[12]H. R. Zeller and I. Giaever, Phys. Rev. 181, 789 (1969).

[13]F. Mezei, Phys. Rev. B 4, 3775 (1971).

[14]N. F. Mott, Phil. Mag. 19, 835 (1969).

[15]V. Ambegaokar, B. I. Halperin, and J. S. Langer, Phys. Rev. B 4, 2612 (1971).

[16]A distribution of activation energies can only produce a deviation from $\log\sigma \sim T^{-1}$ at not too low temperatures.

Theory of Surface Magnetoplasmons in Semiconductors*

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A theory of surface magnetoplasmons in semiconductors is developed with the inclusion of retardation for the geometry in which the magnetic field is parallel to the surface and the direction of propagation is perpendicular to the magnetic field. If the background dielectric constant ϵ_∞ lies in a suitable range for a given value of the magnetic field, gaps appear in the dispersion relation for the surface magnetoplasmons. The possible experimental observation of these gaps is discussed.

The dispersion curve for surface polaritons associated with surface plasmons in n-type InSb has recently been observed by Marschall, Fischer, and Quiesser[1] using the anomalies introduced into the infrared reflectivity by a grating ruled on the surface. At large wave vectors ($k \gg \omega_p/c$) the dispersion curve approaches the asymptotic value $\omega_{sp} \equiv \omega_p(1+1/\epsilon_\infty)^{-1/2}$, where ω_p is the bulk plasma frequency defined by $(4\pi ne^2/m^*\epsilon_\infty)^{1/2}$, n is the free-carrier concentration, m^* is the effective mass, and ϵ_∞ is the high-frequency-background dielectric constant. At small wave vectors ($k < \omega_p/c$) the dispersion curve lies just to the right of the light line $\omega = kc$ and joins the light line at $\omega = 0$.

It is of interest to consider the effects which may arise when an external magnetic field is applied. Chiu and Quinn[2] have investigated this problem for a metal taking $\epsilon_\infty = 1$, a case which does not reveal the interesting effects reported in the present paper. We have developed the theory of surface magnetoplasmons including retardation for the case of semiconductors such as n-type InSb, where the energy band is to a good approximation simple and spherical, and $\epsilon_\infty \gg 1$. The analysis becomes particularly simple in the geometry where the external magnetic field is parallel to the surface and the direction of propagation is perpendicular to the magnetic field, so we restrict ourselves to this case. The material is assumed to be semi-infinite and to fill the half-space specified by $x \geq 0$. The external magnetic field B_0 is taken in the y direction and the wave vector in the z direction. The dielectric tensor has the form

$$\begin{pmatrix} \epsilon_1 & 0 & -i\epsilon_2 \\ 0 & \epsilon_3 & 0 \\ i\epsilon_2 & 0 & \epsilon_1 \end{pmatrix},$$

where $\epsilon_1 = \epsilon_\infty[1+\omega_p^2/(\omega_c^2-\omega^2)]$, $\epsilon_2 = \epsilon_\infty\omega_c\omega_p^2/\omega(\omega^2-\omega_c^2)$, $\epsilon_3 = \epsilon_\infty(1-\omega_p^2/\omega^2)$, and $\omega_c = eB_0/m^*c$. The present case involving a gyrodielectric tensor is formally similar to the case of a magnetic medium with a gyropermeability tensor.[3] Both cases exhibit nonreciprocal effects—a lack of equivalence of positive and negative wave vectors.

Our starting point is Maxwell's equations. After we eliminate the magnetic field from the curl equations, we obtain

$$\operatorname{curl}\operatorname{curl}\vec{E} + (1/c^2)\partial^2\vec{D}/\partial t^2 = 0\,, \qquad (1)$$

where $\vec{E}$ is the electric field and $\vec{D}$ is the displacement. We seek solutions of the form

$$\vec{E} = (E_{ix}, 0, E_{iz})e^{-\alpha x}e^{i(kz-\omega t)}, \quad x \geq 0\,, \qquad (2a)$$

$$\vec{E} = (E_{0x}, 0, E_{0z})e^{\alpha_0 x}e^{i(kz-\omega t)}, \quad x < 0\,. \qquad (2b)$$

Equations (2) form a nontrivial solution to Eq. (1) only if

$$\alpha^2 = k^2 - (\omega^2/c^2)\epsilon_V\,, \qquad (3a)$$

$$\alpha_0^2 = k^2 - \omega^2/c^2\,, \qquad (3b)$$

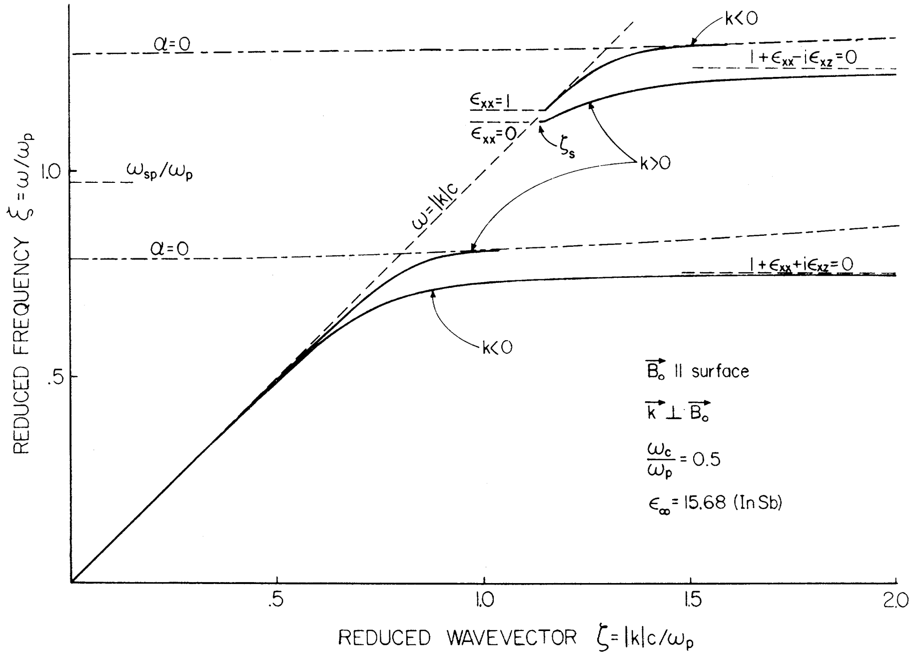

FIG. 1. Dispersion curves for surface polaritons associated with surface magnetoplasmons in n-type InSb. Dashed lines labeled $\alpha=0$, bulk-polariton dispersion curves. ω_{sp} is the zero-field unretarded surface-plasmon frequency defined by $\omega_p(1+1/\epsilon_\infty)^{-1/2}$. ζ_s is defined in the text.

where ϵ_V is the Voigt dielectric constant, given by

$$\epsilon_V = \epsilon_{xx} + \epsilon_{xz}^2/\epsilon_{xx} \tag{3c}$$

$$= \epsilon_1 - \epsilon_2^2/\epsilon_1 . \tag{3d}$$

One can evaluate the amplitude ratios E_{ix}/E_{iz} and E_{0x}/E_{0z} using Eqs. (1) and (3); a simpler result can be obtained, however, if one uses the equation $\nabla\cdot\vec{D}=0$. Then one finds

$$\frac{E_{ix}}{E_{iz}} = \frac{-\alpha\epsilon_{xz} + ik\epsilon_{xx}}{\alpha\epsilon_{xx} + ik\epsilon_{xz}}, \tag{4a}$$

$$E_{0x}/E_{0z} = -ik/\alpha_0 . \tag{4b}$$

The boundary conditions regarding the continuity of the normal component of $\vec{D}$ and the tangential components of $\vec{E}$ at the surface $x=0$ can be written as

$$\epsilon_{xx}E_{ix} + \epsilon_{xz}E_{iz} = E_{0x} , \tag{5a}$$

$$E_{iz} = E_{0z} , \tag{5b}$$

or

$$\epsilon_{xx}(E_{ix}/E_{iz}) + \epsilon_{xz} = E_{0x}/E_{0z} . \tag{6}$$

Substitution of Eqs. (4) into Eq. (6) yields the dispersion relation

$$\alpha + \alpha_0\epsilon_V + ik(\epsilon_{xz}/\epsilon_{xx}) = 0 . \tag{7}$$

We have carried out detailed calculations of the surface-magnetoplasmon dispersion curves for n-type InSb for which $\epsilon_\infty = 15.68$.[4] We assume that the conduction band is spherical and parabolic. One feature, which is immediately obvious from Eq. (7), is that the dispersion curves are different for wave vectors of the same magnitude but opposite signs. Considering first the case $k>0$, and working with the reduced variables $\xi=\omega/\omega_p$, $\zeta=|k|c/\omega_p$, and $\eta=\omega_c/\omega_p$, we plot in Fig. 1 the surface-polariton dispersion curve for the case $\eta=0.5$. A striking effect, not found in the zero-magnetic-field case, is immediately apparent: The dispersion curve consists of two parts with a gap between them. The lower portion starts from the origin and rises just to the right of the light line $\omega=kc$, bends over, and terminates when the curve intersects the dispersion curve for bulk magnetoplasmons (bulk polaritons) defined by $\alpha \equiv k^2 - \omega^2\epsilon_V/c^2 = 0$. The upper branch starts on the line defined by $\epsilon_{xx}=0$, rises, and then approaches the asymptotic frequency for unretarded surface magnetoplasmons found by Pakhomov and Stepanov[5] and defined by the equation $1+\epsilon_{xx} - i\epsilon_{xz}=0$. In the large-wave-vector, unretarded limit, the electric vector for the upper branch executes a retrograde circular motion in

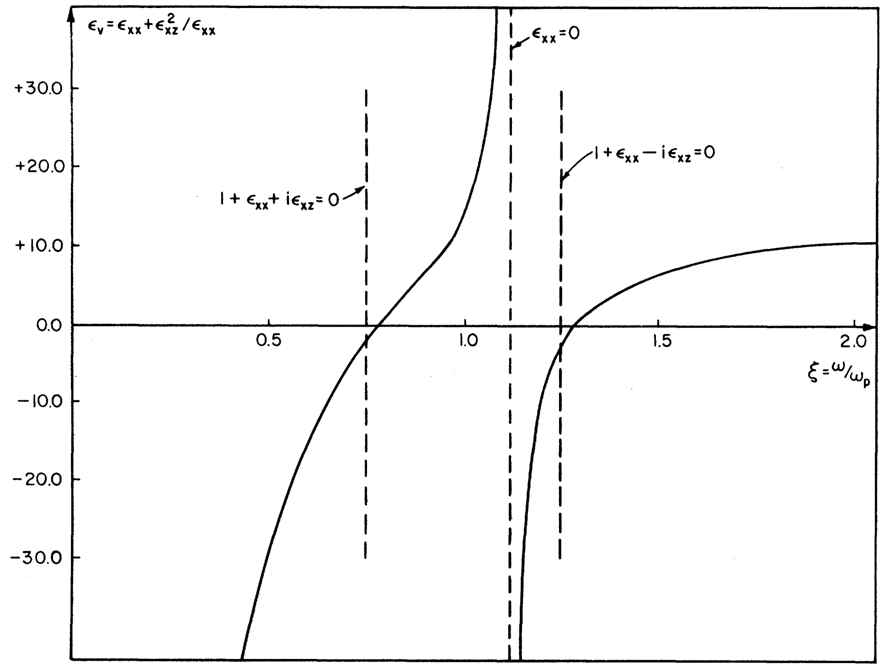

FIG. 2. Dielectric constant ϵ_V plotted as a function of frequency.

the sagittal plane. At the small-wave-vector extremity of this branch, the electric vector is plane polarized perpendicular to the surface. Note that this branch stops before it reaches the light line $\omega = kc$.

The reduced wave vector at which the upper branch starts is specified by the equation

$$\zeta_s^2 = \frac{1+\eta^2}{1-(1+\eta^2)/\epsilon_\infty^2\eta^2}. \tag{8}$$

If ζ_s^2 is to be finite and positive, then ϵ_∞ must satisfy the inequality

$$\epsilon_\infty > (1+\eta^2)^{1/2}/\eta = (\omega_c^2+\omega_p^2)^{1/2}/\omega_c. \tag{9}$$

Recently, Chiu and Quinn[2] studied the surface magnetoplasmon problem with retardation, but did not find a gap in the dispersion curve. The reason is that they did not use a suitable value of ϵ_∞.

That a gap can exist in the dispersion curve is evident from a consideration of Fig. 2, where ϵ_V is plotted against ξ. There is a region below the line $\epsilon_{xx} = 0$ where ϵ_V is large and positive, so that α^2 is negative for finite wave vectors and no surface wave exists. If the unretarded surface magnetoplasmon frequency lies *above* the line $\epsilon_{xx} = 0$, as it does for InSb with $\eta = 0.5$, then a surface wave exists both above and well below this line, and a gap must exist just below this line. To have a surface-wave branch above the line $\epsilon_{xx} = 0$, the ratio ω_c/ω_p must exceed a critical value specified by Eq. (9). For InSb, this critical value is 0.064.

For a value of ϵ_∞ just below the right-hand side of Eq. (9), the upper branch lies below the line $\epsilon_{xx} = 0$ and to the right of the lower bulk dispersion curve. As ϵ_∞ decreases, the gap rapidly closes, and only a single branch remains.

Considering now the case of $k < 0$, we plot also in Fig. 1 the surface-polariton dispersion curves for n-type InSb with $\eta = 0.5$ and $k < 0$. It is clear that the situation is qualitatively different from that where $k > 0$. We now have a complete lower branch running from the origin out to the asymptotic value specified by the equation $1 + \epsilon_{xx} + i\epsilon_{xz} = 0$ which was derived by Pakhomov and Stepanov.[5] In addition, however, we have an upper branch which starts at the light line where $\epsilon_{xx} = 1$, rises to the right of the light line, and ends when it meets the upper bulk-polariton dispersion curve, $\alpha = 0$. The upper branch for $k < 0$ exists when $\epsilon_\infty > 1$ and $\eta > 0$. It may be remarked that, for both $k > 0$ and $k > 0$, there are frequencies at which both a bulk wave and a surface wave can propagate, a situation which does not exist for zero magnetic field.

The gap in the dispersion curve for $k > 0$ and the multiple branches for $k < 0$ should be observable experimentally using the techniques of Marschall, Fischer, and Quiesser.[1] It is necessary to use polarized light with the electric vector perpendicular to the grooves of the grating ruled on the surface. The external magnetic field can then be oriented parallel to the grooves. An alternative and probably more satisfactory procedure is to use the attenuated total reflection technique recently exploited by Marschall and Fischer[6] in their observation of surface optical phonons in GaP.

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[1]N. Marschall, B. Fischer, and H. J. Quiesser, Phys. Rev. Lett. 27, 95 (1971).
[2]K. W. Chiu and J. J. Quinn, to be published. (We wish to thank Professor J. J. Quinn for a copy of this work prior to publication.)
[3]The case of surface polaritons in a magnetic medium will be discussed in A. Hartstein, E. Burstein, R. D. Brewer, A. A. Maradudin, and R. F. Wallis, to be published.
[4]M. Hass and B. W. Henvis, J. Phys. Chem. Solids 23, 1099 (1962).
[5]V. I. Pakhomov and K. N. Stepanov, Zh. Tekh. Fiz. 37, 1393 (1967) [Sov. Phys. Tech. Phys. 12, 1011 (1968)].
[6]N. Marschall and B. Fischer, Phys. Rev. Lett. 28, 811 (1972).

Linewidth of Spontaneous Spin-Flip Light Scattering in InSb†

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The linewidth of spontaneous spin-flip Raman scattering in InSb has been measured as a function of magnetic field and scattering geometry. A phenomenological theory that includes both spin and orbital relaxation gives results in qualitative agreement with experiment.

There has recently been a resurgence of interest in the detailed properties of spin-flip light scattering in semiconductors due to the development of the spin-flip Raman laser.[1-4] In particular, the spontaneous spin-flip linewidth is an important parameter which affects the gain, threshold power, and fine-tuning characteristics[2] of the spin-flip laser. We report the first detailed experimental and theoretical study of the linewidth of spontaneous spin-flip light scattering in indium antimonide. A simple spin-relaxation theory of the linewidth is found to be inconsistent with the experimental data. We have formulated a phenomenological relaxation-time theory, including orbital collisions, which yields motional narrowing of the inhomogeneously broadened spin-flip line, and which gives results in qualitative agreement with experiment.

The experimental arrangement was similar to that previously described.[2,5] A CO laser was used to pump spin-flip scattering in n-type InSb crystals ($2\times2\times6$ mm^3) with an electron density of 1×10^{16} cm^{-3} immersed in superfluid helium ($T\sim2$ K) in the bore of a superconducting magnet. The incident photon energy was 233.5 meV (5.30 μm). In all experiments the incident laser light propagated perpendicular to the magnetic field $\vec{H}$ and was polarized parallel to it. As shown in Fig. 1, the scattered light was collected either along $\vec{H}$ ($\vec{q}\cdot\vec{H}\neq0$ geometry) or in the forward di-

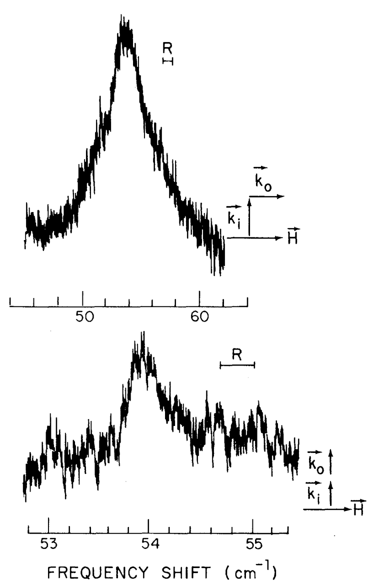

FIG. 1. Line shape of spin-flip Raman scattering ($n=1\times10^{16}$ cm^{-3}, $H=24$ kG, and $T=2$ K). Spectra are shown for the two geometries $\vec{q}\cdot\vec{H}\neq0$ (top) and $\vec{q}\cdot\vec{H}=0$ (bottom).