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Acoustic-Wave Generation in a Low-Pressure Plasma Afterglow

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The afterglow of a weakly ionized helium plasma is investigated using an electrostatic probe. The existence of acoustic waves generated at the onset, or very early afterglow, is confirmed at neutral gas pressures as low as 0.1 Torr.

Evidence of the spontaneous generation and amplification of acoustic waves in weakly ionized plasmas has been reported by several investigators.[1-4] Berlande, Goldan, and Goldstein,[1] investigating the afterglow of low-temperature pulsed helium and neon plasmas, found that the electron density and light emission were periodically modulated. They associated this modulation with acoustic waves generated by the discharge itself. The frequency of modulation was found to be of the order of the fundamental resonance frequency of a lateral acoustic mode of the discharge tube. Strickler and Stewart,[2] while modulating argon and krypton dc glow discharges at pressures ranging from 13.5 to 38.5 Torr, observed a kinking of the discharge path at a series of discrete modulation frequencies. These frequencies were associated with radial and azimuthal acoustic modes of oscillation of the neutral-gas component.

This phenomenon has been treated analytically by Ingard[5] and Ingard and Schultz.[6] According to Ingard,[5] spontaneous generation and amplification of acoustic waves can arise in a weakly ionized plasma from the coherent transfer of energy between the electrons and the neutral-gas component. In the ordinary acoustic mode, the neutrals, electrons, and ions all move in phase with respect to one another, provided the acoustic-wave frequency is less than the plasma frequency and the neutral-neutral collision frequency.[7] By comparing the acoustic-wave rate of growth with that of decay, Ingard[5] has obtained the criterion for the onset of spontaneous oscillations:

$$\frac{1}{4\gamma}\frac{d^2}{l_n l_e}\left(\frac{T_e}{T_n}\right)^{3/2}\left(\frac{m_e}{m_n}\right)^{1/2}\frac{N_e}{N_n}>1.$$

In expression (1), a cylindrical chamber of diameter d is assumed; T_e and T_n are the electron and neutral-particle temperatures; m_e and m_n are the electron and neutral-particle masses; N_e and N_n are the electron and neutral-particle densities; γ is the ratio of the specific heat of the neutral gas at constant pressure to that at constant volume; l_e and l_n are quantities of the order of the mean free path of the electrons and neutral particles, respectively.

In this Letter, the results of an experiment are reported which confirm the existence of acoustic waves in a weakly ionized helium plasma afterglow, at a gas pressure as low as 0.1 Torr, with the chamber walls maintained at a temperature of 293°K. It is shown that the generation of these waves is not directly observable, but only their reflections off of the end walls.

The results were accomplished using a cylindrical Pyrex chamber of inside diameter $d=10.4$ cm and length $L=340$ cm. The plasma was generated by a 10-MHz electric field which was capacitively coupled to the gas by means of two ring electrodes. The electrodes were placed at the center of the chamber and had a separation of 30 cm. This allowed the plasma to be localized to a total length of approximately 70 cm about the center of the chamber. In each case, a balanced steady-state plasma was maintained before the rf power was removed. At the onset of the afterglow, care was taken to assure that no residual voltage remained on the electrodes. A double, floating, planar electrostatic probe was placed on the axis, at the center of the chamber. The probe tips were made of tungsten and each had a diameter of 1 mm. The probe was biased to collect ion satu-

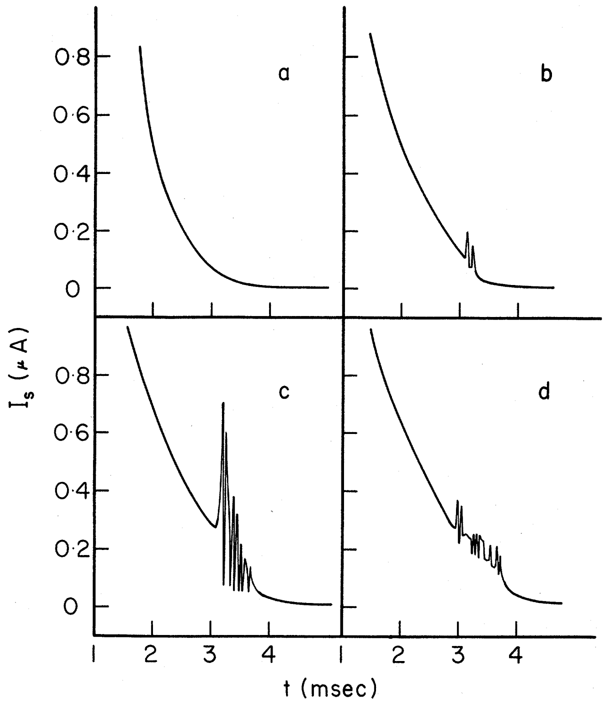

FIG. 1. Ion-saturation current versus time in the afterglow of a helium plasma. (a) $p_g = 0.09$ Torr; (b) $p_g = 0.1$ Torr; (c) $p_g = 0.15$ Torr; (d) $p_g = 0.15$ Torr and end walls lined with fiber glass.

ration current I_s.

Measurements of I_s versus time were made in the afterglow of a helium plasma, over the pressure range from 0.02 to 1.0 Torr. It was not possible to observe I_s during the very early afterglow because of the danger of rf leakage into the probe circuit. For pressures $p_g < 0.1$ Torr, the afterglow was observed to be quiescent, as shown in Fig. 1(a). At a pressure $p_g = 0.1$ Torr, weak oscillations appeared in the afterglow at approximately $t = 3$ msec, as shown in Fig. 1(b). As the pressure was increased, these oscillations increased in amplitude and duration. Above a pressure $p_g = 0.15$ Torr, the amplitude of the oscillations remained constant and the duration approached a time of approximately 1 msec. The oscillations occurred at discrete frequencies which usually ranged from 8 to 12 kHz. At times, oscillations were observed at a frequency of approximately 5 kHz.

In every case, the first 3 msec of the afterglow were observed to be quiescent. At $t = 3$ msec into the afterglow, $T_e \approx T_n$. Acoustic waves could not be generated under this condition. If the oscillations which appear at $t > 3$ msec are of acoustic origin, they must be end-wall reflections of acoustic waves which are generated at the onset of the afterglow. The end walls used in the present experiment were made of polished aluminum, which should give a good reflection. Assuming the acoustic-wave velocity in helium to be $c \approx 10^5$ cm/sec and using the fact that $L = 340$ cm, an acoustic wave generated at the center of the chamber would take 3.4 msec to travel to the end walls and back. The plasma extended a distance of approximately 35 cm on either side of the center of the chamber. This extended source would cause oscillations to occur at the center of the chamber over a period of time between $t \approx 3$ and $t \approx 4$ msec.

In order to determine whether or not these oscillations were caused by acoustic-wave reflections, the end walls were lined with a 1.5-cm-thick layer of fiber glass. Figure 1(c) shows I_s versus time at $p_g = 0.15$ Torr for the case of polished aluminum end walls. Large-amplitude oscillations appeared at approximately $t = 3$ msec. Figure 1(d) shows I_s versus time at $p_g = 0.15$ Torr for the case of fiberglass-lined end walls. Oscillations appeared at approximately $t = 3$ msec, but their amplitudes were greatly reduced from those observed in Fig. 1(c). As a further check, aluminum disks were inserted into the chamber at each end. By moving the disks toward the center of the chamber, the effective length was varied. Measurements were made of the initial time of occurrence of the oscillations as a function of the effective chamber length at $p_g = 0.15$ Torr. As the effective chamber length was decreased, the initial time of occurrence decreased from $t \approx 3$ msec at $L = 340$ cm to $t \approx 2.2$ msec at $L = 255$ cm. The decrease in time was found to be approximately linear with respect to effective chamber length. A further decrease in effective length resulted in a distortion of the plasma column. The frequency of the oscillations was not affected by the decrease in effective length. This fact indicates that the oscillations may be associated with lateral acoustic modes.

The natural frequencies of oscillation in a cylindrical chamber of radius R and length L, assuming rigid walls, are given by the expression

$$f_{nml} = \frac{c}{2\pi}\left[\beta_{nm}^{\;2} + \frac{l^2\pi^2}{L^2}\right]^{1/2}, \qquad (2)$$

where n, m, and l are integers associated with the radial, azimuthal, and longitudinal modes, respectively. The quantity β_{nm} is obtained from the boundary conditon

$$[\partial J_m(\beta r)/\partial r]_{r=R}=0, \tag{3}$$

where $J_m(\beta r)$ is a Bessel function of order m. Assuming $c=10^5$ cm/sec and $R=5.2$ cm, the frequency associated with the lowest order radial mode is $f_{100}\approx 11.7$ kHz, and for the lowest order azimuthal mode is $f_{010}\approx 5.6$ kHz. Both of these values are in approximate agreement with the frequencies observed.

It is apparent that the oscillations are neither generated in the steady-state plasma nor in the observable afterglow. The generation of these oscillations must be associated with power turnoff. It is evident that expression (1) is not satisfied in the steady-state plasma; however, the possibility that it might be satisfied at power turnoff should be considered. If the cooling time for the neutral gas t_n is less than the cooling time for the electrons t_e, the quantity T_e/T_n might increase sufficiently at power turnoff to cause expression (1) to be satisfied. Assuming a weakly ionized plasma,

$$t_n \approx T_{nn} \text{ and } t_e \approx T_{en}/\epsilon,$$

where T_{nn} is the neutral-neutral collision time, T_{en} is the electron-neutral collision time, and ϵ is the fractional energy-transfer coefficient for electron-neutral collisions. At a pressure $p_g = 0.1$ Torr, and assuming that $\epsilon = 2m_e/(m_e+m_n)$, the cooling times at power turnoff are found to be

$$t_n \approx 10^{-6} \text{ and } t_e \approx 10^{-5} \text{ sec.}$$

Thus, an increase in T_e/T_n is possible at power turnoff. Such an increase, however, should be small, since T_n in the steady-state plasma is only slightly greater than its equilibrium value in the neutral gas. If expression (1) is close to being satisfied for the steady-state plasma, a small increase in T_e'/T_n, at power turnoff, may be sufficient to generate oscillations. Assuming $p_g = 0.1$ Torr, the steady-state plasma parameters were estimated to be as follows:

$$N_e/N_n \approx 2\times 10^{-5}, \quad T_e/T_n \approx 180,$$

$$l_e \approx 0.5 \text{ cm}, \quad l_n \approx 0.1 \text{ cm}.$$

Substituting these parameters into the left-hand side of expression (1), one obtains a value of approximately 0.2. Allowing for uncertainties in the above-mentioned parameters, it is conceivable that expression (1) could be on the verge of being satisfied.

It is, therefore, plausible that the Ingard mechanism could be responsible for the generation of the oscillations.

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Multiple-Mirror Confinement of Plasmas*

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Multiple-mirror confinement of high-temperature plasma for fusion application is examined by numerical techniques using a fixed-scattering-center model. It is shown, in contrast to a linear relation found in an earlier study by Post, that the confinement time increases quadratically with the number of mirrors. We present the results of this scaling on the dimensions for a multiple-mirror reactor.

In a previous communication Post[1] explored the properties of multiple-mirror systems for confining plasmas for fusion. He found that such a device, operating in a regime in which the mean free path is comparable to the system length, would have advantages over a long-mean-free-path, single-mirror device both in the scaling laws and in the suppression of velocity-space instabilities. In computer calculations, using a fixed-scattering-center model for the collisions,