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Electron Spin Motion in a Magnetic Mirror Trap

Sara Granger and G. W. Ford
Department of Physics, University of Michigan, Ann Arbor, Michigan 48104
(Received 3 March 1972)

Averaging methods are used to discuss the electron spin motion in a magnetic mirror trap of the sort used in precision $g-2$ experiments. An expression is obtained for the difference frequency correct up to second order in the field nonuniformity. When applied to the experiments this result removes the discrepancy between the last two precision $g-2$ measurements and yields a corrected value for the gyromagnetic anomaly: $a = (1\,159\,656.7 \pm 3.5) \times 10^{-9}$.

In precision $g-2$ experiments[1,2] electrons are confined in a weak magnetic mirror trap, i.e., a nearly uniform axially symmetric magnetic field which increases in strength on either side of a median plane. The electron's motion is a superposition of a rapid rotation (cyclotron motion) about the symmetry axis and a much slower longitudinal oscillation along the axis. The experiments measure the difference frequency ω_D which is the long-time average precession rate of the electron's spin relative to its velocity. Previous analyses of these experiments have used a theoretical expression for ω_D obtained by time averaging the uniform-field result over the field in the trap,[1] a procedure which is not in general correct. Here we give a perturbation expansion for ω_D, obtained using averaging methods, correct up to second order in the field nonuniformity. We then apply this result to the last two precision $g-2$ measurements to obtain corrected values for the gyromagnetic anomaly.

Choosing the field symmetry axis as the z axis, the magnetic field in the trap is of the form

$$\vec{B}(\vec{r}) = B_0[\hat{z} + \vec{b}(\vec{r})], \tag{1}$$

where $\vec{b}$ represents the small perturbation of the uniform field. The electron's orbital equations of motion are

$$d\vec{r}/dt = \vec{v}, \quad d\vec{v}/dt = \vec{\omega} \times \vec{v}, \tag{2}$$

where $\vec{\omega} = e\vec{B}/\gamma mc$ with $\gamma = (1 - v^2/c^2)^{-1/2}$. Since the experiments measure the spin relative to the velocity, it is appropriate to write the spin equations of motion in a coordinate system in which $\vec{v}$ is fixed, i.e., one rotating with instantaneous angular velocity $\vec{\omega}$. The resulting equation is

$$d\vec{S}/dt = \vec{\Omega} \times \vec{S}, \tag{3}$$

where[3]

$$\vec{\Omega} = a[\gamma(\vec{\omega} - \hat{v} \cdot \vec{\omega}\hat{v}) + \hat{v} \cdot \vec{\omega}\hat{v}], \tag{4}$$

with a the gyromagnetic anamoly. In general $\vec{\Omega}$ is time dependent through the dependence of $\vec{B}$, and hence $\vec{\omega}$, upon electron position.

In the uniform-field case ($\vec{b}=0$) we have $\vec{\omega}=\hat{z}\omega_0/\gamma$, where $\omega_0=eB_0/mc$. Since $\vec{\omega}$ is constant in both the lab and rotating frames, $\vec{\Omega}$ is constant in the rotating frame. The spin motion in this frame is therefore a uniform precession about $\vec{\Omega}$ with angular velocity $|\vec{\Omega}|$. Hence, the difference frequency is[4]

$$\omega_D=|\vec{\Omega}|=a\omega_0(1-\beta_z^{\,2})^{1/2}, \tag{5}$$

where $\beta_z=\hat{z}\cdot\vec{v}/c$. In the trapping field, $\vec{b}\neq 0$ and $\vec{\Omega}$ is a function of electron position. We must therefore first discuss the equations of motion (2) in order to determine the time dependence of $\vec{\Omega}$. To do this we introduce dimensionless guiding-center coordinates,

$$\vec{\xi}=\rho_c^{\,-1}\vec{r}+\hat{z}\times\hat{v}, \tag{6}$$

where $\rho_c=\gamma v/\omega_0$ is the cyclotron radius for an electron moving in a plane perpendicular to a uniform field B_0. We use subscripts $\parallel$ and $\perp$ to denote vector components perpendicular and parallel to $\hat{z}$ and express the electron velocity in the form

$$\vec{v}=v_\parallel\hat{z}+v_\perp\hat{t}, \tag{7}$$

where $v_\perp=(v^2-v_\parallel^{\,2})^{1/2}$ and

$$\hat{t}=\hat{x}\cos\beta+\hat{y}\sin\beta. \tag{8}$$

The equations of motion for the variables $\vec{\xi}$, $v_\parallel$, and β, obtained using (2), are

$$\frac{d\vec{\xi}}{dt}=\frac{\omega_0}{\gamma}\left(\frac{v_\parallel}{v}\hat{z}-\frac{v_\perp}{v}b_\parallel\hat{t}+\frac{v_\parallel}{v}\vec{b}\right), \tag{9a}$$

$$\frac{1}{v}\frac{dv_\parallel}{dt}=-\frac{\omega_0}{\gamma}\frac{v_\perp}{v}\hat{z}\times\hat{t}\cdot\vec{b}, \tag{9b}$$

and

$$\frac{d\beta}{dt}=\frac{\omega_0}{\gamma}\left(1+b_\parallel-\frac{v_\parallel}{v_\perp}\hat{t}\cdot\vec{b}\right). \tag{9c}$$

Since $b\ll 1$, electrons are confined within the trap only if $v_\parallel\ll v$. For this case the Eqs. (9) are of the form appropriate for the method of rapidly rotating phase[5]: The rate of change of the cyclotron phase β is large compared to the rates of change of the other variables, while all the rates have rapidly oscillating components of relative order b arising from the cyclotron motion. The method separates the mean motion of the variables from the small (order b) oscillations about the mean produced by the oscillating terms in the rates. The result is that up to relative order b^2 the equations for the mean motion are obtained by replacing the variables of (9) by their means, which we indicate by a bar, and averaging the resulting equations over the mean phase angle $\bar{\beta}$. Hence, this mean phase angle (the rapid phase) does not appear explicitly in the equations of the mean motion. In addition, using axial symmetry and the fact that $v_\parallel/v\ll 1$, it can be shown that up to relative order b^2 the mean motion of the components of the guiding center perpendicular to the symmetry axis is a slow rotation which does not affect the motion of $\bar{\xi}_\parallel$ and $\bar{v}_\parallel$. This last motion (the longitudinal motion) is periodic with $\bar{\xi}_\parallel$ oscillating between two "mirror points." The angular frequency $\omega_\parallel$ of this motion is of order $b^{1/2}\omega_0/\gamma$ and the amplitude of $\bar{v}_\parallel/v$ is of order $b^{1/2}$. These are the essential features of the orbital motion we need for investigating the spin motion.

To discuss this spin motion we must first express $\vec{\Omega}_D$ in a frame rotating with instantaneous angular velocity $\vec{\omega}$. We choose as the basis of this frame

$$\hat{e}_1=\hat{v},\quad \hat{e}_2=(\cos\varphi)\hat{z}\times\hat{t}+(\sin\varphi)\hat{v}\times(\hat{z}\times\hat{t}),\quad \hat{e}_3=\hat{e}_1\times\hat{e}_2, \tag{10}$$

which satisfy $\hat{e}_i=\vec{\omega}\times\hat{e}_j$ provided we require that

$$\frac{d\varphi}{dt}=\frac{\omega_0}{\gamma}\frac{v}{v_\perp}\hat{t}\cdot\vec{b}. \tag{11}$$

In this frame

$$\vec{\Omega}_D=a\omega_0\Big(\frac{1}{\gamma}\left[\frac{v_\parallel}{v}(1+b_\parallel)+\frac{v_\perp}{v}\hat{t}\cdot\vec{b}\right]\hat{e}_1+\left\{\left[\frac{v_\perp}{v}(1+b_\parallel)-\frac{v_\parallel}{v}\hat{t}\cdot\vec{b}\right]\sin\varphi+\hat{z}\times\hat{t}\cdot\vec{b}\cos\varphi\right\}\hat{e}_2$$
$$+\left\{\left[\frac{v_\perp}{v}(1+b_\parallel)-\frac{v_\parallel}{v}\hat{t}\cdot\vec{b}\right]\cos\varphi-\hat{z}\times\hat{t}\cdot\vec{b}\sin\varphi\right\}\hat{e}_3\Big). \tag{12}$$

Equations (3) and (11) depend upon time only through the motion variables. Hence, since $a\ll 1$ we may simply append these equations to the equations of motion (9) when we apply the method of rapidly rotating phase. As before, the equations of the mean motion up to relative order b^2 are obtained by replacing the variables in (3) and (11) by their means and averaging over the mean cyclotron phase angle $\bar{\beta}$. The equation for the mean of φ is

$$\frac{d\bar{\varphi}}{dt}=\frac{\omega_0}{\gamma}\frac{v}{\bar{v}_\perp}\frac{1}{2\pi}\int_0^{2\pi}d\beta\,\hat{t}\cdot\vec{b}=0, \tag{13}$$

since the integral is proportional to a line integral over a closed circular path which vanishes since curl $\vec{b}=0$. Hence $\bar{\varphi}$ is constant up to relative order b^2 and we use the freedom in the initial orientation of the coordinates $\hat{e}_i$ to choose $\bar{\varphi}=0$. The equation of the mean spin motion, obtained by averaging (3) with $\vec{\Omega}_D$ given by (12), becomes

$$d\bar{\vec{S}}/dt=a\omega_0(\hat{e}_3+\vec{\Delta})\times\bar{\vec{S}}, \tag{14}$$

where, using $\bar{v}_\perp/v=1-\frac{1}{2}(\bar{v}_\parallel/v)^2+O(b^2)$,

$$\vec{\Delta}=\frac{1}{\gamma}\frac{\bar{v}_\parallel}{v}(1+\langle b_\parallel\rangle)\hat{e}_1+\langle\hat{z}\times\hat{t}\cdot\vec{b}\hat{e}_2\rangle+\left(\langle b_\parallel\rangle-\frac{1}{2}\frac{\bar{v}_\parallel{}^2}{v^2}\right)\hat{e}_3. \tag{15}$$

Here the angular brackets indicate the average over $\bar{\beta}$. Since $\vec{\Delta}$ is a function only of the mean motion variables, it is a periodic function of time with angular frequency $\omega_\parallel$. Also since $\bar{v}_\parallel/v$ is of order $b^{1/2}$, $\vec{\Delta}$ contains terms of order $b^{1/2}$, b, and $b^{3/2}$. To calculate the effects of the small oscillating term $\vec{\Delta}$ on the mean spin motion, we transform to a coordinate frame rotating with constant angular velocity $a\omega_0$ about the $\hat{e}_3$ axis.[6] In this frame the equation for the mean spin motion becomes

$$d\bar{\bar{\vec{S}}}/dt=a\omega_0\vec{\Delta}(t)\times\bar{\bar{\vec{S}}}, \tag{16}$$

where

$$\vec{\Delta}(t)=\hat{e}_3\cdot\vec{\Delta}\hat{e}_3+(\vec{\Delta}-\hat{e}_3\cdot\vec{\Delta}\hat{e}_3)\cos a\omega_0 t+\hat{e}_3\times\vec{\Delta}\sin a\omega_0 t \tag{17}$$

is a doubly periodic function of time with frequencies $\omega_\parallel$ and $a\omega_0$. Equation (16) is in the standard form for the application of the method of averaging: The time rate of change of $\bar{\bar{\vec{S}}}$ is small compared to the frequencies $\omega_\parallel$ and $a\omega_0$ of the oscillations of this rate.[7] As in the method of rapidly rotating phase, this method separates the slow mean motion of $\bar{\bar{\vec{S}}}$ (itself a mean with respect to the cyclotron motion) from the small-amplitude doubly periodic oscillations about this mean. Since $\vec{\Delta}$ is of order $b^{1/2}$ and we want to describe the motion up to order b^2, the method must be carried through third order. The result we find is that the mean spin motion when referred back to the velocity-fixed frame is a uniform precession about the $\hat{e}_3$ axis with angular velocity

$$\omega_D=a\omega_0\left\{1+[\langle b_\parallel\rangle]-\frac{1}{2}\sum_n|\alpha_n|^2\left(1+\frac{1}{\gamma^2}\frac{a^2\omega_0{}^2+2a\gamma^2\omega_\parallel{}^2n^2}{n^2\omega_\parallel{}^2-a^2\omega_0{}^2}\right)+O(b^2)\right\}. \tag{18}$$

Here the square brackets denote the average over a period of the longitudinal motion and the coefficients α_n come from the Fourier expansion

$$\bar{v}_\parallel/v=\sum_n\alpha_n\exp(-in\omega_\parallel t). \tag{19}$$

We identify ω_D given by (18) with the experimentally observed difference frequency.

The expansion (18) simplifies when the frequency of the longitudinal oscillations of the electron in the trap is either very high or very low compared to the difference frequency itself. When $\omega_\parallel\gg a\omega_0$, we find

$$\omega_D\approx a\omega_0\{1+[\langle b_\parallel\rangle-\frac{1}{2}(\bar{v}_\parallel/v)^2]\}, \tag{20}$$

where we have used the result

$$[(\bar{v}_\parallel/v)^2]=\sum_n|\alpha_n|^2. \tag{21}$$

The expression (20) is just what is obtained if the expression (12) for $\vec{\Omega}_D$ is first averaged over the cyclotron motion and then averaged over the longitudinal motion. When $\omega_\parallel\ll a\omega_0$ we find

$$\omega_D\approx a\omega_0\{1+[\langle b_\parallel\rangle-\frac{1}{2}\gamma^{-2}(\gamma^2-1)(\bar{v}_\parallel/v)^2]\}, \tag{22}$$

where we have again used (21). This expression is just what is obtained when (12) is first averaged over the cyclotron motion and then the *magnitude* of the resulting precession rate is averaged over the longitudinal motion. It is equivalent to averaging the uniform-field result (5) over the longitudinal motion and is the expression used in the most recent analyses of the $g-2$ experiments.

These results allow us to resolve the serious discrepency between the results of the last two $g-2$ measurements. The first of these (g-III), when analyzed using (22), gave a result $3\frac{1}{2}$ standard deviations below that of the second (g-IV).[1] But $\omega_{\parallel}/a\omega_0 \approx 5.5$ in g-III, so the limit (20) is applicable. When we reanalyze the g-III data using (20), we find $a_{g\text{-III}} = (1\,159\,690 \pm 30) \times 10^{-9}$, which is about 1 standard deviation above the g-IV value.

For g-IV the limit (22) does not quite apply since $\omega_{\parallel}/a\omega_0 \approx 0.4$ is not negligibly small. When we reanalyze the g-IV data using the general expression (18), we find $a_{g\text{-IV}} = (1\,159\,656.7 \pm 3.5) \times 10^{-9}$, which is about one part per million below the previously reported value.[1] This is to be compared with the latest theoretical value,[8] $a_{\text{th}} = (1\,159\,655 \pm 2) \times 10^{-9}$.

Details will appear in a forthcoming publication. We want to thank A. Rich and J. Wesley for making their data available to us and for many helpful conversations.

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Chiral Symmetry Realization in the Quark Model*

Heinz Pagels
Rockefeller University, New York, New York 10021
(Received 15 March 1972)

Chiral symmetry realizations of the quark-gluon model are considered in the absence of self-energy insertions for the gluon and photon, so the bare fermion mass vanishes. We exhibit the conditions which realize the chiral symmetry with Nambu-Goldstone bosons as bound states. A formal argument indicates that in this model electromagnetic corrections to the strong interactions are then finite to leading order and possibly all orders.

It has been appreciated for some time that chiral SU(3)⊗SU(3) may be an approximate symmetry of the strong-interaction Hamiltonian provided that the vacuum state is just SU(3) invariant. Such a realization of the exact symmetry requires an octet of Nambu-Goldstone bosons (NGB) corresponding to π, K, and η.

A popular model from the point of view of the algebraic simplicity of the structure of the currents and light-cone algebras is the quark-gluon model. This model is essentially a generalization of quantum electrodynamics (QED) to include a triplet of quark fermions and a massive neutral vector meson. In this model, however, the possibility of a Goldstone realization is obscure.

What we propose here is to investigate as a viable model for the strong interactions the quark-gluon model with emphasis on the Goldstone realizations of the symmetry. This undertaking is necessarily nonperturbative in the gluon coupling g_0 because all the hadrons and in particular the ground-state mesons must emerge as bound states of the quarks. The proposed approach is along the lines of investigations originally persued by Nambu and Jona-Lasinio.[1] However, their model was characterized by a strong cutoff dependence which is absent in this model. The model we propose is not new, but the emphasis on the solutions exhibiting collective phenomena is.

Two important features of this model emerge from this preliminary investigation. First, if the bare fermion mass vanishes, and we ignore self-energy insertions in the photon and gluon propagators, so that the physical mass is generated by the interaction, then radiative corrections to the strong interactions are damped. To leading order in the electromagnetic coupling the