by using different formulas, and hence, data on different hypernuclei. But, in all the cases we have discussed the discrepancy is less than 0.3 MeV (apart from the uncertainties in the experimental B_Λ's used).

(ii) If there are no three-body ΛNN forces present, the four-term formula (3) should work well. The deviation of the four-term formula can be as large as 1.5 MeV, and is much larger than the deviation of less than 0.3 MeV obtained with the six-term formulas. This may be taken as an evidence for three-body ΛNN forces.[2]

(iii) As was noted before, our prediction implies large differences between the B_Λ's of mirror hypernuclei, such as (${}^{6}_{\Lambda}$Li, ${}^{6}_{\Lambda}$He), (${}^{7}_{\Lambda}$Be, ${}^{7}_{\Lambda}$He), (${}^{9}_{\Lambda}$B, ${}^{9}_{\Lambda}$Li), and (${}^{10}_{\Lambda}$B, ${}^{10}_{\Lambda}$Be). If this is confirmed, this will pose a difficult theoretical problem.[8,11] The large charge asymmetry predicted hinges on the experimental B_Λ values of ${}^{6}_{\Lambda}$He and ${}^{10}_{\Lambda}$B. For example, consider the hypothetical case of $B_\Lambda({}^{6}_{\Lambda}\mathrm{He})$ being 0.5 MeV larger than the currently accepted experimental value. Then the derived value of $B_\Lambda({}^{6}_{\Lambda}\mathrm{Li})$ would be 0.5 MeV less, giving a less pronounced charge asymmetry.

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Conservation of the Newman-Penrose Conserved Quantities*

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Using an example of Press and Bardeen, Newman-Penrose quantities of the electromagnetic field in a Schwarzschild background are related to a differential conservation law and hence changes result from a flux. There is no discontinuity in the quantities resulting from a sudden change in dipole moment; there is no singular surface which moves out at $\frac{1}{3}$ the speed of light. It is suggested that for Newman-Penrose quantities to exist, multipole moments must approach a limit as $1/u$, $u \to -\infty$.

In a recent Letter,[1] Press and Bardeen (PB) suggest that Newman-Penrose quantities (NPQ) need not be constant when defined at finite distances rather than at null infinity where the NP constants[2,3] are defined. The particularly intriguing aspect of their discussion is the apparent existence of a surface which moves out at $\frac{1}{3}$ the speed of light, across which the NPQ may change discontinuously from one static value to another. In their example, PB write the electromagnetic field as a power series in $1/r$, hence as a Taylor series in the vicinity of infinity. The NPQ are then found through the coefficient of the appropriate term in this series. In general, the series has only a finite radius of convergence and that radius determines the above surface of discontinuity. It is the purpose of this note to point out that there exists an alternate definition which leads to the NP constants at null infinity, but at finite distances the corresponding NPQ undergo a continuous change. Thus with the alternate definition, the example of PB does not result in a $c/3$ surface of discontinuity.

A constant of the motion is ordinarily associated with a differential conservation law. In a series of papers,[4-7] the relationship between the NP constants and such a conservation law was established. For linear field equations, the conservation law is related to the linear superposition of solutions. In general relativity, the argument is more complicated, but need not concern us here. The details are presented in Refs. 6 and 7.

Press and Bardeen limit their discussion to the electromagnetic field in a Schwarzschild metric, and we shall deal with their example. However, we shall take results and notation from Ref. 7. The components of electromagnetic field and the vector potential may be expressed in terms of a null tetrad[8] ($l\mu n^\mu = m_\mu \bar{m}^\mu = 1$, all other contractions vanishing) as follows[9]:

$$F^{\mu\nu} = \varphi_2 l^{[\mu}m^{\nu]} - \varphi_1(l^{[\mu}n^{\nu]} - m^{[\mu}\bar{m}^{\nu]}) - \varphi_0 n^{[\mu}\bar{m}^{\nu]} + \text{c.c.},$$
$$A^\mu = A_0 l^\mu + A_2 m^\mu + A_3 \bar{m}^\mu,$$
$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu, \tag{1}$$

where $l^{[\mu}m^{\nu]} \equiv l^\mu m^\nu - l^\mu m^\nu$. The gauge is chosen so that $A_1 = A_\mu l^\mu = 0$. Similar expressions hold for $\tilde{\delta}F^{\mu\nu}$ and $\tilde{\delta}A^\mu$ when they appear.

Since $r\varphi_1$ satisfies a generalized scalar wave equation, it is convenient to introduce, following PB, the quantity

$$r^2\varphi_1 = \Psi$$

and to express all other quantities in terms of Ψ. To do so explicitly, expand all quantities in a series of appropriate spin-weighted spherical functions. In particular,

$$\Psi = \sum_{l,m} \Psi_{lm0} Y_{lm}. \tag{2}$$

We require the results only for $l=1$, so we shall suppress and write simply $\Psi_{1m} = \Psi_m$. Then in an obvious extension of this notation for $l=1$, we obtain from Maxwell's equations and Eqs. (1)

$$r\varphi_{0,m} = \partial_r \Psi_m,$$
$$r^2\varphi_{1,m} = \Psi_m,$$
$$r\varphi_{2,m} = -[\partial_0 - \tfrac{1}{2}(1-2M/r)\partial_r]\Psi_m, \tag{3a}$$

and

$$rA_{3,m} = (-1)^{m+1} r\bar{A}_{2,-m} = \Psi_m,$$
$$A_0 = [\partial_0 - \tfrac{1}{2}(1-2M/r)\partial_r][\Psi_m + (-1)^m \bar{\Psi}_m]. \tag{3b}$$

By applying Green's identity to Maxwell's equations we find a conserved vector density

$$t^\rho = \tfrac{1}{2}(-g)^{1/2}\{\tilde{\delta}A_\mu F^{\mu\rho} - A_\mu \tilde{\delta}F^{\mu\rho}\},$$
$$F^{\mu\rho}{}_{;\rho} = (\tilde{\delta}F^{\mu\rho})_{;\rho} = 0 \Rightarrow t^\rho{}_{,\rho} = 0. \tag{4}$$

The varied quantities can be determined in accordance with Eq. (3) in terms of a $\tilde{\delta}\Psi_m$. To obtain NPQ, choose $\tilde{\delta}\Psi_m$ to be an incoming dipole field which in flat space is different from zero only on a past null cone and in curved space vanishes at least behind the surface[8] $u + 2r^* = 2R$,

$$\tilde{\delta}\Psi_m = r^2\partial_r(B_m/r^2) + O(r^{-3}),$$
$$B_m(u+2r^*) = a_m r_0{}^4 \delta(u + 2r^* - 2R); \tag{5}$$

R is a constant which specifies the incoming null surface, and r_0 is defined by $u + 2r_0{}^* - 2R = 0$ for fixed u and R.

Then we define the NPQ as ($l_\rho = u_{,\rho}$)

$$\bar{a}_m \mathfrak{F}_m(u, r_0) = \int_{u=\text{const}} t^\rho l_\rho \, dr\, d\theta\, d\varphi,$$

or by using explicitly Eqs. (2), (3), and (5)

$$\mathfrak{F}_m(u,r) = r^2[\partial_r r^2 \partial_r \Psi_m + O(r^{-3})],$$

where the subscript has been dropped from r_0. In Ref. 7 we show that

$$\lim_{r\to\infty} \mathfrak{F}_m(u,r) \equiv F_m$$

are constants of the motion (the NP constants) in an asymptotically flat space-time and for fields which together with two derivatives exist at null infinity. In general $\mathfrak{F}_m(u,r)$ will not be constant. Furthermore, the limit $u \to \infty$ will in general lead to quite a different value than the limit $r \to \infty$. To complicate matters, it is not clear, at this time, which, if either, limit is measurable in a given physical situation. However, because of the differential conservation law $t^\rho{}_{,\rho} = 0$ the difference in the $\mathfrak{F}_m$ evaluated on two surfaces, $u = u_1$ and $u = u_2$, will be given by a flux integral[6,7] over a timelike surface connecting the two null surfaces for $r > r_0$.

Let us now consider the example of PB to see whether a discontinuity appears on the surface $u = 2r$. They consider a situation which is initially static; a sharp pulse of dipole radiation is emitted along the null surface $u = 0$; the electric dipole moment becomes constant once again. To first order in M, they write the solution as

$$\Psi_m = \frac{D_m}{r} + \frac{3}{2}\frac{MD_m}{r^2} + O\left(\frac{M^2}{r^3}\right), \quad u<0;$$
$$\Psi_m = \frac{D_m'}{r} + \frac{3}{2}\frac{MD_m}{r^2} + \frac{3}{2}\frac{M(D_m'-D_m)}{r^2}\frac{u(u+8r/3)}{(u+2r)^2} + O(M^2/r^3), \quad u>0. \tag{7}$$

Substituting this solution into (6), we obtain

$$\mathfrak{F}_m(u,r) = 3MD_m + O(M^2/r), \quad u<0;$$
$$\mathfrak{F}_m(u,r) = 3MD_m + 3M(D_m'-D_m)u^2/(u+2r)^2 + \cdots, \quad u>0,$$

where the dots indicate terms which vanish in both the limits, $r\to\infty$ and $u\to\infty$.

For $u<0$, $\mathfrak{F}_m$ is constant and to first order in M it is just the NP constant $F_m = 3MD_m$. For $u>0$, $\mathfrak{F}_m$ is no longer constant but rather it increases uniformly from its previous constant value. Nothing unusual occurs at $u=2r$ and the limit $u\to\infty$ gives a measure of the new dipole moment D_m' as desired by Press and Bardeen.

Thus it appears that no physical discontinuity moves out at a velocity $\frac{1}{3}c$ as suggested by Press and Bardeen. However, there is one other point to discuss. The argument used by PB shows that if the dipole moment does not become static in the infinite past, NP constants may not exist at all. Since their argument depends on the pile up, at null infinity, of ($\frac{1}{3}c$) surfaces of discontinuity, it is interesting to see how this behavior appears in the present context.

The existence of NP constants depends on satisfying stronger conditions at null infinity than those required for proof of the peeling theorem.[10] For peeling to occur, it is sufficient that $\varphi_0 \sim O(r^{-3})$; NP constants require the first two terms of an asymptotic expansion:

$$r^3\varphi_0 = \varphi_0{}^0 + \varphi_0{}^1/r + O(r^{-2}),$$

In flat space the NP constants arise from the incoming field. For the dipole fields

$$\tilde{\Psi}_m(v,r) = \dot{\bar{f}}_m(v) - \bar{f}_m(v)/r, \quad v = u+2r.$$

Therefore, in order that NP constants exist we must require $\bar{f}_m \sim v^{-1}$, $v\to\infty$. In fact, all multipole moments of the incoming radiation field must die out like v^{-1} in order that the constants exist. At null infinity, the limit $u\to-\infty$ and $v\to+\infty$ merge; in flat space they are identical, in curved space they tend to the singular point I_0. Since in curved space, the outgoing field is scattered to produce an incoming field, one suspects that NP constants exist only if the outgoing dipole field is characterized by functions $f_m(u)$ and constants $f_{0,m}$ such that

$$f_m(u) - f_{0,m} \sim u^{-1}, \quad u\to-\infty.$$

At present, there is no proof of this conjecture. Nonetheless, $\mathfrak{F}_m(u,r)$ [Eq. (16)] and both of its limits may exist under weaker conditions.

To conclude this discussion, by defining the NPQ through a differential conservation law, discontinuities such as discussed by PB do not occur. The discontinuities arise in the work of Press and Bardeen because they identify the constants or quantities with coefficients in a power series. This power series does not, in general, have an infinite radius of convergence, but one should not attach physical significance to this mathematical fact. Furthermore, one would expect that should the NPQ become measurable, hence physically important, their relationship to a conservation law would likewise be important.

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$$u = t - r^*,$$
$$r^* = r + 2M\ln(r/2M - 1).$$

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