Elementary Pion with Transverse-Momentum Cutoff

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The puzzle of peaks in π-exchange processes is resolved in terms of an elementary pion with transverse momentum cutoff. In np charge exchange, for instance, this implies keeping both t- *and* u-channel π exchanges. A *quantitative* fit is obtained for this process maintaining ρ-A_2 exchange degeneracy. This quality of the fit is in general comparable with (if not better than) the strong-cut model with less than half the number of parameters.

Pion-exchange process are quite mysterious. In all such processes the energy dependence of the differential cross section [at least for small momentum transfers, $|t| \lesssim 1$ (GeV/c)^2] is quite consistent with the exchange of a $J=0$ object.[1] However, since the pion-exchange contribution is supposed to vanish in the forward direction (because of its pseudoscalar nature) we should observe a pronounced dip (or a zero) in the forward direction. Such is not always the case; pn charge exchange and π^+ photoproduction are celebrated counter examples in that instead they have a prounced peak in the forward direction.

Wolf[2] has made *quantitative* fits to all π-exchange processes which do possess forward dips (e.g., $\bar{p}p \to \bar{\Delta}\Delta$, $\pi^- p \to \rho^0 n$) using elementary pions with suitable form factors. For processes which have forward peaks there have been two alternative explanations, both using moving pions: π conspiracy and the Reggeized absorption model. Since in nature we have so far no direct evidence for a moving pion[1,3] we would like to start with the assumption that the pion *is* elementary. This implies in particular that for those processes which allow pion exchange in two channels, both exchanges ought to be included. For example, in $np \to pn$ charge exchange, for which usually only the t-channel pion is kept (which does vanish in the forward direction), the u-channel pion should also be included. At this point the reader may object that the usual form factor (say, of the type e^{bu}) will make the latter contribution vanishingly small for large energies. However, the experimental data strongly suggest that the cutoff is really in the transverse momentum variable, $P_\perp^2 = -tu/(t+u)$. For u large and t small and fixed, this gives a form factor (for *both* exchanges) of the type e^{bt}. Thus, our hypothesis of elementary pion exchange with a transverse momentum cutoff has the virtue of a *nonvanishing* forward pion contribution as well as the correct energy dependence.[4] This is our proposed explanation for the forward peak dilemma in pion-exchange processes.

In Table I we list our predictions for some pion-exchange processes. For meson-baryon processes (rows 1, 2, and 3), since there is only t-channel pion exchange, we expect a dip at $t=0$. This is experimentally verified.[5] For baryon-baryon processes (rows 4 and 5) there are u-channel π exchanges and hence we expect *peaks* in agreement with the experimental data.[6,7] Processes in rows 6 and 7[8] posses s-channel pions and hence *a priori* we do not expect a dip. However, we do not have a reliable model for the form factor with large *timelike* momentum and at present we make no prediction for these processes. The last process, $\gamma p \to \pi^+ n$, is also complicated because of gauge invariance requirements. But

TABLE I. Prediction and experimental situation regarding the forward $d\sigma/dt$ for pion exchange processes.

Process	Prediction	Experiment
$\gamma p \to \pi^- \Delta^{++}$	Dip	Yes
$\pi^- p \to \rho^0 n$	Dip	Yes
$\pi^+ p \to \rho^0 \Delta^{++}$	Dip	Yes
$np \to pn$	Peak	Yes
$pp \to n\Delta^{++}$	Peak	Yes
$\bar{p}p \to \bar{n}n$	?	Not certain, perhaps a peak
$\bar{p}p \to \bar{\Delta}\Delta$	?	Dip
$\gamma p \to \pi^+ n$	(Born term gives a peak)	Pronounced peak

if one uses the simple Born term, one does obtain a peak.[9]

To make a more quantitative verification of our hypothesis we selected $np \to pn$ charge exchange since it has the most accurate data for lab momenta P_L up to 24 GeV/c.[6] ρ and A_2 Regge trajectories are exchanged in the t channel and we have π's in both t and u channels. In order not to complicate the analysis we have assumed exact exchange degeneracy between ρ and A_2. This should be a reasonable hypothesis since pp and pn total cross sections are equal (within the experimental errors) at high energies. We have used the following normalization for the t-channel helicity amplitudes in terms of the (spin averaged) differential cross section:

$$d\sigma/dt = (1/32\pi k^2 s)[|f_{++;++}|^2 + |f_{++;--}|^2 + |f_{+-;+-}|^2 + |f_{+-;-+}|^2 + 4|f_{++;+-}|^2],$$

where $s=(P_1+P_2)^2 = 4(k^2+m^2)$.

The contributions from various exchanges to the amplitude have been parametrized as follows:

$$f_{++;++} = g^2\pi_t + \beta_{++}{}^2(t)(s/m^2)^{\alpha(t)}, \quad f_{++;--} = -g^2(\pi_t - \tfrac{1}{2}\pi_u) + \beta_{++}{}^2(t)(s/m^2)^{\alpha(t)}, \quad f_{+-;-+} = -\beta_{+-}{}^2(t)(s/m^2)^{\alpha(t)},$$

$$f_{+-;+-} = -\tfrac{1}{2}g^2\pi_u + \beta_{+-}{}^2(t)(s/m^2)^{\alpha(t)}, \quad f_{++;+-} = +\beta_{++}(t)\beta_{+-}(t)(s/m^2)^{\alpha(t)},$$

where

$$\alpha(t) = \tfrac{1}{2} + \alpha' t \quad (\alpha' = 1\ \mathrm{GeV}^{-2}), \quad \beta_{++}{}^2(t) = X_1(1 - X_2\alpha' t), \quad \beta_{+-}{}^2(t) = \alpha' t X_3(1 - X_4\alpha' t), \quad g^2/4\pi = 14.73,$$

$$\pi_t = \frac{-t}{\mu^2 - t}\,\frac{t - X_5\mu^2}{X_6 t - (X_5 + X_6 - 1)\mu^2}\exp\left[b\,\frac{(t-\mu^2)(u-\mu^2)}{t+u}\right], \quad \pi_u = \pi_t(t \leftrightarrow u).$$

Some variation in the Regge residue functions $\beta_{\lambda\mu}(t)$ as well as in the pion form factor has been allowed, to obtain more quantitative fits. We required $X_5 > 1$ and $0 < X_6 < 1$ in order to avoid spurious poles or zeros in the spacelike region.

We have performed a least squares fit to the $7 \leq P_L \leq 11.75$-GeV/c data of Miller *et al.*[6] for $|t| < 0.46$ (GeV/c)^2. The results are presented in Fig. 1.[10] For 246 data points, we have $\chi^2 = 624$. Lest the reader be dismayed by such a large χ^2, we wish to point out that the only other "successful" fits (using the strong-cut absorption model[11,12] with about seventeen parameters) are in fact worse, as can be seen in Fig. 2 where we present a recent calculation by Reay.[13]

Using the previously obtained parameters, we have evaluated our *predictions* for $P_L = 19.2$ and 24 GeV/c, and in Fig. 3 we compare it with the data of Engler *et al.* There is an uncertainty of 30% in the absolute normalization of the data (Engler *et al.*, Ref. 6). In fact, in Fig. 3 the 19.3-GeV/c data have been renormalized by +30% and the 24-GeV/c data by +11%.

From the above analysis, we find our fit to be comparable with and in fact slightly better than that of the strong-cut model and hence it lends partial support to our conjectures. It is not unlikely that a much more refined fit could be obtained, breaking the exchange degeneracy between ρ and A_2 (as was done for instance in Ref. 11) and thus introducing more parameters. In fact, to obtain a nonvanishing polarization[14] one is required to do so.

A more direct test of the elementarity of the pion should be in terms of its renormalization constant Z. Recently, West[15] has done an analysis regarding the compositeness of the proton using deep-inelastic ep data and has given strong arguments in its support. A similar analysis for the pion would require data regarding the pion electromagnetic form factor at high q^2 as well as pion Compton scattering data, which are of course not available.

If the pion is elementary it is natural to assume that the other members of the pseudoscalar nonet are also (viz., K, η, η'). Data regarding these latter exchanges are scarce and hence we are unable to present any evidence pro or con regarding their elementarity.

We have also applied our model to inclusive processes and find a striking difference between our predictions and that of the Reggeized pion model. Consider, for example, reactions of the type projectile $+p \to n +$ anything. For fixed small t (the momentum transfer between proton and neutron), the (Reggeized) pion contribution, through the triple Regge term,[16] yields a power-law growth of the cross section with the missing mass. In our model with elementary pions and a transverse momentum cutoff, a straightforward calculation shows that for fixed t, the cross section has an extremely sharp rise (~exponential) with the missing mass. We believe that the

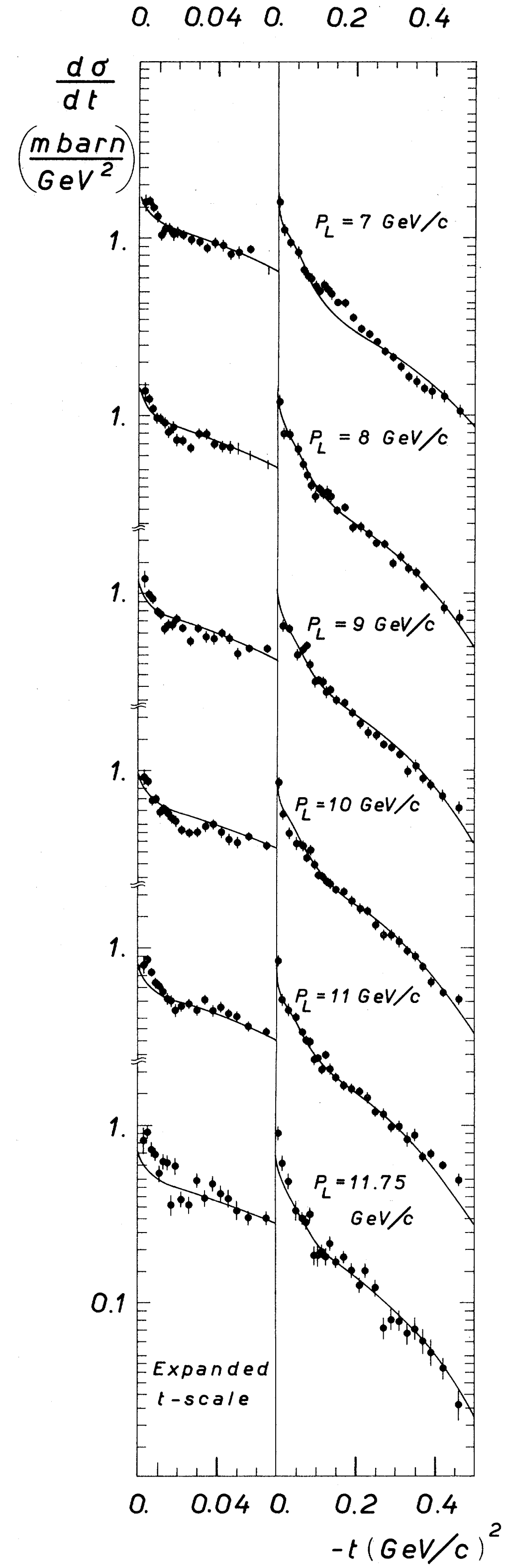

FIG. 1. Comparison of our model with the experimental data of Miller *et al.* (Ref. 6). The left-hand side of this figure shows the very forward points on an expanded scale.

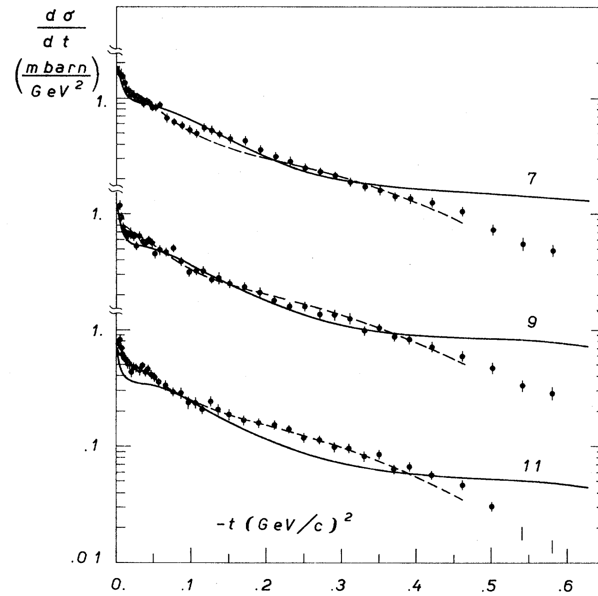

FIG. 2. Comparison of our fit (dashed line) with SCRAM fit done by Reay (Ref. 13), along with the experimental data of Miller *et al.* (Ref. 6).

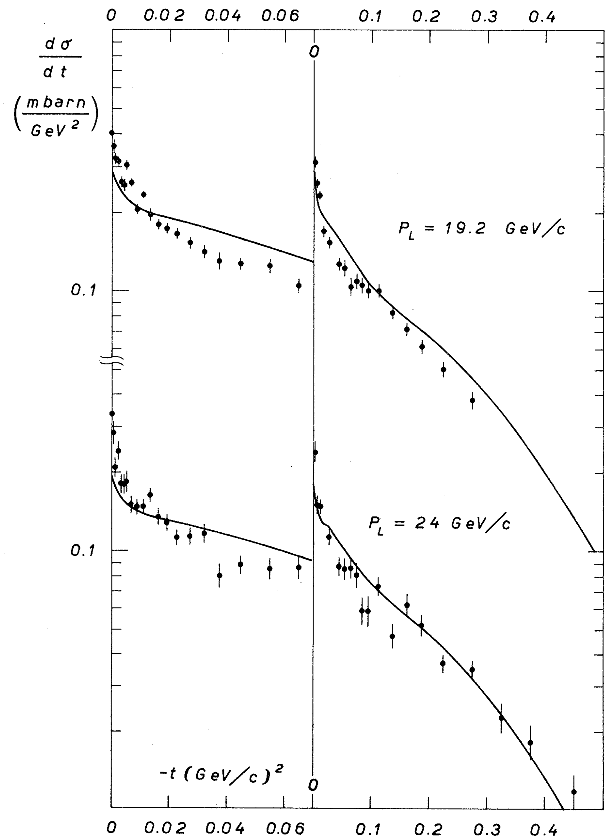

FIG. 3. Our prediction at $P_L = 19.2$ and 24 GeV/c as compared with the data of Engler *et al.* (Ref. 6), normalized as explained in the text. The left-hand side of this figure shows the very forward points on an expanded scale.

two behaviors are sufficiently different to allow for a clear test of our model. The neutron spectra have not yet been measured in hadronic reactions.[17] At the Stanford Linear Accelerator Center, a measurement of the reaction $e+p\rightarrow e'+n+x$ can also help decide regarding our model, since arguments similar to those above are also applicable here. We hope that such measurements shall be forthcoming soon.

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[1]A shrinking forward peak for the process $\pi^+p\rightarrow\rho^0\Delta^{++}$ has often been quoted as a "proof" that the pion trajectory is a moving one [see, for instance, B. Haber, U. Maor, G. Yekutieli, and E. Gotsman, Phys. Rev. 168, 1773 (1968)]. However, G. Wolf [Phys. Rev. Lett. 19, 925 (1967), and Phys. Rev. 182, 1538 (1969)] has convincingly demonstrated that the data are in very good agreement with an elementary pion exchange *provided* proper account of the resonance widths in ρ and Δ is taken. See G. Chew, Comments Nucl. Particle Phys. 1, 187 (1967), for details.

[2]Wolf, Ref. 1.

[3]Chew, Ref. 1.

[4]A moving pion trajectory $\alpha_\pi(u)$, even with a transverse momentum cutoff, gives practically no contribution from u-channel π exchange because of the factor $s^{\alpha_\pi(u)}$. Hence, $\alpha_\pi(u)\equiv 0$ is essential in our model.

[5]See Ref. 2 and the following more recent data. $\pi^-p\rightarrow\rho^0n$ at 11 GeV/c (with low statistics): C. Caso *et al.* Nuovo Cimento 62A, 755 (1969). $\pi^+p\rightarrow\rho^0\Delta^{++}$ at 11.7 GeV/c: R. Maddock *et al.*, Nuovo Cimento 5A, 457 (1971).

[6]$np\rightarrow pn$: E. Miller *et al.*, Phys. Rev. Lett. 26, 984 (1971); E. Miller, thesis, Ohio State University, 1971 (unpublished); J. Engler *et al.*, Phys. Lett. 34B, 528 (1971).

[7]$pp\rightarrow n\Delta^{++}$: H. Dehne *et al.*, Nuovo Cimento 53, 232 (1968). In Ref. 2, Wolf has made a good fit using only the t-channel pion, whence he obtains a dip. Experimentally there is no indication of a dip.

[8]$\bar{p}p\rightarrow\bar{n}n$: P. Astbury *et al.*, Phys. Lett. 22, 537 (1966), and 23, 160 (1960). $\bar{p}p\rightarrow\bar{\Delta}\Delta$: V. Alles-Borelli *et al.*, Nuovo Cimento 48, 360 (1967).

[9]B. Richter, in *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN Scientific Information Service, Geneva, 1968); J. Jackson and C. Quigg, Nucl. Phys. B22, 301 (1970).

[10]Values of the parameters are $b=14$ (GeV/c)$^{-2}$, $X_1=4.24$, $X_2=1.10$, $X_3=74$, $X_4=-1.32$, $X_5=30$, and $X_6=0.75$. With these values of X_5 and X_6, the variation in the pion "form factor" between $t=\mu^2$ and $t=0$ is $\approx 12\%$.

[11]F. Henyey *et al.*, Phys. Rev. 182, 1579 (1969); M. Ross, F. Henyey, and G. Kane, Nucl. Phys. B23, 269 (1970); G. Kane *et al.*, Phys. Rev. Lett. 25, 1519 (1970).

[12]J. Froyland and G. Winbow, CERN Report No. Th1362, 1971 (to be published).

[13]N. W. Reay, private communication to E. Miller, quoted in the thesis of Miller (Ref. 6). The results of his fit, shown in our Fig. 2, were taken from Miller's Fig. 30 (Ref. 6). No χ^2 are available for this fit but Ross quotes a value of 340 for 100 BB charge-exchange data points. See, M. Ross, in Proceedings of the Irvine Conference on Regge Poles, Irvine, California, 1969 (unpublished).

[14]P. Robrish *et al.*, Phys. Lett. 31B, 617 (1970).

[15]G. West, Phys. Rev. Lett. 27, 762 (1971).

[16]C. De Tar *et al.*, Phys. Rev. Lett. 26, 675 (1971).

[17]At the CERN intersecting storage rings, a preliminary determination of the neutron spectra in proton-proton scattering has been reported by J. Engler *et al.* (CERN-Karlsruhe collaboration). However, these measurements are not able to provide data regarding the (t, M_x^2) distribution. Such experiments are underway.