples to a conserved current. We assume such differences can be ignored if we consider the large momentum behavior for which we can ignore the gluon mass.

If we surpress internal-symmetry considerations, the space-time structure of this model is that of QED with zero bare fermion mass. It is known that in massive QED if we ignore self-energy insertions for the vector meson so that the coupling g_0 satisfies a Gell-Mann–Low eigenvalue condition,[3] which we assume is satisfied, then the physical fermion mass is generated purely from the interactions and the self-energy is finite.[4]

One can always pick a gauge so that the gauge-dependent vertex and wave-function renormalization constants $Z_1=Z_2$ are finite (Z_3 is finite on the assumption that the Gell-Mann–Low eigenvalue condition is satisfied). In the Landau gauge in which Z_2 is finite, to leading order in an expansion in the fermion propagator, the Schwinger-Dyson equation (SDE) for the self-energy $\Sigma(p)$ is[4]

$$\Sigma(p^2)=i3g_0{}^2\int\frac{d^4l\,\Sigma(l^2)}{(l^2-m^2)[(l-p)^2-\mu^2]}. \tag{1}$$

The solution of this equation has a convergent asymptotic behavior $\Sigma(p^2)\to(1/p^2)^{3g_0{}^2/(4\pi)^2}$, $p^2\to\infty$, implying a finite self-energy. The physical fermion mass, which is an imput into the theory, specifies the boundry condition on the SDE, $\Sigma(m^2)=m$.

Without any such approximations one can show[5,6] that in massive QED, in the Landau gauge, the renormalized Feynman propagator

$$\tilde{S}_F{}^{-1}(p)=C(p^2)\gamma\cdot p-\Sigma(p^2)$$

as $p^2\to\infty$ behaves like $C(p^2)\to$ const, $\Sigma(p^2)\to$ const $\times(1/p^2)^{\epsilon(g_0{}^2)}$, where $\epsilon(g_0{}^2)=3g_0{}^2/(4\pi)^2+\frac{3}{2}g_0{}^4/(4\pi)^4+\cdots$. We will assume $\epsilon=\epsilon(g_0{}^2)>0$.

It has been pointed out that in spite of the formal γ_5 invariance of massless QED that the matrix elements of the divergences of the axial-vector currents are not conserved and hence there are no NGB. Here we point out that discussions indicating the absence of NGB beg the question. Namely, while there exist solutions to the theory which have no NGB, there also exist solutions which have them. Presumably the former solution applies to QED while the latter applies to the strong interactions.

Arguments against the coupling of the Goldstone mode have proceeded from the axial-vector Ward identities.[5] The axial-vector current transforming like $\gamma_\mu\gamma_5\lambda_0$ has an anomaly (with $\mathcal{L}'=0$) and requires separate treatment, but the other eight axial currents do not and satisfy

$$(p'-p)_\mu\Gamma_\mu{}^5(p',p) =2m_0(\Lambda)\Gamma^5(p',p)+S_F{}^{-1}(p')\gamma_5+\gamma_5S_F{}^{-1}(p).$$

Here $m_0(\Lambda)$ is the bare fermion mass in a cutoff theory and vanishes as $\Lambda\to\infty$.[5] The unrenormalized quantities are rendered finite by multiplication,[7] $\tilde{\Gamma}_\mu{}^5=Z_A\Gamma_\mu{}^5$, $\tilde{\Gamma}^5=Z_D\Gamma^5$, $\tilde{S}_F{}^{-1}=Z_2S_F{}^{-1}$. Defining $m\overline{\Gamma}^5=Z_Am_0(\Lambda)\Gamma^5=Z_Am_0(\Lambda)\tilde{\Gamma}^5/Z_D$ we have

$$(p'-p)_\mu\tilde{\Gamma}_\mu{}^5(p',p) =2m\overline{\Gamma}{}^5(p',p)+(Z_A/Z_2)[\tilde{S}_F{}^{-1}(p')\gamma_5+\gamma_5\tilde{S}_F{}^{-1}(p)].$$

Now one can show by varying with respect to the cutoff Λ that Z_A/Z_2 and $Z_Am_0(\Lambda)/Z_D$ are cutoff independent.[7] By a gauge choice Z_2 is finite; hence Z_A is cutoff independent. Further, the SDE for Γ^5 implies

$$m\overline{\Gamma}^5=Z_Am_0(\Lambda)\gamma_5+\int m\overline{\Gamma}_5\tilde{S}_F\tilde{S}_F\tilde{K},$$

and we conclude, since $Z_Am_0(\Lambda)\to0$ as $\Lambda\to\infty$, that $\overline{\Gamma}^5$ satisfies a homogeneous equation $\overline{\Gamma}^5=\int\overline{\Gamma}^5\tilde{S}_F\tilde{S}_F\tilde{K}$.

If $\tilde{\Gamma}_\mu{}^5$ has a NGB pole, then

$$\tilde{\Gamma}_\mu{}^5(p',p)\to\gamma_5G(p)(p'-p)_\mu/(p'-p)^2$$

as $p'\to p$. Using $\tilde{S}_F{}^{-1}(p)=C(p)\gamma\cdot p-\Sigma(p)$, the Ward identity reads, as $p'\to p$,

$$\gamma_5G(p)=2m\overline{\Gamma}^5(p,p)-(Z_A/Z_2)\gamma_52\Sigma(p).$$

If $G(p)=0$ and the NGB does not couple, then we have the boundary condition

$$m\overline{\Gamma}^5(p,p)=(Z_A/Z_2)\gamma_5\Sigma(p)\neq0$$

and we cannot have a trivial solution to the homogeneous SDE. So the axial currents are not conserved in spite of the formal γ_5 invariance. Alternatively, if we chose the trivial solution $\overline{\Gamma}^5=0$ to the homogeneous equation corresponding to conserved axial currents, then

$$G(p) = -2(Z_A/Z_2)\Sigma(p),$$

a Goldberger-Treiman relation, and the NGB couples. We conclude, at least in the absence of anomalies in the Ward identity, that we can have solutions with NGB.[8]

Willey[9] has studied the Bethe-Salpeter equation (BSE) for the elastic off-shell fermion-fermion scattering amplitude $T_{abcd}(p',p,k)$. In this model he exhibited that the BSE has solutions for T with no NGB pole at $k^2=0$. If, however, one writes

$$T_{abcd}(p',p,k) = g(p')g(p)(\gamma_5)_{ab}(\gamma_5)_{cd}/k^2 + T_{abcd}{}^R(p',p,k)$$

with T^R regular at $k=0$, then the BSE can be shown to imply that $g(p)$, the coupling of the NGB, satisfies Eq. (1) and hence $g(p) = C\Sigma(p)$ with C a constant. $C=0$ corresponds to Willey's solution, and the γ_5 projection of T^R then satisfies his equation. However, the point of this discussion is that there exist solutions with $C \neq 0$ provided (1) is satisfied. Then T^R satisfies an equation *different* from that considered by Willey. Hence there can exist solutions with NGB.

An intriguing feature of this model without vector-meson self-energy insertions is that radiative corrections to strongly interacting processes may be finite. We add to the strong-interaction Lagrangian the usual minimally coupled electromagnetic field. The crucial assumption is that eigenvalue conditions are satisfied so that we will ignore photon and gluon self-energy insertions. To leading order in $\alpha_0 = e_0^2/4\pi$ in the gluon model the divergent part of the electromagnetic interaction is[10]

$$\mathcal{L}_{\text{em}}{}^{\text{div}} = \frac{ie_0^2}{(2\pi)^4}\int \frac{d^4x}{(x^2-i\epsilon)^2}\,\partial_\mu \overline{V}_\mu(x\,|\,0)\big|_{x=0},$$

where $\overline{V}_\mu(x\,|\,0)$ is a bilocal operator appearing in the light-cone algebra.[11]

To examine for the presence of a divergence we will introduce a cutoff Λ^2 for large momentum or equivalently Λ^{-2} in coordinate space to define our expressions. Then we want to see if

$$\mathcal{L}_{\text{em}}{}^{\text{div}}(\Lambda) = \frac{ie_0^2}{(2\pi)^4}\int \frac{d^4x}{(x^2-\Lambda^{-2}-i\epsilon)^2}\,\partial_\mu \overline{V}_\mu(x\,|\,0;\Lambda)$$

is finite as $\Lambda \to \infty$. In the gluon model the bilocal operator is specified by[10]

$$4\partial_\mu \overline{V}_\mu(x\,|\,0;\Lambda) = m_0(\Lambda)[:\overline{q}(x)Q^2q(0):+:\overline{q}(0)Q^2q(x):]$$

for $x \simeq 0$, where $q(x)$ is the fermion triplet field.

Here $m_0(\Lambda)$, in the cutoff version of the model we are considering, does not vanish for finite Λ and behaves like[5] $m_0(\Lambda) \to (\Lambda^2)^{-\epsilon} \to 0$ as $\Lambda \to \infty$, if we use the gauge in which Z_2 is finite. In the cutoff model the Wilson expansion[12] we consider is for

$$:\overline{q}(x)Q^2q(y): \simeq [(x-y)^2 - \Lambda^{-2} - i\epsilon]^{-\delta}O(x) + \cdots$$

with $(x-y)^2$ near the light cone. Let us define $j(x,y,\Lambda) = m_0(\Lambda):\overline{q}(x)Q^2q(y):$. Then we can determine the power δ and $O(x)$ noting that $j(x,x,\Lambda) = j_s(x)$ is just the local scalar vertex operator which is known to be cutoff independent.[6] As $\Lambda \to \infty$, $j_s(x) \sim (\Lambda^2)^{-\epsilon}(\Lambda^2)^{\delta}O(x)$ and we conclude since $j_s(x)$ is independent of Λ, that $\delta = \epsilon$ and $j_s(x) \sim O(x)$. Then the bilocal operator of interest,

$$4\partial_\mu \overline{V}_\mu(x\,|\,0;\Lambda) = j(x,0,\Lambda) + j(0,x,\Lambda),$$

is also specified,

$$4\partial_\mu \overline{V}_\mu(x\,|\,0;\Lambda) = m_0(\Lambda)[x^2 - \Lambda^{-2} - i\epsilon]^{-\epsilon}[j_s(x) + j_s(0)] \quad (3)$$

for $x^2 \simeq 0$. Since $j_s(x)$, a local operator, has well-defined cutoff-independent behavior, and is nonsingular at $x=0$, we have isolated the potentially dangerous part of the integral in the c-number factor in (3). Performing our Wick-rotated integral,

$$\mathcal{L}_{\text{em}}{}^{\text{div}}(\Lambda) \sim e_0^2\int_0 \frac{x^2\,dx^2}{(x^2+\Lambda^{-2})^{2+\epsilon}}\, m_0(\Lambda) j_s(0),$$

and then letting $\Lambda \to \infty$,

$$\mathcal{L}_{\text{em}}{}^{\text{div}}(\Lambda) \sim (e_0^2/\epsilon)m_0(\Lambda)(\Lambda^2)^{\epsilon}j_s(0) \sim (e_0^2/\epsilon)j_s(0)$$

is finite. So there is no divergence to leading order in e_0^2. We conjecture this is true to all orders.

This argument is essentially a formal argument and could break down. To see this damping of the divergence another way we consider the self-energy $\Sigma(p^2)$ of a charged fermion of charge e_0. Then, to the same leading order approximation for which (1) is valid, $\Sigma(p^2)$ satisfies homogeneous SDE in the presence of both gluon and photon couplings. Then

$$\Sigma(p^2, g_0, e_0) \to (1/p^2)^{3(g_0^2+e_0^2)/(4\pi)^2},$$

and

$$\Sigma(p^2, g_0, e_0^2) - \Sigma(p^2, g_0, 0) \simeq [3e_0^2/(4\pi)^2]\ln(p^2)(1/p^2)^{3g_0^2/(4\pi)^2} + \cdots$$

will obey an unsubtracted dispersion relation to each order in e_0^2 (but not g_0^2). A full proof of the fact that $\Sigma(p^2)$ obeys an unsubtracted dispersion relation requires a treatment of the Callen-Symanzik equations for both photons and gluons analogous to that given for one vector field in Ref. 6.

On the other hand, we know of no reason why ordered products of currents $[j_\mu(x)j_\nu(0)]_+$ should not exhibit canonical behavior on the light cone in conformity with scaling experiments. However, this analysis suggests that the delicate cancelations required in the Cottingham formula to render the self-energy finite[13] do in fact occur in spite of the individual structure functions scaling.

The treatment of the divergences of the weak interactions is associated with symmetry breaking. The weak interaction and a theory of symmetry breaking which imitates the strong and electromagnetic interactions by introducing an additional vector-meson $W_\mu{}^8$ coupling to $\bar{q}\lambda^8\gamma_\mu q$, we defer to a future study. It should be pointed out that if to leading order the weak interactions exhibit characteristic quadratic divergences, then the power behavior damping of the strong interactions suggested by this study would fail to eliminate this divergence. So a modified theory of the weak interactions may be required.

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