Thirdly, one might attempt to use the new trial wave function to improve the agreement between experimental and theoretical values of the liquid structure function in helium, in particular in the region about the first peak. Preliminary results indicate that four-particle functions may be needed in the case of a weakly interacting Bose gas, if exact agreement is to be achieved for $S(k)$ up to $O(\lambda^2)$. This observation, however, need not be relevant in the case of helium.[11]

Finally we wish to remark that, in view of possible improvements yet to be made on helium calculations, the statements which have recently been appearing in literature[12] concerning He-He potentials based on Jastrow-type calculations are in our opinion premature.

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# Higher-Order Corrections due to the Order Parameter to the Flux-Flow Conductivity of Dirty, Type-II Superconductors

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We study theoretically the higher-order corrections in $\Delta(\vec{r})$, the superconducting order parameter, to the flux-flow conductivity of dirty, type-II superconductors. It is shown that the Thompson term (i.e., the anomalous term) in the flux-flow conductivity is extremely sensitive to the higher-order corrections and decreases rapidly in the vortex state, while the Caroli-Maki term is affected only slightly by the higher-order corrections.

The flux-flow conductivity of type-II superconductors in the vortex state is of particular interest, since this involves the time-dependent variation of the superconducting order parameter. An early theory proposed by Caroli and Maki (CM)[1] appeared to describe the flux-flow resistivity of dirty, type-II superconductors rather well. Recently, however, Thompson[2] and Takayama and Ebisawa[3] have shown that in the calculation of CM some diagrams were neglected which describe the additional dissipation due to the presence of the order parameter. Since this correction term is of the same order of magnitude as the CM term, in the vicinity of the transition temperature[2,3] at least, the inclusion of this new term certainly destroys the good agreement so far found between theory and experiment.[4,5] In order to avoid this difficulty, it was suggested that the anomalous term may be suppressed rapidly in the vortex state.[6]

Very recently, Kim[7] reported a direct measurement of the field derivative of the flux-flow resistance, $\partial R/\partial H$ ($H$ being the applied magnetic field), in an NbMo-alloy system. It was found that if $\partial R/\partial H$ is extrapolated linearly to the upper critical field $H_{c2}$, the observed $\partial R/\partial H$ at $H_{c2}$ agrees with the expression from the flux-flow conductivity, which contains the Thompson term. On the other hand, it was also observed that $\partial R/\partial H$ decreases rapidly to the value expected by the CM theory as the magnetic field decreases below $H_{c2}$.

More recently, in the study of the fluctuation-induced electric conductivity above the transition temperature, it was shown that the anomalous term[2,8] due to the fluctuation is regularized rapidly because of a strong modification of the interaction vertex due to the fluctuation itself.[9-11] Because of the similarity between the anomalous terms in the fluctuation-induced conductivity and in the flux-flow conductivity, we expect that the vertex associated with the order parameter will be modified strongly, in the vortex state as well, by the presence of the finite order parameter itself. The purpose of this paper is to study systematically the higher-order corrections in $\Delta(\vec{r})$ to the flux-flow conductivity in dirty, type-II superconductors.

We will denote by $\Lambda(\vec{q}, \omega_n)$ the vertex function associated with $\Delta_{\vec{q}}$, where $\Delta_{\vec{q}}$ is the Fourier transform of $\Delta(\vec{r})$, and first calculate the corrected vertex function by perturbation in $\Delta_{\vec{q}}$. The lowest-order diagrams in $\Delta_{\vec{q}}$ are given in Fig. 1, which yield for $\Lambda(\vec{q}, \omega_n)$ in the dirty limit

$$\Lambda(\vec{q}, \omega_n) = |\tilde{\omega}_n|[|\omega_n| + \tfrac{1}{2}D\vec{q}^2 + \textstyle\sum_{\vec{q}'}|\Delta_{\vec{q}'}|^2/(|\omega_n| + \tfrac{1}{2}D\vec{q}^2)]^{-1}, \tag{1}$$

where $\tilde{\omega}_n = \omega_n(1 + 1/2\tau|\omega_n|)$, $\omega_n = 2\pi(n+\frac{1}{2})T$, with $\tau$ the lifetime of electrons due to impurity scattering. It should be noted that Eq. (1) is equivalent to the renormalized vertex calculated by Keller and Korenman,[9] if $|\Delta_{\vec{q}'}|^2$ under the summation sign is replaced by $\mathfrak{D}(\vec{q}', 0)$, the fluctuation propagator of the order parameter above the transition temperature. We note also that the coefficient of the term proportional to $|\Delta_{\vec{q}'}|^2$ in Eq. (1) is larger than the one expected from the gap equation by a factor of 2. This is because, in the present problem, we have two different ways of inserting external vertices carrying the momentum $\vec{q}$ in the corresponding diagram for the gap equation. In the presence of magnetic fields, the order parameter $\Delta(\vec{r})$ in the vortex state is expressed in terms of the Abrikosov solution (at least in the vicinity of $H = H_{c2}$):

$$\Delta(\vec{r}) = \textstyle\sum_n C_n \exp(inky)\exp[-eH(x - nk/2eH)^2], \tag{2}$$

where we have assumed that the magnetic field is parallel to the $z$ axis. In this case Eq. (1) is easily rewritten as[12]

$$\Lambda(\vec{q}, \omega_n) = |\tilde{\omega}_n|[|\omega_n| + \tfrac{1}{2}\epsilon_0 + |\Delta|^2/(|\omega_n| + \tfrac{1}{2}\epsilon_0)]^{-1}, \tag{3}$$

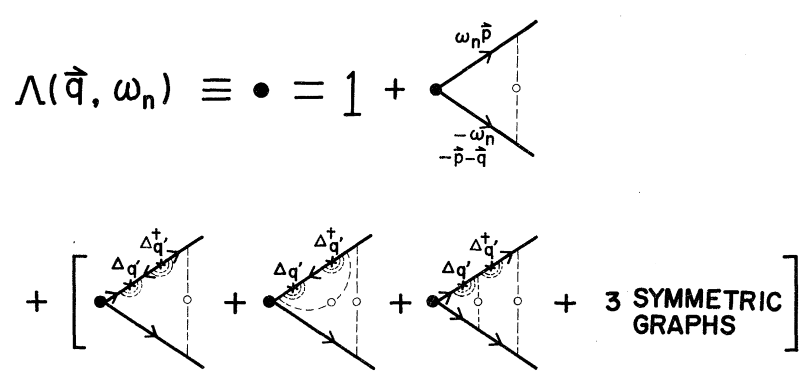


FIG. 1. Lowest-order corrections to $\Lambda(\vec{q},\omega_n)$ in $\Delta_{\vec{q}'}$. Broken and solid lines, impurity scattering and the electron propagator, respectively. In the perturbational calculation the zeroth-order vertex function in $|\Delta_q|^2$ is used for the vertex $\Delta_{\vec{q}'}$ and $\Delta_{\vec{q}'}^\dagger$ in the diagrams. The ladder-type impurity corrections for these vertices are expressed simply by two broken lines. Furthermore, we have considered only the momentum-independent part of the vertex corrections (see Ref. 9), by which we mean that only the combination like $\Delta_{\vec{q}'}\Delta_{\vec{q}'}^\dagger$ is taken into account.

where $\epsilon_0 = 2eDH$, $|\Delta|^2$ is the space average of $|\Delta(\vec{r})|^2$, and $D=\frac{1}{3}lv$ is the diffusion constant. Note that $\epsilon_0$ plays a role of pair-breaking energy[2] and is a universal function of temperature, determined by[12,13]

$$-\ln(T/T_{c0}) = \psi(\tfrac{1}{2}+\epsilon_0/4\pi T) - \psi(\tfrac{1}{2}), \tag{4}$$

where $T_{c0}$ is the transition temperature in the absence of magnetic field and $\psi(z)$ is the digamma function.

In the following calculation, it is necessary to determine $\Lambda(\vec{q}, \omega_n)$ valid for all orders in $|\Delta|^2$. This can be done in principle by analyzing the equation for the vertex function $\Lambda$ in a way similar to the case of the current vertex function,[14] in which the true $G$ and $F$ functions for the vortex state are employed. However, it is difficult to determine the true Green's functions for the vortex state. On the other hand, in the BCS state, where the order parameter is constant in space, we can determine the full vertex function $\Lambda_{\text{BCS}}$, which is given by

$$\Lambda_{\text{BCS}}(\vec{q}, \omega_n) = 1 + \frac{1}{2\tau(\omega_n{}^2+\Delta^2)^{1/2}}\left(1 - \frac{\Delta^2}{2(\omega_n{}^2+\Delta^2)}\right). \tag{5}$$

Comparing Eq. (3) with Eq. (5), we may conjecture that $\Lambda(\vec{q}, \omega_n)$ in the vortex state is given by

$$\Lambda(\vec{q}, \omega_n) = 1 + \frac{1}{2\tau[(|\omega_n|+\frac{1}{2}\epsilon_0)^2+|\Delta|^2]^{1/2}}\left(1 - \frac{|\Delta|^2}{2[(|\omega_n|+\frac{1}{2}\epsilon_0)^2+|\Delta|^2]}\right) \tag{6}$$

$$\cong \frac{|\tilde{\omega}_n|}{[(|\omega_n|+\frac{1}{2}\epsilon_0)^2+|\Delta|^2]^{1/2}} \quad \text{for } \tau\omega_n \ll 1. \tag{6$'$}$$

In fact, if we expand Eq. (6) in powers of $|\Delta|^2$, it does reproduce the correct expansion of $\Lambda^{-1}(\vec{q}, \omega_n)$ at least up to the terms of the order $|\Delta|^4$. In going from (6) to (6′), we have neglected the terms which give rise to higher-order corrections in $|\Delta|^2/\Delta_{00}{}^2$, where $\Delta_{00}$ is the energy gap at $T=0$ K and $H=0$ [or more precisely $\Delta_{00}$ here is given as $\Delta_{00} \cong \max(T, \epsilon)$]. These corrections appear in the CM term as well as the Thompson term. If the above $\Lambda$ is inserted in the anomalous term, Eq. (6) gives rise to the higher-order corrections in $|\Delta|^2$ of the combination $|\Delta|^2/\epsilon_0{}^2$, instead of $|\Delta|^2/\Delta_{00}{}^2$, as we will see below. In fact, it was suggested previously by Thompson[2] that his calculation of the anomalous term may have corrections of order $\Delta/\epsilon_0$.

First, let us consider the anomalous term in the current-current correlation function, which is written to the lowest order in $|\Delta|^2$ as[2]

$$\mathcal{Q}_a(\omega_\nu) = \frac{\sigma|\Delta|^2}{2}\sum_{n=0}^{\nu-1}\left\{\frac{1}{2}\left[\frac{1}{(|\omega_n|+\frac{1}{2}\epsilon_0)^2} + \frac{1}{(|\omega_{\nu-n}|+\frac{1}{2}\epsilon_0)^2}\right] + \frac{1}{(|\omega_n|+\frac{1}{2}\epsilon_0)(|\omega_{\nu-n}|+\frac{1}{2}\epsilon_0)}\right\}, \tag{7}$$

where $\sigma$ is the conductivity in the normal state and $\omega_\nu = 2\pi\nu T$. The higher-order corrections in $|\Delta|^2$ to the anomalous term is calculated by simply replacing $(|\omega_n|+\frac{1}{2}\epsilon_0)^{-1}$ by $[(|\omega_n|+\frac{1}{2}\epsilon_0)^2+|\Delta|^2]^{-1/2}$, and we will have

$$\mathcal{Q}_a'(\omega_\nu) = \frac{\sigma|\Delta|^2}{2}\sum_{n=0}^{\nu-1}\left\{\frac{1}{2}\left[\frac{1}{(|\omega_n|+\frac{1}{2}\epsilon_0)^2+|\Delta|^2} + \frac{1}{(|\omega_{\nu-n}|+\frac{1}{2}\epsilon_0)^2+|\Delta|^2}\right]\right.$$
$$\left. + \frac{1}{[(|\omega_n|+\frac{1}{2}\epsilon_0)^2+|\Delta|^2]^{1/2}[(|\omega_{\nu-n}|+\frac{1}{2}\epsilon_0)^2+|\Delta|^2]^{1/2}}\right\} \tag{8}$$

$$= \frac{\sigma}{2\pi T}|\Delta|^2\left\{\tfrac{1}{2}[\psi^{(1)}(\tfrac{1}{2}-i\omega/2\pi T+\rho) - \psi^{(1)}(\tfrac{1}{2}+\rho)] + \frac{2}{\pi}\int_0^{\pi/2} d\varphi \frac{2\pi T}{[(-i\omega+\epsilon_0)^2+4|\Delta|^2\sin^2\varphi]^{1/2}}\right.$$
$$\left.\times\left[\psi\left(\frac{1}{2}-\frac{i\omega}{2\pi T}+\rho\right) - \psi(\tfrac{1}{2}+\rho)\right] + O\left(\frac{|\Delta|^2}{\Delta_{00}{}^2}\right)\right\}, \tag{9}$$

where $\rho = \epsilon_0/4\pi T$ and $\psi^{(1)}(z)$ is the trigamma function. In Eq. (9) we continued $\omega_\nu$ analytically to a real frequency $\omega$. On the other hand, a similar analysis of the CM term[1] shows that the higher-order corrections to the CM term are always of the order of $|\Delta|^2/\Delta_{00}{}^2$ in dirty superconductors. We will assume in the following that the corrections of the order of $|\Delta|^2/\Delta_{00}{}^2$ are negligible in the range where the static magnetization has a linear dependence on $H_{c2}-H$, but that the corrections of the order of $|\Delta|^2/\epsilon_0^2$, peculiar to dynamical properties, are important even in the above field range, since $\epsilon_0 \ll \Delta_{00}$ in the vicinity of the transition temperature. Making use of Eq. (9), we calculate easily the dc conductivity in the vortex state,

$$\sigma_s = \sigma - (eM/2\pi T)C[|\Delta|, t], \tag{10}$$

$$C[|\Delta|, t] = \rho^{-1} + \left[\frac{\psi^{(2)}(\frac{1}{2}+\rho)}{\psi^{(1)}(\frac{1}{2}+\rho)} + \rho^{-1}\Phi\left(\frac{4|\Delta|^2}{\epsilon_0^{\,2}}\right)\right], \tag{11}$$

$$\Phi(x) = \frac{2}{\pi}(1+x)^{-1/2}K\left(\frac{x}{1+x}\right)^{1/2}, \tag{12}$$

where $t = T/T_{c0}$ and $K(z)$ is the complete elliptic integral. Here we made use of a relation[12]

$$-4eM = (\sigma/\pi T)\psi^{(1)}(\tfrac{1}{2}+\rho)|\Delta|^2, \tag{13}$$

where $M$ is the magnetization. The universal function $\Phi(x)$ is shown in Fig. 2. Since $|\Delta|^2$ is proportional to $H_{c2}-H$, we see easily that the contribution from the anomalous term decreases rapidly as the field decreases. In particular, the field region, where the anomalous term is appreciable, is given by $4|\Delta|^2/\epsilon_0^{\,2} \lesssim 10$, which can be rewritten as

$$1 - H/H_{c2} \lesssim 1 - t. \tag{14}$$

Therefore, we may conclude that in the vicinity of the transition temperature where $1-t$ is small, the anomalous term is suppressed very rapidly in the vortex state. Equation (9) indicates that the anomalous term also decreases in the case of complex conductivity with a finite frequency $\omega$. This may account for the reason why most experiments agree rather well with the CM theory without the anomalous term. The gauge-invariant treatment of the anomalous term as well as details of the present analysis will be published elsewhere.

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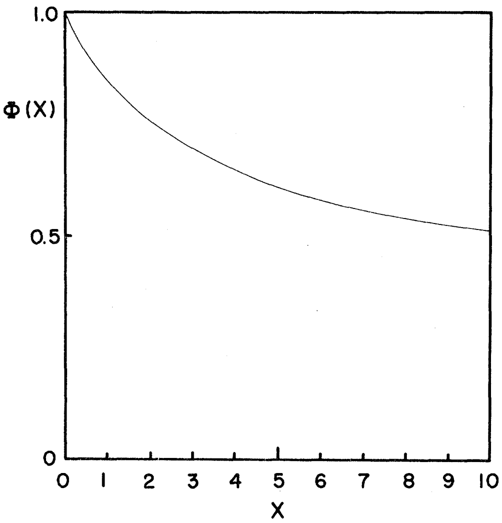


FIG. 2. Universal function $\Phi(x)$, which describes the nonlinear field dependence of the anomalous term.

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