

linearly with the number of rf stoppers. In our calculation the density in the central mirror also increases linearly with the number of mirror sections. However, the additional mirror sections also usefully contain particles, such that the total containment increases quadratically.

It should be emphasized that, in addition to the containment times as calculated here, there are additional considerations that bear on the usefulness of the multiple-mirror configuration as a plasma containment device. One important consideration is that, for a fully ionized plasma, the important scattering mechanism is ion-ion self-scattering in which the scattering centers have an average drift in the direction of the particle loss. These drifts due to finite loss rates have not been included in the present computational model. The drifts corresponding to the loss times reported in Tables I and II are small compared to the thermal velocity; but, nevertheless, the effect of drifts might have a serious effect on the scaling of confinement time if they were included in a self-consistent way.

A second consideration is that in some density regimes the confinement time is proportional to the density, and a fluctuation in density may cause a change in the confinement time that enhances the fluctuation. The scattering-center density distribution in the numerical model, however, is fixed.

Another important consideration is that radial losses have not been included in the numerical

computations. While hydrodynamic stability may be possible without absolute minimum \vec{B} ,⁵ micro-instabilities may cause radial losses in a physical system in competition to end loss.

Although additional work is required to determine the feasibility of a multiple-mirror system, it is nonetheless instructive to evaluate the computational results by estimating parameters for a D-T multiple-mirror reaction required to have a ratio (fusion thermal power generated)/(plasma kinetic energy flow) = 10. For lack of space, we report only the results: $kT_i = kT_e = 5$ keV, $n = 3 \times 10^{16}$ cm⁻³ (center), $M_{\text{eff}} = 2$ (center)–13 (ends), $l_c = 6.5$ m, $L = 250$ m. The assumptions used in the reactor calculation have been previously reported.⁶

*Research sponsored by the National Science Foundation under Grant No. GK-27538, and the U. S. Air Force Office of Scientific Research under Grant No. AF-AFOSR-69-1754.

¹R. F. Post, Phys. Rev. Lett. **18**, 232 (1967).

²L. Spitzer, Jr., *Physics of Fully Ionized Gases* (Interscience, New York, 1956), p. 78.

³B. G. Logan, A. J. Lichtenberg, and M. A. Lieberman, Bull. Amer. Phys. Soc. **15**, 1432 (1970).

⁴J. B. Taylor, private communication.

⁵H. P. Furth and M. N. Rosenbluth, Phys. Fluids **7**, 764 (1964).

⁶B. G. Logan, Plasma Research, Electronics Research Laboratory, University of California, Berkeley, Fourth Quarterly Report, 1969 (unpublished), p. 2.

Electromagnetic Generation of Transverse Acoustic Waves in the Gigahertz Range Using Indium Films

Y. Goldstein and A. Zemel

The Racah Institute of Physics, The Hebrew University, Jerusalem, Israel

(Received 17 November 1971)

Transverse acoustic waves were generated electromagnetically at 9 GHz using In films with a conversion efficiency α of 5×10^{-5} . A good agreement is found between theory and experiment and we obtain the value of 620 Å for the superconducting penetration depth λ at $0.4T_c$ and 5×10^{10} Ω⁻¹ cm⁻² for the ratio of the conductivity to the mean free path for In. From the temperature dependence of α below the superconducting transition temperature it is concluded that pair scattering at the surface does not conserve momentum.

It was found by Abeles¹ that an electromagnetic field in the gigahertz region can generate a transverse acoustic wave at the surface of a clean indium film. Electrons in the metal are accelerated by the electromagnetic field within the penetration depth. For a thin film at low tempera-

tures, scattering in the bulk can be neglected and the momentum gained by the electrons is transferred to the lattice by surface scattering. In the gigahertz region this momentum transfer is coherent and gives rise to a transverse acoustic wave. The theory is extremely sensitive to the

value of the electromagnetic penetration depth and to the fraction n/n_0 of the conduction electrons that can be scattered by the surface. In the normal state this fraction is equal to 1. In the superconducting state, however, if pair scattering at the surface conserves momentum, n/n_0 is expected to decrease exponentially with temperature. In addition, in the superconducting state the electromagnetic penetration depth, too, becomes temperature dependent, decreasing with decreasing temperatures. Another contribution to the acoustic-wave generation results from the direct interaction of the electromagnetic field with the lattice ions, but at our frequencies this contribution is negligible. The experiments of Abeles were performed in the vicinity of the superconducting transition temperature T_c of indium. Above T_c the conversion efficiency α of electromagnetic to acoustic power was found to be very small (3×10^{-7}). Below T_c the conversion efficiency decreased even further and the signal disappeared in noise.

We succeeded in obtaining a conversion efficiency of 5×10^{-5} , 2 orders of magnitude higher than Abeles. Because of these high conversion efficiencies we were able to study the temperature dependence of α down to $0.4T_c$. A good agreement was found between theory and experiment with respect to both the value and the temperature dependence of α . From the temperature dependence of α below T_c we conclude that pair scattering at the surface does not conserve momentum. This conclusion is in agreement with our previous results² and with results of microwave absorption experiments.^{3,4}

The experimental arrangement was similar to that in Ref. 1. A thin ($d \approx 1200 \text{ \AA}$) indium film was vacuum deposited on an optically polished high-purity silicon rod. The axis of the rod was along the $[110]$ direction and its end faces were parallel within 6 sec of arc. The silicon face with the indium on it was pressed against a 0.5-cm-diam hole in the bottom wall of a high-Q resonant rectangular microwave cavity. The cavity was excited with 9.02-GHz microwave pulses of 1- μ sec duration and 150-W peak power. The power reflected and/or radiated from the cavity was detected by a sensitive microwave detector and displayed on an oscilloscope. Following the initial excitation pulse, several echoes were observed delayed by times corresponding to the flight of fast and slow shear waves in silicon.

In the temperature range of 4.2 to 3.4°K, the transition temperature of indium, we observed

seven to eight echoes. The echo envelope decreased exponentially, with 7 dB attenuation between consecutive echoes. Both the amplitudes of the echoes and the attenuation were temperature independent in the above range. To obtain the conversion efficiency α we measured the ratio of the amplitude of the first echo to that of the excitation pulse. In the absence of attenuation this ratio is equal to α^2 . However, because of the attenuation, we extrapolated the echo envelope to the time $t=0$, which resulted in a 7-dB correction. This corrected value was defined as the insertion loss (IL).

Above T_c the insertion loss was independent of temperature and power over 4 orders of magnitude of microwave power. Below T_c both the insertion loss and the attenuation were strongly power dependent. The insert in Fig. 1 shows a typical plot of the dependence of IL on microwave power. It can be seen from the figure that over the range of 8 dB of microwave power IL changes by 27 dB. Furthermore, IL does not decrease monotonically but exhibits local minima and maxima. Only at low powers does IL become independent of microwave power. That this behavior is

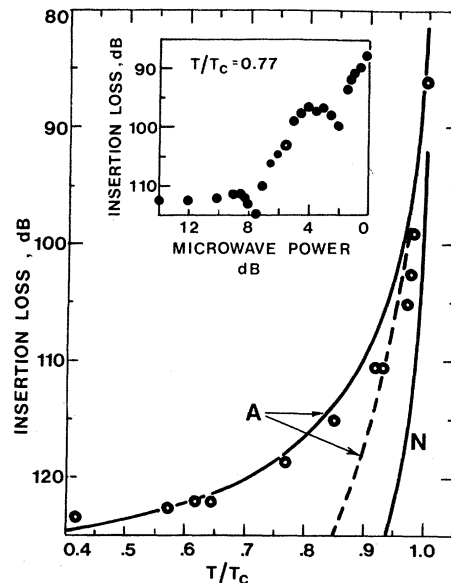


FIG. 1. The measured IL (circles) versus the reduced temperature T/T_c , together with the theoretical curves. Curve N was calculated for the normal and curves A for the extreme anomalous limit. The two solid curves were calculated on the assumption that $n/n_0 = 1$. In calculating the dashed curve, a two-fluid model was assumed for the temperature dependence of n/n_0 . The insert shows the dependence of IL on microwave power; 0 dB corresponds to the highest power (150 W).

connected with the superconductivity of the indium film was ascertained by the application of an external magnetic field (perpendicular to the film surface). A small field, less than 70 Oe (its exact value depending on the temperature), was adequate to restore the insertion loss and the attenuation to their values above T_c , independent of power.

To understand the peculiar power dependence of IL below T_c , one has to keep in mind that IL is determined by two processes. First, transverse acoustic waves are generated by the initial microwave pulse with a conversion efficiency α_1 . Next, the acoustic wave, after reflection at the second end of the rod and a corresponding time delay, generates an electromagnetic pulse with a conversion efficiency α_2 . Because of the symmetry of the two processes, $\alpha_1 = \alpha_2$, provided the indium film is in the same state (normal or superconducting) at the time of each process. However, it is possible that during the first process the indium film is in the normal or intermediate state, while during the second process the film is in the intermediate or superconducting state. The analysis of the data for IL versus power showed that such is indeed the case. It was found that at 0 dB at all temperatures the value of α was equal to the conversion efficiency of normal indium, α_N . The value of α_2 at 0 dB was also found to be larger than the conversion efficiency of superconducting indium, α_S . As the power was reduced a little, α_2 decreased and approached α_S , while α_1 usually remained unchanged at the value α_N . At $T = 0.77T_c$ this occurred around 2 dB. As the power was further reduced, α_1 also decreased and at sufficiently low powers attained the value of α_S (around 8 dB in the figure). At the powers where α_1 and α_2 attained the value of α_S we found local maxima (and minima) in the data for IL versus power. We associate these maxima with the increase of the Q of the cavity due to the transition of the indium film from the intermediate into the superconducting state. Such an increase in the Q (at the time of the initial microwave pulse) was also observed directly.

The mechanism responsible for the "quenching" of the indium film is not clear at the present time. The estimated⁵ value of the microwave magnetic field seems to be insufficient to drive the indium film into the normal state and thus does not account even for the power dependence of α_1 . The possibility of heating the indium film by the microwave pulse seems to be ruled out as well. The indium film was in intimate thermal

contact with both the silicon crystal and the cavity wall. In addition, the film was immersed in the liquid helium bath and the behavior was similar above and below the λ point of helium.

To obtain the correct α_S for superconducting indium we used the value of IL at low powers where it becomes power independent. To compare this value with the insertion loss of normal indium we also had to take into account the changes in the Q of the cavity, which resulted in a small correction, increasing IL. In Fig. 1 we plot the temperature dependence of the insertion loss at low powers (corrected for the change in Q). To calculate the theoretical temperature dependence of IL, we expressed the penetration depth $|1/K|$ in terms of the conductivity σ using the theory of skin effect. In the normal-state limit we obtain

$$K_N = (4\pi i \omega \sigma)^{1/2}, \quad (1)$$

while in the extreme anomalous limit we obtain⁶

$$K_A = (8\pi^2 \omega \sigma / \sqrt{3} c^2 l)^{1/3} e^{i\pi/3}, \quad (2)$$

where ω is the (angular) frequency, l the electronic mean free path, c the velocity of light, and $i = (-1)^{1/2}$. Equation (1) is expected to be a reasonable approximation at high temperatures where $|1/K| \gg d$. At low T/T_c , $|1/K|$ decreases towards the superconducting penetration depth λ and the anomalous limit would seem to be more appropriate.

The theoretical curves in Fig. 1 labeled A and N were calculated from the theory for α^2 derived by Abeles⁷ using for K the expressions for K_A and K_N . The temperature dependence of σ in the superconducting state was calculated from Mattis and Bardeen's⁸ theory for σ/σ_N , where σ_N is the normal conductivity. In calculating the two solid curves we assumed that the fraction of electrons, n/n_0 , that can be scattered at the surface is 1. In calculating the dashed curve we assumed, according to the two-fluid model,⁹ that $n/n_0 = (T/T_c)^4$. Inspecting Fig. 1, we see that the temperature dependence of the experimental data agrees well with the solid curve A, obtained for the anomalous limit and with $n/n_0 = 1$. Thus the whole temperature dependence of this curve arises from the temperature dependence of K_A .

Because the anomalous limit becomes a better approximation the lower the temperature, we fitted (the solid) curve A to the experimental results at low temperatures. The fit yields the value $\lambda = 620 \text{ \AA}$ for the superconducting penetration depth of indium at $T/T_c = 0.4$. This value is in

reasonable agreement with those published in the literature.^{1,10,11} In addition we obtain the value of the ratio $\sigma_n/l = 4.8 \times 10^{10} \Omega^{-1} \text{ cm}^{-2}$ which, to our knowledge, is the first published value of σ_n/l for indium. This value, obtained from the measured insertion loss, agrees well with the value obtained¹² for diffuse surface scattering from the measured conductivity of the indium film, $\sigma_f \approx 1 \times 10^6 \Omega^{-1} \text{ cm}^{-1}$, and the estimated value of the mean free path, $l \approx 10\,000 \text{ \AA}$. At higher temperatures the experimental data deviate somewhat from curve A and are bracketed by the two solid theoretical curves. This is expected because with increasing temperatures the approximation of the extreme anomalous limit becomes progressively worse.

In conclusion, the good agreement between theory and experiment shows that the simplified model used for the calculation of α is basically sound. To fit the theory to experiment we had to assume that $n/n_0 = 1$, and thus we conclude that pair scattering at the surface does not conserve momentum. We would also like to point out that because of the relatively high conversion efficiencies and the momentum transfer by pair scattering at the surface, it may very well be that transverse phonon generation is one of the important loss mechanisms in very high- Q superconducting cavities. In the case of indium, for example, the residual microwave surface resistance at 10 GHz due to this mechanism is calculated to be $2 \times 10^{-7} \Omega$.

We would like to thank Dr. B. Abeles for many stimulating discussions, Mr. S. Kiryati for help in setting up the experiment, and Mr. I. Aharonov

for help in preparation of the samples.

¹B. Abeles, Phys. Rev. Lett. **19**, 1181 (1967).

²Y. Goldstein and B. Abeles, in *Proceedings of the Eleventh International Conference on Low Temperature Physics, St. Andrews, 1968* (University of St. Andrews, St. Andrews, Scotland, 1968), p. 965.

³W. V. Budzinski and M. P. Garfunkel, in *Proceedings of the Tenth International Conference on Low Temperature Physics, Moscow, 1966*, edited by M. P. Malkov (VINITI Publishing House, Moscow, U. S. S. R., 1967), Vol. IIB, p. 243.

⁴G. Fischer and R. Klein, Phys. Rev. **165**, 578 (1968).

⁵The magnitude of the microwave magnetic field can be obtained from the value of the Q of the cavity (about 5000) and the incident microwave power. For 0 dB, the estimated value was 90 Oe.

⁶P. D. Southgate, J. Appl. Phys. **40**, 22 (1969).

⁷See Eq. (4) of Ref. 1. We assumed one electron per atom for n_0 and used the value of C_{66} for the shear stiffness constant of indium [D. R. Wider and C. S. Smith, J. Phys. Chem. Solids **4**, 128 (1958)]. The validity of Eq. (4) is restricted to thin films, $d \ll l \ll v_F/\omega$, where v_F is the Fermi velocity, and it assumes that the acoustic impedance of the substrate does not differ appreciably from that of the metal film. These conditions were fulfilled in our indium films.

⁸D. C. Mattis and J. Bardeen, Phys. Rev. **111**, 412 (1958).

⁹M. Tinkham, in *Low Temperature Physics*, edited by C. Dewitt, B. Dreyfus, and P. G. de Gennes (Gordon and Breach, New York, 1962).

¹⁰A. M. Toxen, Phys. Rev. **127**, 382 (1962).

¹¹J. I. Gittleman, S. Bozowski, and B. Rosenblum, Phys. Rev. **161**, 398 (1967).

¹²See, for instance, A. H. Wilson, *The Theory of Metals* (Cambridge Univ. Press, Cambridge, England, 1953), 2nd ed., p. 245.

Riedel Singularity in Sn-Sn-Oxide-Sn Josephson Tunnel Junctions*

S. A. Buckner, T. F. Finnegan,[†] and D. N. Langenberg

Department of Physics and Laboratory for Research on the Structure of Matter,
University of Pennsylvania, Philadelphia, Pennsylvania 19104

(Received 15 November 1971)

An experimental study has been made of the Riedel singularity in Sn-Sn-oxide-Sn Josephson tunnel junctions. The data indicate that gap anisotropy rather than quasiparticle damping is the dominant factor in rounding off the observed singularity. However, it appears feasible to use the singularity as a probe of quasiparticle damping effects.

The amplitude of the Josephson tunneling supercurrent calculated within the BCS theory displays a logarithmic singularity which is related to the singularity in the BCS quasiparticle density of states at the edge of the superconducting energy gap.¹⁻³ A consequence of this singularity is that

alternate radiation-induced steps in the dc current-voltage (I - V) characteristic of a Josephson tunnel junction should show amplitude singularities when one of the steps lies at the gap voltage $2\Delta/e$, where Δ is the gap parameter. The Riedel singularity will be rounded off by quasiparticle