Prediction of Binding Energies of Light Hypernuclei*

R. K. Bhaduri and Y. Nogami
Department of Physics, McMaster University, Hamilton, Ontario, Canada
(Received 20 March 1972)

It is pointed out that Garvey and Kelson's mass formulas, which relate masses of adjoining nuclides, hold remarkably well for Λ separation energies of light hypernuclei. The binding energies of a number of hypernuclei in the p shell are predicted, including the controversial cases of ${}^{6}_{\Lambda}\mathrm{Li}$ and ${}^{9}_{\Lambda}\mathrm{B}$.

Garvey and Kelson[1] have developed a method of exploiting known nuclear masses to predict the masses of adjoining nuclides. The method consists of constructing a difference equation of the form $\sum_i C_i \times M(N_i, Z_i)=0$, where $C_i=\pm 1$, $M(N_i, Z_i)$ is the mass of the ith nuclide with N_i neutrons and Z_i protons, and the number of terms in the equation is even. The sum over a set of neighboring nuclides is taken so as to cancel the two-body (n,n) bonds among themselves, and likewise for the (p,p) and (n,p) bonds. The simplest nontrivial formulas are

$$M(N+2, Z-2)+M(N+1, Z)+M(N, Z-1)=M(N+2, Z-1)+M(N+1, Z-2)+M(N, Z), \tag{1}$$

and

$$M(N+2, Z)+M(N+1, Z-2)+M(N, Z-1)=M(N+2, Z-1)+M(N+1, Z)+M(N, Z-2). \tag{2}$$

One can easily check that the number of (n,n), (p,p), and (n,p) bonds separately cancel out in the above formulas. Equations (1) and (2) have been tested for several hundred nuclides with $A>16$ and $N>Z$, and if $N=Z$, N may not be odd. It was found that the mean deviation was less than 200 keV, a remarkable accuracy in view of the simplicity of the method. The restrictions on N and Z are imposed to take account of isospin dependence of the symmetry energy, a complication not present in the Λ separation energy B_Λ. For light nuclides with $A<16$, Eqs. (1) and (2) are much less accurate, because the strength of the two-nucleon bond is sensitive to the relevant orbits of the nucleons and the cancelation of these is much less complete.

The purpose of this note is to demonstrate that the mass formulas (1) and (2) can be applied directly to the Λ separation energies of hypernuclei, *including the light ones*, without any constraints on N, Z, and A, and a number of interesting predictions can be made.

If we assume that the Λ binding in a hypernucleus is due only to two-body ΛN forces, and require the cancelation of the Λn and Λp bonds separately, the simplest nontrivial equation is

$$B_\Lambda(N_1, Z_1)+B_\Lambda(N_2, Z_2)=B_\Lambda(N_1, Z_2)+B_\Lambda(N_2, Z_1), \tag{3}$$

where B_Λ is the separation energy of the Λ. One can choose $N_1=N_2+1$ or N_2+2, and similarly $Z_1=Z_2+1$ or Z_2+2. But as was suggested before, contribution of three-body ΛNN forces should not be ignored in the Λ binding.[2] Hence we have to introduce a constraint so that the Λnn, Λpp, and Λnp bonds separately cancel, which implies $\sum C_i N_i Z_i=0$. Then Eq. (3) becomes trivial: $N_1=N_2$ and $Z_1=Z_2$, and we are led to Eqs. (1) and (2), in which M is replaced by B_Λ, as the simplest nontrivial relations. One can construct recurrence relations using Eqs. (1) and/or (2) repeatedly. For example,

$$B_\Lambda(N+3, Z+1)+B_\Lambda(N+2, Z)+B_\Lambda(N+1, Z-2)+B_\Lambda(N, Z-1)$$
$$=B_\Lambda(N+3, Z)+B_\Lambda(N+2, Z+1)+B_\Lambda(N+1, Z-1)+B_\Lambda(N, Z-2). \tag{4}$$

In Fig. 1 we show a chart of hypernuclei in the s and p shells. Those in the area surrounded by the bold solid line are well-established species.[3] Although the existence of ${}^{7}_{\Lambda}\mathrm{He}$ and ${}^{8}_{\Lambda}\mathrm{He}$ have been established,[4] their B_Λ's are rather uncertain. As one can see from Fig. 1, we cannot test Eqs. (1) and (2) with known B_Λ's. However, we can test Eq. (4) for $(N,Z)=(1,3)$. Then the left-hand side (LHS) of Eq. (4) corresponds to ${}^{9}_{\Lambda}\mathrm{Be}$, ${}^{7}_{\Lambda}\mathrm{Li}$, ${}^{4}_{\Lambda}\mathrm{H}$, and ${}^{4}_{\Lambda}\mathrm{He}$, while the right-hand side (RHS) corresponds to ${}^{8}_{\Lambda}\mathrm{Li}$, ${}^{8}_{\Lambda}\mathrm{Be}$, ${}^{5}_{\Lambda}\mathrm{He}$, and ${}^{3}_{\Lambda}\mathrm{H}$, and their B_Λ's add up to

$$\mathrm{LHS}=16.52\ (\pm 0.09),\quad \mathrm{RHS}=16.75\ (\pm 0.11). \tag{5}$$

Here, and in the following, the energies are giv-

Z \ N	1	2	3	4	5	6
6				${}^{11}_{\Lambda}$C (8.76) stable	${}^{12}_{\Lambda}$C (11.87) stable	${}^{13}_{\Lambda}$C 11.32 ± .15
5			${}^{9}_{\Lambda}$B (7.09,7.26)	${}^{10}_{\Lambda}$B 8.62 ± .20	${}^{11}_{\Lambda}$B 10.19 ± .10	${}^{12}_{\Lambda}$B 11.06 ± .10
4		${}^{7}_{\Lambda}$Be 5.09 ± .12	${}^{8}_{\Lambda}$Be 6.81 ± .07	${}^{9}_{\Lambda}$Be 6.63 ± .04	${}^{10}_{\Lambda}$Be (9.62) stable	${}^{11}_{\Lambda}$Be (8.95) stable
3	${}^{5}_{\Lambda}$Li (3.07) unstable	${}^{6}_{\Lambda}$Li (5.55) stable	${}^{7}_{\Lambda}$Li 5.56 ± .06	${}^{8}_{\Lambda}$Li 6.80 ± .05	${}^{9}_{\Lambda}$Li 8.25 ± .13	${}^{10}_{\Lambda}$Li (9.00) stable
2	${}^{4}_{\Lambda}$He 2.31 ± .03	${}^{5}_{\Lambda}$He 3.08 ± .02	${}^{6}_{\Lambda}$He 4.28 ± .15	${}^{7}_{\Lambda}$He (3.81) stable	${}^{8}_{\Lambda}$He (6.68) stable	${}^{9}_{\Lambda}$He (5.89)
1	${}^{3}_{\Lambda}$H 0.06 ± .06	${}^{4}_{\Lambda}$H 2.02 ± .05	${}^{5}_{\Lambda}$H (1.51) unstable	${}^{6}_{\Lambda}$H (2.23) stable		

FIG. 1. Chart of hypernuclei. Those in the area surrounded by the bold solid line are well-established species. The figures are the Λ separation energies (B_Λ) in MeV. Those with uncertainties are experimental ones taken from Ref. 3, whereas those in parentheses are predicted values. For $B_\Lambda({}^4_\Lambda\text{H})$ two different experimental values have been reported, 2.28 ± 0.03 and 2.02 ± 0.05, which were obtained from two-body and three-body decays. We take the latter value for the reasons given in the text.

en in MeV. The uncertainty shown in parentheses is the root-mean-square uncertainty of experimental B_Λ's. Equation (5) shows that Eq. (4) holds remarkably well. In Eq. (5) we took $B_\Lambda({}^4_\Lambda\text{H}) = 2.02 \pm 0.05$, but the other experimental value $B_\Lambda({}^4_\Lambda\text{H}) = 2.28 \pm 0.03$ leads to LHS = 16.78 (± 0.08), almost a perfect agreement. However, we prefer the value 2.02 rather than 2.28 for two reasons: (i) The comparison of the B_Λ values of ${}^4_\Lambda$He and ${}^4_\Lambda$H, both obtained from three-body decay modes, gave $B_\Lambda({}^4_\Lambda\text{He}) - B_\Lambda({}^4_\Lambda\text{H}) = 0.29 \pm 0.06$.[5] This comparison is free from the possible ambiquity concerning the range-energy relation. (ii) $B_\Lambda({}^4_\Lambda\text{H}) = 2.02$, when used to predict other B_Λ's, gives better overall consistency of the relations.

Is the excellent agreement shown above fortuitous? We believe it is not. Being in a shallower well and free from the restriction due to the Pauli principle, the Λ can move around in the core nucleus quite freely, and hence the overlaps of the wave functions of Λ and N vary little from one N orbit to the next. The effect of three-body ΛNN forces is as sensitive to the nucleon orbits as the two-body NN force, but the major part of the Λ binding arises due to two-body ΛN forces.

Now let us exploit Eqs. (1), (2), and/or (4) to make predictions. In the following we will refer to Eqs. (1), (2), and (4) as $S_1(N,Z)$, $S_2(N,Z)$, and $S_4(N,Z)$, respectively. In the chart shown in Fig. 1, those numbers within the dashed line can be obtained by using one of the three formulas once. Let us discuss the individual cases.

For ${}^6_\Lambda$Li, B_Λ can be obtained in two ways,

$$B_\Lambda({}^6_\Lambda\text{Li}) = \begin{cases} 5.55(\pm 0.18) \\ 5.78(\pm 0.18) \end{cases} \text{from} \begin{cases} S_2(1,3) & (6) \\ S_2(2,4). & (7) \end{cases}$$

Here, we took $B_\Lambda({}^4_\Lambda\text{H}) = 2.02 \pm 0.05$. Note that both the values are well above the lower limit of 5.50 ± 0.02 which follows from the stability against the decay ${}^6_\Lambda\text{Li} \to {}^5_\Lambda\text{He} + p$. Therefore, we predict that ${}^6_\Lambda$Li is stable. On the other hand, the upper limit of $B_\Lambda({}^6_\Lambda\text{Li})$ is 5.69 ± 0.12 because ${}^7_\Lambda$Be is known to be stable. Therefore, we take Eq. (6) rather than Eq. (7).

${}^6_\Lambda$Li has been a controversial hypernucleus, since an event was observed which was attributed to the decay ${}^{6}_{\Lambda}\mathrm{Li} \to {}^{4}_{\Lambda}\mathrm{He} + p + p + \pi^-$, with $B_\Lambda({}^{6}_{\Lambda}\mathrm{Li}) = 5.89 \pm 0.37$.[6] As was pointed out by Harmsen,[6] the existence of stable ${}^{6}_{\Lambda}\mathrm{Li}$ implies a large violation of charge symmetry of the ΛN interaction and/or a considerable rearrangement of the core nucleus, because the mirror hypernucleus has a much smaller B_Λ, $B_\Lambda({}^{6}_{\Lambda}\mathrm{He}) = 4.28 \pm 0.15$. Both of these possibilities were examined by Lovitch, Rosati, and Dalitz.[7] Their conclusion is that the ${}^{6}_{\Lambda}\mathrm{Li}$ is unlikely to be bound. Coremans *et al.*[8] pointed out that there are alternative interpretations for the event reported by Harmsen.[6]

For ${}^{5}_{\Lambda}\mathrm{Li}$,

$$B_\Lambda({}^{5}_{\Lambda}\mathrm{Li}) = 3.07\ (\pm 0.16) \text{ from } S_2(1,4). \tag{8}$$

This is unstable against the decay ${}^{5}_{\Lambda}\mathrm{Li} \to {}^{4}_{\Lambda}\mathrm{He} + p$.

For ${}^{10}_{\Lambda}\mathrm{Be}$,

$$B_\Lambda({}^{10}_{\Lambda}\mathrm{Be}) = 9.62\ (\pm 0.25) \text{ from } S_2(3,5). \tag{9}$$

This is consistent with

$$B_\Lambda({}^{10}_{\Lambda}\mathrm{Be}) - B_\Lambda({}^{6}_{\Lambda}\mathrm{Li}) = 3.84\ (\pm 0.28) \text{ from } S_4(2,4), \tag{10}$$

with $B_\Lambda({}^{6}_{\Lambda}\mathrm{Li}) = 5.55$. This hypernucleus is stable.

For ${}^{11}_{\Lambda}\mathrm{Be}$,

$$B_\Lambda({}^{11}_{\Lambda}\mathrm{Be}) = 8.95\ (\pm 0.20) \text{ from } S_2(4,5), \tag{11}$$

which is stable.

For ${}^{12}_{\Lambda}\mathrm{C}$,

$$B_\Lambda({}^{12}_{\Lambda}\mathrm{C}) = 11.87\ (\pm 0.23) \text{ from } S_4(3,5). \tag{12}$$

This is stable against the decay into ${}^{11}_{\Lambda}\mathrm{B} + p$, and also stable against the decay into ${}^{11}_{\Lambda}\mathrm{C} + n$, as can be seen from $B_\Lambda({}^{11}_{\Lambda}\mathrm{C})$, which we estimate later.

Using the above obtained B_Λ's we can extend our prediction to neighboring species.

For ${}^{7}_{\Lambda}\mathrm{He}$,

$$B_\Lambda({}^{7}_{\Lambda}\mathrm{He}) = \begin{cases} 3.81(\pm 0.28) \\ 3.98(\pm 0.33) \end{cases} \text{from} \begin{cases} S_1(2,4) & (13) \\ S_2(3,4). & (14) \end{cases}$$

Two experimental values for $B_\Lambda({}^{7}_{\Lambda}\mathrm{He})$ have been reported[4]: 3.75 ± 0.28 and 6.09 ± 0.54. The former is consistent with our prediction. In the chart we show our prediction (13) rather than the experimental values. Note that the difference between the B_Λ's of the mirror species, ${}^{7}_{\Lambda}\mathrm{He}$ and ${}^{7}_{\Lambda}\mathrm{Be}$, is quite large.

For ${}^{8}_{\Lambda}\mathrm{He}$,

$$B_\Lambda({}^{8}_{\Lambda}\mathrm{He}) = 6.68\ (\pm 0.32) \text{ from } S_1(3,4) \text{ and Eq. (13).} \tag{15}$$

An event for the decay ${}^{8}_{\Lambda}\mathrm{He} \to {}^{8}\mathrm{Li} + \pi^-$, with $B_\Lambda({}^{8}_{\Lambda}\mathrm{He}) = 7.16 \pm 0.70$, has been reported.[4] It was noted that this B_Λ may have been overestimated by ignoring the possibility that the ${}^{8}\mathrm{Li}$ nucleus may sometime be emitted in its first excited state.

For ${}^{9}_{\Lambda}\mathrm{B}$,

$$B_\Lambda({}^{9}_{\Lambda}\mathrm{B}) = \begin{cases} 7.09(\pm 0.30) \\ 7.26(\pm 0.36) \end{cases} \text{from} \begin{cases} S_2(2,5) & (16) \\ S_1(3,5). & (17) \end{cases}$$

Here we have used the predicted B_Λ's of ${}^{6}_{\Lambda}\mathrm{Li}$ and ${}^{10}_{\Lambda}\mathrm{Be}$ in Eqs. (16) and (17), respectively. The stability against ${}^{9}_{\Lambda}\mathrm{B} \to {}^{8}_{\Lambda}\mathrm{Be} + p$ requires that $B_\Lambda({}^{9}_{\Lambda}\mathrm{B}) > 6.67 \pm 0.07$. Our estimates of $B_\Lambda({}^{9}_{\Lambda}\mathrm{B})$ show that ${}^{9}_{\Lambda}\mathrm{B}$ is likely to be stable. Indeed, the experimental analyses of Bohm *et al.*[9] list six five-body decays which would be identified either as ${}^{9}_{\Lambda}\mathrm{B}$ or as ${}^{10}_{\Lambda}\mathrm{B}$. The former interpretation was rejected on the grounds that $B_\Lambda({}^{9}_{\Lambda}\mathrm{B})$ in all the above cases turned out to be at least 1 MeV less than the known $B_\Lambda({}^{9}_{\Lambda}\mathrm{Li})$, indicating large charge asymmetry. Inspection of Table I of their paper[9] reveals that three of the events are grouped around $B_\Lambda({}^{10}_{\Lambda}\mathrm{B}) \approx 8.3$, while the other three are around $B_\Lambda({}^{10}_{\Lambda}\mathrm{B}) \approx 9.2$. From analyses of these events they arrived at the experimental value of $B_\Lambda({}^{10}_{\Lambda}\mathrm{B}) \approx 8.62$. We propose that the three events grouped around 9.2 MeV are possibly due to the decay of ${}^{9}_{\Lambda}\mathrm{B}$; this interpretation would yield $B_\Lambda({}^{9}_{\Lambda}\mathrm{B}) \approx 7.1$ in agreement with our estimate (16). Furthermore, the spread in the experimental values of $B_\Lambda({}^{10}_{\Lambda}\mathrm{B})$ would be reduced, giving a somewhat lower value of $B_\Lambda({}^{10}_{\Lambda}\mathrm{B}) \approx 8.3$.

One can predict more exotic hypernuclei. We list some of them in the following without quoting the uncertainties of B_Λ's:

$$B_\Lambda({}^{5}_{\Lambda}\mathrm{H}) = 1.51 \text{ (unstable) from } S_2(2,3); \tag{18}$$

$$B_\Lambda({}^{6}_{\Lambda}\mathrm{H}) = 2.23 \text{ (stable) from } S_1(2,3). \tag{19}$$

The hypernucleus ${}^{6}_{\Lambda}\mathrm{H}$ is stable because of the large binding energy of the core nucleus: $B({}^{5}\mathrm{H}) = 10.12$ as compared with $B({}^{4}\mathrm{H}) = 3.28$.

$$B_\Lambda({}^{9}_{\Lambda}\mathrm{He}) = 5.89 \text{ from } S_1(4,4), \tag{20}$$

$$B_\Lambda({}^{10}_{\Lambda}\mathrm{Li}) = 9.00 \text{ (stable) from } S_1(4,5), \tag{21}$$

$$B_\Lambda({}^{11}_{\Lambda}\mathrm{C}) = 8.76 \text{ (stable) from } S_1(4,6). \tag{22}$$

The existence of ${}^{6}_{\Lambda}\mathrm{He}$ and ${}^{9}_{\Lambda}\mathrm{He}$ was also suggested by Dalitz and Levi-Setti.[10] Obviously, further application of our formulas will make more predictions, but with accumulating uncertainties. It seems premature to go beyond what we have done at the present time.

Finally, we would like to emphasize the following:

(i) In some cases, like ${}^{6}_{\Lambda}\mathrm{Li}$, B_Λ can be obtained by using different formulas, and hence, data on different hypernuclei. But, in all the cases we have discussed the discrepancy is less than 0.3 MeV (apart from the uncertainties in the experimental B_Λ's used).

(ii) If there are no three-body ΛNN forces present, the four-term formula (3) should work well. The deviation of the four-term formula can be as large as 1.5 MeV, and is much larger than the deviation of less than 0.3 MeV obtained with the six-term formulas. This may be taken as an evidence for three-body ΛNN forces.[2]

(iii) As was noted before, our prediction implies large differences between the B_Λ's of mirror hypernuclei, such as (${}^6_\Lambda$Li, ${}^6_\Lambda$He), (${}^7_\Lambda$Be, ${}^7_\Lambda$He), (${}^9_\Lambda$B, ${}^9_\Lambda$Li), and (${}^{10}_\Lambda$B, ${}^{10}_\Lambda$Be). If this is confirmed, this will pose a difficult theoretical problem.[8,11] The large charge asymmetry predicted hinges on the experimental B_Λ values of ${}^6_\Lambda$He and ${}^{10}_\Lambda$B. For example, consider the hypothetical case of $B_\Lambda({}^6_\Lambda\text{He})$ being 0.5 MeV larger than the currently accepted experimental value. Then the derived value of $B_\Lambda({}^6_\Lambda\text{Li})$ would be 0.5 MeV less, giving a less pronounced charge asymmetry.

*Work supported by the National Research Council of Canada.

[1]G. T. Garvey and I. Kelson, Phys. Rev. Lett. 16, 197 (1966); G. T. Garvey, W. J. Gerace, R. L. Jaffe, and I. Talmi, Rev. Mod. Phys. 41, Part II, S1 (1969).

[2]R. K. Bhaduri, B. A. Loiseau, and Y. Nogami, Ann. Phys. (New York) 44, 57 (1967); A. Gal, Phys. Rev. Lett. 18, 568 (1967); A. Gal, J. M. Soper, and R. H. Dalitz, Ann. Phys. (New York) 63, 53 (1971), and to be published; K. F. Chong and J. Law, Can. J. Phys. 49, 224 (1971).

[3]The data are taken from the review of D. H. Davis and J. Sacton, in *Proceedings of the International Conference on Hypernuclear Physics*, edited by A. R. Bodmer and L. G. Hyman (Argonne National Laboratory, Argonne, Ill., 1969), p. 159.

[4] For ${}^7_\Lambda$He, G. Bohm *et al.*, Nucl. Phys. B4, 511 (1968); for ${}^8_\Lambda$He, M. Jurie *et al.*, Nucl. Phys. B35, 160 (1971).

[5]See Ref. 3, p. 160. Bohm *et al.* (Ref. 4) obtained $B_\Lambda({}^4_\Lambda\text{He}) - B_\Lambda({}^4_\Lambda\text{H}) = 0.28 \pm 0.07$. See also W. Gajewski *et al.*, Nucl. Phys. B1, 105 (1967).

[6]D. M. Harmsen, Phys. Rev. Lett. 19, 1186, 1409 (1967); W. A. Barletta and D. M. Harmsen, Nucl. Phys. B10, 99 (1969). Also, evidence for ${}^6_\Lambda$Li with $B_\Lambda = 4.40 \pm 0.40$ was reported by S. N. Ganguli [Ref. 2 in G. Coremans *et al.*, Nucl. Phys. B4, 580 (1968).]

[7]L. Lovitch, S. Rosati, and R. Dalitz, Nuovo Cimento 53A, 301, 1059 (1968).

[8]Coremans *et al.*, Ref. 6; Barletta and Harmsen, Ref. 6.

[9]G. Bohm *et al.*, Nucl. Phys. B12, 1 (1969). They included one event found by C. Mayeur *et al.*, Nuovo Cimento 43, 180 (1966).

[10]R. H. Dalitz and R. Levi-Setti, Nuovo Cimento 30, 489 (1963).

[11]Charge asymmetry of ΛN interactions has been discussed in relation to the differences $B_\Lambda({}^4_\Lambda\text{He}) - B_\Lambda({}^4_\Lambda\text{H})$ by R. H. Dalitz and F. von Hippel, Phys. Lett. 10, 153 (1964); B. W. Downs, Nuovo Cimento 43, 454 (1966).

Conservation of the Newman-Penrose Conserved Quantities*

Joshua N. Goldberg
Syracuse University, Syracuse, New York 13210
(Received 14 February 1972)

Using an example of Press and Bardeen, Newman-Penrose quantities of the electromagnetic field in a Schwarzschild background are related to a differential conservation law and hence changes result from a flux. There is no discontinuity in the quantities resulting from a sudden change in dipole moment; there is no singular surface which moves out at $\frac{1}{3}$ the speed of light. It is suggested that for Newman-Penrose quantities to exist, multipole moments must approach a limit as $1/u$, $u \to -\infty$.

In a recent Letter,[1] Press and Bardeen (PB) suggest that Newman-Penrose quantities (NPQ) need not be constant when defined at finite distances rather than at null infinity where the NP constants[2,3] are defined. The particularly intriguing aspect of their discussion is the apparent existence of a surface which moves out at $\frac{1}{3}$ the speed of light, across which the NPQ may change discontinuously from one static value to another. In their example, PB write the electromagnetic field as a power series in $1/r$, hence as a Taylor series in the vicinity of infinity. The NPQ are then found through the coefficient of the appropriate term in this series. In general, the series has only a finite radius of convergence and that radius determines the above surface of discon-