Asymmetry and Leading Particles*

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(Received 27 March 1972)

This work demonstrates a remarkable regularity for the asymmetry parameter $\langle\gamma_s/\gamma_c\rangle_a$ of all charged secondaries in multiparticle processes, i.e., $\langle\gamma_s/\gamma_c\rangle_a = n_s^{-0.58}g(E_p)$, where n_s and E_p are, respectively, the number of shower particles and the incident energy for a given multiparticle event. We show a simple derivation of $g(E_p)$ in terms of the emission model and discuss the implications of our results as they relate to the concept of leading particles.

In connection with the estimation of primary energy E_p by Castognoli's method, it is well known that the estimator E_c overestimates E_p. The basic assumptions of the Castagnoli method[1] are as follows: (i) spectrum independence,

$$\tilde{\beta}_i/\beta_c = \tilde{\rho}_i = 1;$$

(ii) forward-backward symmetry,

$$\sum_{i=1}^{n_s} \frac{\sin\tilde{\theta}_i}{\tilde{\rho}_i + \cos\tilde{\theta}_i} = 1;$$

where $\tilde{\beta}_i$ and $\tilde{\theta}_i$ are the respective c.m. velocity and emission angle of the ith shower particle in a given multiparticle event, and β_c is the velocity of the c.m. system. Since only the information of the laboratory emission angles θ_i of the charged secondaries is available, the Lorentz factor obtained from the above assumptions is γ_s, which is the Lorentz factor of the system of charged secondaries, instead of γ_c, the Lorentz factor of the c.m. system, i.e.,

$$\gamma_s = -\langle \ln \tan\theta \rangle. \quad (1)$$

E_c is then related to γ_s by

$$E_c = M_t\{(\gamma_s^2 - 1) + \gamma_s[\gamma_s^2 - 1 + (M_i/M_t)^2]^{1/2}\}, \quad (2)$$

where M_i and M_t are, respectively, the masses of the incident and target particles.

Assumption (i) is, in general, not fulfilled, and its breakdown leads to overestimation of γ_c by a factor of ~ 1.4.[2] Assumption (ii) is, on the average, fullfilled if n_s includes only those pionization products. In practice, however, (ii) is not fulfilled, because the c.m. energetic secondary particle with $\tilde{\theta} \simeq 180°$ may appear in the lab system with $\theta \gtrsim 90°$ and thus has to be neglected in estimating γ_s by Eq. (1). Nevertheless, it has been shown that the degree of overestimation can be reduced as E_p increases, because the effective mass of the pionization charged secondaries increases with E_p. Using the emission model,[3] we can estimate $\langle\gamma_s/\gamma_c\rangle_p$, where the subscript p reminds us of the pionization. The theoretical predictions were shown to agree quantitatively with the experimental data.[3]

If n_s includes all the charged secondaries, i.e., pionization products and leading particles, $\langle\gamma_s/\gamma_c\rangle_a$ should not decrease with E_p, where the subscript a reminds us of all charged secondaries. In fact, since the mean inelasticity of the charge pionization products, $\langle\tilde{K}_{\rm ch}\rangle$, decreases in the high-energy region, and the charged leading particles appear unlikely as a shower particle in laboratory backward sphere, we expect $\langle\gamma_s/\gamma_c\rangle_a$ to increase with E_p.

In this paper, we present an investigation of $\langle\gamma_s/\gamma_c\rangle_a$. We first show that $\langle\gamma_s/\gamma_c\rangle_a$ posesses a remarkable regularity as a function of n_s and E_p, i.e.,

$$\langle\gamma_s/\gamma_c\rangle_a = f(n_s)g(E_p). \quad (3)$$

We then derive $g(E_p)$ on the basis of the emission model. Finally, we discuss the implications of our results as they relate to the concepts of leading particles, etc.

To evaluate γ_s/γ_c, we have to know γ_c. For accelerator data, γ_c is known. For the cosmic-ray data, we estimate E_p by $E_{\rm ch}$, i.e.,

$$E_{\rm ch} = (0.4 \text{ GeV}) \sum_{i=1}^{n} \frac{1}{\sin\theta_i}, \quad (4)$$

and then obtain γ_c by a formula similar to Eq. (2).

We make use of cosmic-ray events of the International Cooperative Emulsion Flight Collaboration[4] and of Barkow *et al.*[5] Since the selection criteria are $N_h \leqslant 5$ and $n_s \geqslant 7$, Eq. (4) is quite reliable.[6] We classify the events into two classes, one with $E_{\rm ch} \geqslant 10^3$ GeV and the other with $10^3 > E_{\rm ch} \geqslant 10^2$ GeV. We obtain 55 events for the former which consists mainly of secondary events. (Among them, two events with $E_c/E_{\rm ch} > 100$ were considered as abnormal and so were deleted.)

The results of our analysis together with those from accelerator data by Anzon *et al.*[7] are shown

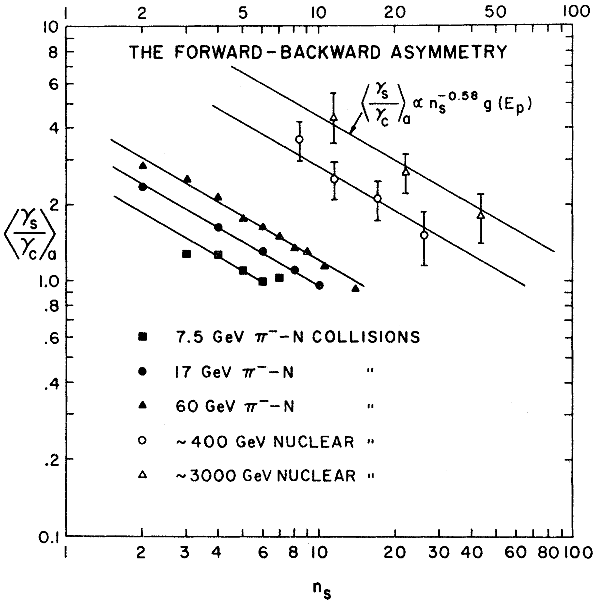

FIG. 1. Asymmetry parameter $\langle\gamma_s/\gamma_c\rangle_a$ as a function of n_s in hadron collisions with different incident energies E_p.

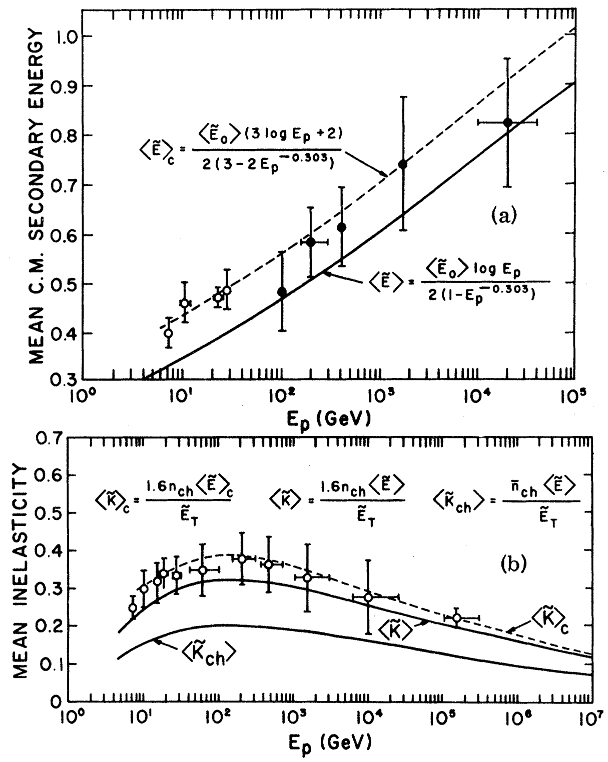

FIG. 2. (a) Average c.m. energy of the pionization products, $\langle\tilde{E}\rangle$, as a function of E_p. $\langle\tilde{E}_c\rangle$ is $\langle\tilde{E}\rangle$ corrected for the production of a neutral secondary from an incompletely excited level. For details, see Ref. 3. (b) Average c.m. inelasticity of the pionization products, $\langle\tilde{K}\rangle$, as a function of E_p. $\langle\tilde{K}_{ch}\rangle$ is the $\langle\tilde{K}\rangle$ corresponding to $\langle\tilde{E}_c\rangle$. For details, see Ref. 3.

in Fig. 1. We note that the γ_s of the accelerator events were estimated with an independent method other than Eq. (1). The errors associated with cosmic-ray data are only those of statistical Poisson type.

It is easily seen from Fig. 1 that Eq. (3) should follow, with $f(n_s)$ given by

$$f(n_s)\propto n_s^{-0.58},\quad 2\lesssim n_s\lesssim n_{\max},\qquad f(n_s)\simeq \text{const},\quad n_s>n_{\max},\tag{5}$$

where $n_{\max}$ is given by

$$n_{\max}=f^{-1}(1/g(E_p)).\tag{6}$$

What is the functional form of $g(E_p)$? We note that $\langle\tilde{E}\rangle$, which is the average c.m. energy of the charged pionization products (for N-N collisions), is given by

$$\langle\tilde{E}\rangle=\langle\tilde{E}_0\rangle\ln E_p/2(1-E_p^{-0.303}),\tag{7}$$

and $\langle\tilde{K}\rangle_c$, which is the mean inelasticity of all pionization products, is given by

$$\langle K\rangle_c\propto E_p^{-0.197}\ln E_p \text{ for large } E_p\tag{8}$$

on the basis of the emission model,[3] where $\langle\tilde{E}_0\rangle\simeq 0.35$ GeV. (From now on, E_p is in units of GeV.)

The theoretical predictions of Eqs. (7) and (8) and their comparison with the experimental data are shown in Fig. 2. Since $\langle\tilde{K}\rangle_c$ is small in magnitude and decreases slowly with E_p, the c.m. energy of leading particles is

$$\tilde{E}_l\propto(1-\langle\tilde{K}\rangle_c)(E_p+1)^{0.5}\simeq(E_p+1)^{0.5}.\tag{9}$$

$\langle\gamma_s/\gamma_c\rangle_a$ for a given n_s, i.e., $g(E_p)$, should be *roughly* given by

$$g(E_p)\propto\frac{\tilde{E}_l+b}{\langle\tilde{E}\rangle}\propto\frac{[(E_p+1)^{0.5}+b](1-E_p^{-0.303})}{\ln E_p},\tag{10}$$

where b represents the contribution of pionization products.

With the help of Fig. 1, and the functional form of $f(n_s)$ and $g(E_p)$ as given by Eqs. (5) and (10), respectively, we estimate b to be ~ 1. We can write $\langle\gamma_s/\gamma_c\rangle_a$ more explicitly as

$$\langle\gamma_s/\gamma_c\rangle_a=1.45n_s^{-0.58}[(E_p+1)^{0.5}+1](1-E_p^{-0.303})/\ln E_p,\quad 2\lesssim n_s\lesssim n_{\max},\qquad =\sim 1,\quad n_s>n_{\max};\tag{11}$$

and $n_{max}(E_p)$ is given by

$$n_{max} = \{1.4[(E_p+1)^{0.5}+1](1-E_p^{-0.303})/\ln E_p\}^{1/0.58}. \quad (12)$$

The n_{max}'s at $E_p = 7.5, 17, \cdots$ have been calculated and compared with the values suggested from Fig. 1 in Table I.[8] The agreements are quantitatively satisfactory in view of the possible errors.

We proceed now to discuss the result and its implications.

(a) $\langle\gamma_s/\gamma_c\rangle_a$ or $f(n_s)$ in particular is not defined for $n_s \lesssim 2$, because resonance production is copious there and Eq. (9) may not hold. In general, $\langle\gamma_s/\gamma_c\rangle_a$ should be flatter in such a low-n_s region because the decay products of the leading particle share $\tilde{E}_l$ among themselves. It will be interesting to study how $\langle\gamma_s/\gamma_c\rangle_a$ deviates from Eqs. (11) or (5) in relating to the rate of resonance production or final-state interactions.

(b) As n_s approaches n_{max}, the leading particles become less energetic and lose their "leading" status. For $n_s < n_{max}$, the leading particles are kinematically indistinguishable from others. Therefore, the concept of leading particles is valid only in the region where $n_s < n_{max}$.

(c) Since there are no leading particles for $n_s > n_{max}$, $\langle\gamma_s/\gamma_c\rangle_a$ should be unity. This should have been the region where Fermi conceived his statistical model.[9] In plain words, the collisions here are head on and central. It will be interesting to explore further the theoretical meaning of n_{max} in this respect.

(d) In the high-energy limit, Eq. (12) becomes

$$n_{max} = 1.9\left[\frac{E_p^{0.5}}{\ln E_p}\right]^{1.725} \text{ for large } E_p. \quad (13)$$

Comparing with $\bar{n}_{ch}$, which is the mean charged multiplicity of pionization products,

$$\bar{n}_{ch} = 2(E_p^{0.303} - 1), \quad (14)$$

we have

$$\lim_{E_p\to\infty} \frac{\bar{n}_{ch}}{n_{max}} = 0. \quad (15)$$

Equation (15) implies that the collisions become more elastic and, in the high-energy limit, $\langle\tilde{K}\rangle_c \to 0$, which is consistent with Eq. (8).

(e) Let us generalize Eq. (13) to

$$n_{max} = \text{const}\,[E_p^{0.5}/\ln E_p]^{1/\delta}. \quad (16)$$

Then, δ seems larger than 0.58 for the cosmic-ray data as can be seen from Fig. 1. This may be a genuine trend or just a systematic error in the estimation of E_p by E_{ch} due to Eq. (4) and intranuclear cascade effects.[6] In case that δ should increase and we conjecture that the inelasticity should decrease but approach somehow a certain constant in the high-energy limit, then

$$\lim_{E_p\to\infty} \delta = \frac{0.5}{0.303} = \frac{1}{2\ln 2} \quad (17)$$

(f) If we accept Eq. (13) or (11) to hold strictly, then the logarithmic increase of $\langle n_s\rangle$ applies a much faster decrease of the inelasticity than Eq. (8). This seems unlikely in view of the present data.

(g) Finally, we discuss briefly the asymmetry of the pionization products, i.e., $\langle\gamma_s/\gamma_c\rangle_p$. Using the emission model, the typical c.m. velocity of the system of charged pionization products was shown to be[3]

$$\tilde{\beta}_{s,p} = 2/(1.5\ln E_p + 2), \quad (17a)$$

and therefore the typical $\gamma_{s,p}/\gamma_c$ is

$$\gamma_{s,p}/\gamma_c = \tilde{\gamma}_{s,p}(1 \pm \tilde{\beta}_{c,p}) \simeq \tilde{\gamma}_{s,p}(1 \pm \tilde{\beta}_{s,p}), \quad (17b)$$

where $\tilde{\gamma}_{s,p} = (1-\tilde{\beta}_{s,p})^{-1/2}$. We have, for the forward plus branch,[10]

$$\langle\gamma_s/\gamma_c\rangle_p = \begin{cases} 1.80, & E_p = 17 \text{ GeV}; \\ 1.45, & E_p = 200 \text{ GeV}. \end{cases} \quad (18a)$$

This is to be compared with the experimental results[10]

$$\gamma_{s,p}/\gamma_c = \begin{cases} \sim 1.85, & E_p = 17 \text{ GeV}; \\ \sim 1.55, & E_p = \sim 200 \text{ GeV}; \end{cases} \quad (18b)$$

which can be seen from Figs. 3(a) and 3(b). Indeed, $\langle\gamma_s/\gamma_c\rangle_p$ should decrease with E_p as was

TABLE I. n_{max} for different incident energies E_p (GeV).

E_p	n_{max} Exptl.	n_{max} Theor.
7.5	6.0	6.5
17	9.0	8.8
60	16.5	14.0
~400	60 ± 12	51.5
~3000	135 ± 35	190

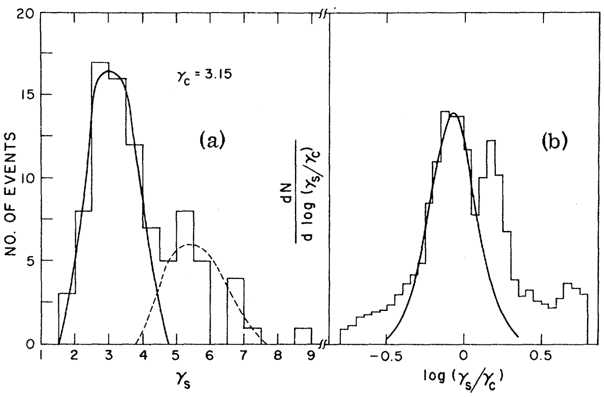

FIG. 3. (a) Distribution of values of $\gamma_{s,p}$ for 17-GeV π^--N collisions. Curves are free-hand fits of the experimental histogram of the author's work in Ref. 2. (b) Distribution of values of $\ln(\gamma_s/\gamma_c)$ for nuclear interactions of ~200-GeV cosmic rays in graphite observed by Erofeeva *et al.*, Ref. 11.

pointed out earlier.

In conclusion, we emphasize that the simple regularity of $\langle\gamma_s/\gamma_c\rangle_a$ is still not fully understood. However, the plot may provide us with an effective diagnosis for multiparticle processes.

The author wishes to thank Professor K. Kang and other members of the High Energy Physics Group at Brown University for various discussions.

*Work supported in part by the U. S. Atomic Energy Commission.

[1]C. Castagnoli *et al.*, Nuovo Cimento 10, 1539 (1953).

[2]M. L. Shen and M. F. Kaplon, Ann. Phys. (New York) 32, 452 (1965).

[3]M. L. Shen, Progr. Theor. Phys. 45, 1817 (1971), and "Quantization of Secondary Energy and Model of Multiple Particle Production" (to be published).

[4]International Cooperative Emulsion Flight Collaboration, Nuovo Cimento Suppl. 1, 1039 (1963).

[5]A. G. Barkow *et al.*, Phys. Rev. 122, 617 (1961).

[6]To be more exact, Eq. (4) should be written as

$$E_{\rm ch} = \frac{0.35\ \text{GeV}}{\langle N_h\rangle^{1/4}\langle K_a\rangle}\sum_{i=1}^{n_s}\frac{1}{\sin\theta_i}, \qquad (4')$$

where $\langle K_a\rangle$ is the mean inelasticity of all charged secondaries which include the mean pionization products and those relating to leading particles, and is equal to ~0.75. $\langle N_h\rangle^{1/4}$ is introduced to correct for the effect of intranuclear cascade. Since $\langle n_s\rangle_{N_h\geq 1} = N_h^{1/4}\langle n_s\rangle_{N_h=0}$ and $\langle N_h\rangle = 2.4$, Eqs. (4) and (4′) are equivalent [M. L. Shen, Nucl. Phys. B 3, 77 (1968)]. On the other hand $\langle N_h\rangle \propto n_s$, Eq. (4′) implies that $E_{\rm ch}$ of lower n_s events may underestimate E_p, while $E_{\rm ch}$ of larger n_s events may overestimate E_p.

[7]V. Anzon *et al.*, Yad. Fiz. 10, 991 (1970) [Sov. J. Nucl. Phys. 10, 570 (1970)].

[8]$n_{\rm max}$ of ~3000 GeV seems somewhat overestimated by Eq. (12). The discrepancy could be explained partly by the difference in the incident particles, i.e., N-N collisions versus π-N collisions of others.

[9]E. Fermi, Phys. Rev. 81, 681 (1951).

[10]We consider only the plus branch, because the minus branch is usually insignificant as a result of the more probable neglect of charged secondaries with $\theta \gtrsim 90°$. For details, see Ref. 3.

[11]The 17-GeV π^--N data were taken from Ref. 2 and the ~200-GeV nuclear collision data were quoted from I. N. Erofeeva *et al.*, Can. J. Phys. 46, S681 (1968).

Neutrinos of Nonzero Rest Mass*

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Some implications of neutrinos having nonzero rest masses and having finite lifetimes are considered.

All explanations of the anomalously low counting rate in the experiment of Davis and co-workers[1] to detect solar neutrinos which ascribe unusual properties to the neutrinos depend on the neutrinos having nonzero masses. Recently, we proposed a theory in which neutrinos are predicted to have nonzero masses; in particular, the muon and the electron neutrino are predicted to have masses 2.5 keV and 12 eV, respectively.[2] In this Letter we show that if the neutrinos have the masses as given above then there are severe, sometimes fatal, constraints on some of the possible explanations of the results of Davis and co-workers. In particular, we show that if neutrinos have the above masses, then (1) neutrino oscillations $\nu_e \rightleftarrows \nu_\mu$ suggested by Gribov and Pontecorvo[3] as a possible explanation for the results from Ref. 1 can be ruled out; (2) limits for the decay rates $\nu_e \to \nu_1 + \nu_2 + \bar{\nu}_3$ and $\nu_e \to \nu_1 + \gamma$ are $\Gamma_1 \leq 10^{-6}$ yr^{-1} and $\Gamma_2 \leq 3\times10^{-13}$ yr^{-1}, respectively