Can Synchrotron Gravitational Radiation Exist?*

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A complete relativistic analysis for gravitational radiation emitted by a particle in circular orbit around a Schwarzschild black hole is presented in the Regge-Wheeler formalism. For completeness and contrast we also analyze the electromagnetic and scalar radiation emitted by a suitably charged particle. The three radiation spectra are drastically different. We stress some important consequences and astrophysical implications.

It has been recently suggested by Misner,[1] Misner *et al.*,[2] and Campbell and Matzner[3] that in the emission process of gravitational radiation, high beaming due to synchrotron effects could take place in extremely relativistic regimes. Indeed the existence of this phenomenon would be of great importance for experimental detection of gravitational radiation. The required total energy corresponding to an observed event may be very much smaller than usually estimated if (in order of importance!) (a) the beaming effect exists; (b) a privileged plane of emission is found for the beamed radiation; and (c) the detector happens to be in that plane. The failure to fulfill one of these three circumstances would make the phenomenon largely uninteresting. In this Letter we address ourselves to condition (a). We analyze the gravitational radiation emitted by a particle moving in the field of a Schwarzschild black hole in stable ($r > 6M$) as well as unstable ($3M \leq r \leq 6M$) circular orbits (geometrical units, $G = c = 1$). To compare and contrast the results we also give the explicit analytic formulas and the energy fluxes for the cases of a charged particle emitting electromagnetic radiation (for details see Ruffini, Tiomno, and Vishveshwara,[4] Ruffini and Tiomno,[5] and Denardo, Ruffini, and Tiomno[6]) and a particle[7] emitting scalar radiation in the same orbit. As a biproduct of our results it will become evident that an extrapolation from the results obtained in the case of scalar radiation to the case of gravitational synchrotron radiation does not properly account for the complexity of the tensor field. The complete treatment and details of these works will be published in later papers.[5,6,8]

Before giving the main results of our treatment let us recall that the circular orbits between $3M \leq r \leq 6M$ are all *unstable* and therefore unphysical.[9] The captured star will in fact spiral in down to $r = 6M$ ($r = M$ in the case of a co-rotating star in the extreme Kerr geometry) in the family of stable circular orbits and then plunge in. However, these stable configurations are uninteresting from the point of view of beaming of the radiation because the velocity of the particle is too small to have any beaming effect.[10,11] We here consider the orbits $3M \leq r \leq 6M$ only to explore the more fundamental question of the physics of the emission of gravitational radiation in a case where the particle does indeed reach a velocity comparable to the velocity of light. We have to use a fully relativistic formalism in the sense that (a) it be valid even for $v/c \sim 1$; (b) it takes into proper account the contribution from the given background. Thus we use the Regge-Wheeler[12] formalism with the Green's function techniques as previously developed by Zerilli,[13] Davis and Ruffini,[14] Davis, Ruffini, Press, and Price,[15] and Davis, Ruffini, and Tiomno.[16]

The power emitted from a particle in a circular orbit around a Schwarzschild black hole is given in all the three cases (scalar, vector, tensor) by

$$P(\omega) = \sum_{l,m>0} \frac{\omega^2}{2\pi} (|R_{(M)}{}^{lm}|^2 + |R_{(E)}{}^{lm}|^2). \quad (1)$$

Here $\omega = m\omega_0$ is the frequency of the radiation, with $\omega_0 = (M/r_0{}^3)^{1/2}$, r_0 being the radial coordinate of the circular orbit. The functions $R_{(M)}{}^{lm}$ (nonexistent in the scalar case) have parity $(-1)^{l+1}$;

TABLE I. The magnetic and electric components of the power radiated from a particle in circular orbit are here given in the case of scalar, vector, and tensor radiation. M is the mass of the black hole, m_0 the mass of the particle, $\gamma = dt/ds = (1-3M/r_0)^{-1/2}$, $r^* = r + 2M\ln(r/2M - 1)$, and r_0 and r_0^* are the orbit's coordinates. W is the Wronskian of the two independent solutions $u(r)$ and $v(r)$, and $\omega_0 = (M/r_0^3)^{1/2}$.

	$(-1)^{\ell}$ electric	$(-1)^{\ell+1}$ magnetic
Scalar	$R_{(E)}^{\ell m}(n\omega_o, r^*) = 4\pi s Y_{\ell}^{m}(\frac{\pi}{2},0)\,\frac{u(r^*)v(r_o^*)}{\gamma r_o W}\,\delta_n^m$	none
Vector	$R_{(E)}^{\ell m}(n\omega_o, r^*) = \frac{4\pi q Y_{\ell}^{m}(\frac{\pi}{2},0)\delta_n^m}{[\ell(\ell+1)]^{\frac{1}{2}}W}\,u(r^*)\,\frac{d}{dr_o^*}\,v(r_o^*)$	$R_{(M)}^{\ell m}(n\omega_o, r^*) = -\,4\pi q\omega_o Y_{\ell}^{m+1}(\frac{\pi}{2},0)\delta_n^m C_m^{\ell}\,\frac{u(r^*)v(r_o^*)}{W}$
Tensor	$R_{(E)}^{\ell m}(n\omega_o, r^*) = 4\pi m_o\gamma D_m^{\ell} Y_m^{\ell}(\frac{\pi}{2},0)\delta_n^m\,u(r^*)\,\frac{1}{W}\times$ $\times\left[\alpha(r_o)v(r_o^*) + \frac{1}{\lambda}\frac{d}{dr_o^*}\left(\beta(r_o)v(r_o^*)\right)\right]$ $\lambda = \frac{1}{2}(\ell-1)(\ell+2)$ $\beta(r_o) = \left(1-\frac{2M}{r_o}\right)\left(1+\frac{3M}{\lambda r_o}\right)^{-1}$ $D_m^{\ell} = ((\ell-1)(\ell+2))^{\frac{1}{2}}(\ell(\ell+1))^{-\frac{1}{2}}$ $\alpha(r_o) = \frac{r_o - 2M}{(r_o+\frac{3M}{\lambda})^2}\left(1+\frac{1}{\lambda}+\frac{M}{\lambda r_o}-\frac{3M}{\lambda^2 r_o}+\frac{6M^2}{\lambda^2 r_o^2}\right) -$ $-\,(2m^2-\ell(\ell+1))(\ell(\ell+1)-2)^{-1}\,\omega_o^2 r_o$	$R_{(M)}^{\ell m}(n\omega_o, r^*) = 4\pi m_o\gamma C_m^{\ell} Y_{m+1}^{\ell}(\frac{\pi}{2},0)\omega_o\,\frac{\delta_n^m u(r^*)}{\sqrt{(\ell-1)(\ell+2)}\,W}\times$ $\times\frac{d}{dr_o^*}[r_o v(r_o^*)]$ $C_m^{\ell} = (\ell(\ell+1) - m(m+1))^{\frac{1}{2}}(\ell(\ell+1))^{-\frac{1}{2}}$

the functions $R_{(E)}{}^{lm}$ have parity $(-1)^l$. They are computed in the asymptotic region $r \to +\infty$. Their structure is different depending on whether they refer to the scalar, electromagnetic, or gravitational case as summarized in Table I. They are expressed in terms of the two radial functions $u(r_*)$ and $v(r_*)$ (purely outgoing wave at infinity and ingoing wave at the black-hole surface, respectively) which are solutions of Schrödinger-type equations:

$$d^2u/dr_*{}^2 + (\omega^2 - V_{\rm eff})u = 0. \qquad (2)$$

Here $\omega = n\omega_0$, $r_* = r + 2M\ln(r/2M - 1)$, and $V_{\rm eff}(r)$ will depend on the particular field under examination. In the limit of high l, the potential approaches $(1-2M/r)l(l+1)/r^2$ for all three fields. We have integrated Eq. (2) numerically for the three cases.[17] The powers radiated at the orbit $r = 3.05M$ are given in Fig. 1. The difference in the three cases is manifest. Only for scalar radiation can we find something similar to what is usually called synchrotron radiation: *most* of the radiation concentrated at high multipoles. The contributions of lowest multipoles is significant in the case of electromagnetic radiation and very important in the case of gravitational radiation.

To complete the analysis of gravitational radiation from circular orbits, we have summarized some of the main results in Fig. 2. The energy emitted is here plotted again as a function of the harmonic index l for different circular orbits corresponding to different radii. In particular it is clear from these results that for the gravitational radiation (1) the beaming (high-l component) is insignificant for orbits up to the last stable circular orbit; (2) for orbits $3M \leqslant r \leqslant 6M$ we have indeed an enhancement of high-l components. *However*, the lower multipoles continue to give very substantial contributions.

The analysis of the physical reasons for the drastic differences in the three cases is under examination and will be presented elsewhere. It is evident that the "water-sprinkler effect" of the radiation arises because even zero-mass quanta in extremely relativistic orbits $(r \to 3M)$

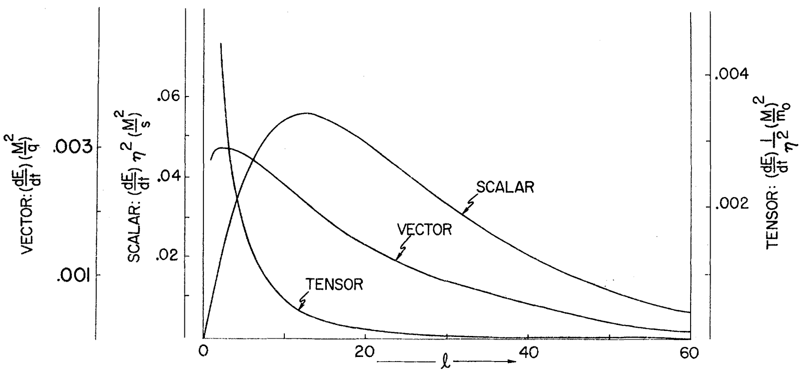

FIG. 1. Scalar, vector, and tensor radiation as functions of l compared and contrasted. The particle is assumed to be in a circular orbit at $r=3.05M$ with $\eta^2=(1-3M/r_0)^{-1}=61$. Here, m is the mass, q the electric charge, and s the scalar charge of the particle. Since most of the radiation (~97%) comes from the $l=m=n$ modes, this plot is in fact a power spectrum.

are strongly influenced by the background geometry.[18] In this connection it has been proposed[18] that a particle *not* in a geodesic orbit but suitably propelled by nongravitational forces to acquire relativistic velocities should emit "synchrotron"-like gravitational radiation. We do not think however that this effect has any realistic astrophysics application. We are currently considering the corresponding effect in the case of electromagnetic radiation by analyzing the motion of a relativistic particle in a Reissner-Nordstrom background.[6] We summarize our results as follows: (a) No gravitational synchrotron radiation can be expected from stable circular orbits in Schwarzschild geometry (this result has been generalized to the stable Kerr orbits both corotating and counter-rotating[10]); (b) for unstable circular orbits a large amount of the radiation is still emitted at low multipoles, and thus at larger angles. In computing the effect of the enhancements of the highest multipoles we have to compensate for this defocusing effect.

These results raise serious questions about the effectiveness of the so-called gravitational

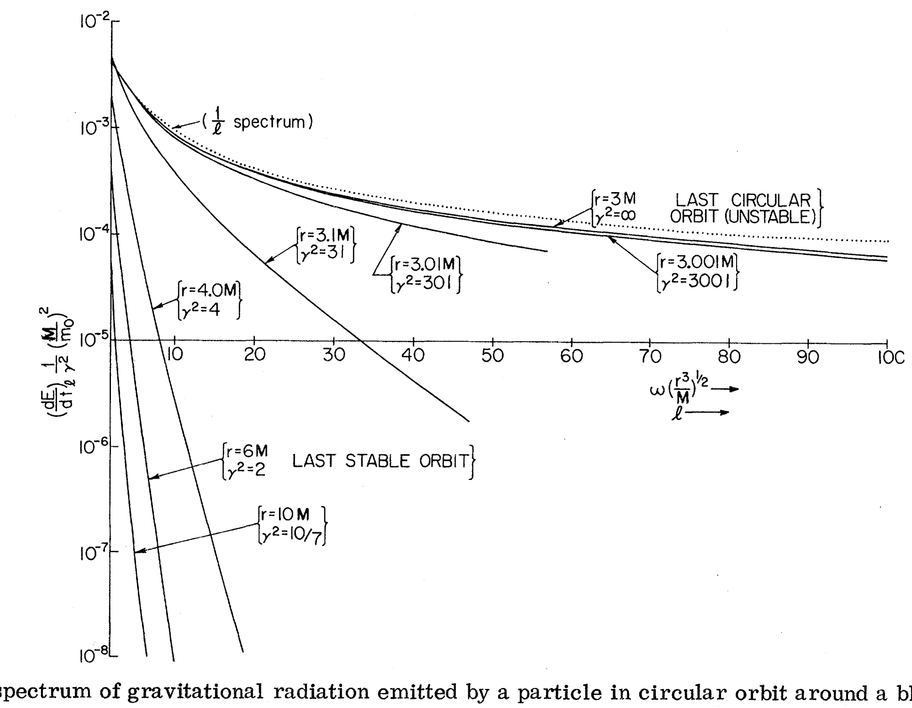

FIG. 2. The spectrum of gravitational radiation emitted by a particle in circular orbit around a black hole for selected values of the orbital radius. Beaming effects are totally negligible up to the last stable circular orbit ($6M$). When $r\to 3M$, higher multipoles are enhanced; however, the amount of energy radiated at lower multipoles is far from negligible.

synchrotron radiation mechanism in concentrating energy into a plane. We think, however, that quite apart from any discussion of gravitational synchrotron radiation a detailed analysis of the polarization, intensity, and angular distribution of the electromagnetic and gravitational radiation emitted by material falling into a collapsed object can greatly help in the understanding of the physical processes and in the design of the most descriminating detectors.

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[3]G. A. Campbell and R. A. Matzner, to be published.

[4]R. Ruffini, J. Tiomno, C. V. Vishveshwara, Nuovo Cimento Lett. 3, 211 (1972).

[5]R. Riffini and J. Tiomno, to be published (derivation of equation in Table I for the electromagnetic case, with details of polarization and radiation features).

[6]G. Denardo, R. Ruffini, and J. Tiomno, to be published (radiation features for very highly bound orbits in the Reisner-Nordstrom field).

[7]Details of this problem can be found in Ref. 2. In Table I we have adapted the formulas of Misner *et al*. for the scalar case to our Green's function approach.

[8]M. Davis, R. Ruffini, and J. Tiomno, to be published (derivation of gravitational radiation equations given in Table I, and a detailed analysis of the polarization of the radiation).

[9]For analysis of geodesics in the Schwarzschild and Kerr cases see R. Ruffini and J. A. Wheeler, in *The Significance of Space Research for Fundamental Physics*, edited by A. F. Moore and V. Hardy (European Space Record Organization, Paris, 1971).

[10]R. Ruffini, Bull. Amer. Phys. Soc. 17, 449 (1972).

[11]Moreover, tidal effects have to be taken into account as shown and computed by Ruffini and Wheeler (see Ref. 9) and B. Mashoon (to be published). The validity of the "particle" approximation should be carefully analyzed for any astrophysical application!

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[17]It has been shown that for very large values of l it is possible to give analytic solutions of Eq. (2) which are the same in all three cases (see Ref. 2 and R. A. Breuer, P. L. Chrzanowski, H. G. Hughes, and C. W. Misner, to be published). This result is currently being used to give asymptotic formulas ($l > 100$) for the electromagnetic and gravitational case by R. A. Breuer, R. Ruffini, J. Tiomno, and C. V. Vishveshwara (to be published).

[18]R. Ruffini, unpublished.

[19]The spectrum of bursts of gravitational radiation emitted by a pair of particles in narrow elliptical orbits about their mutual center of gravity has been analyzed in the linearized approximation by Ruffini and Wheeler (Ref. 9). They point out a clear enhancement of high multipoles. Quite independently from the present work on synchrotron radiation, we are currently analyzing this interesting process in a completely relativistic approach.