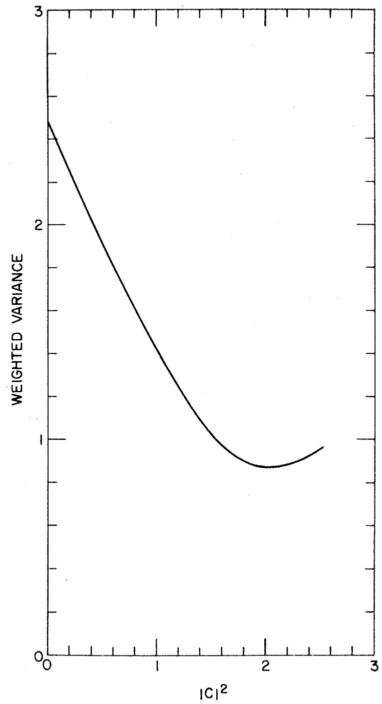

FIG. 3. Weighted variance versus $|C|^2$ for the 13.6-MeV data.

finds a single minimum with about the same weighted variance.

The value of $|C|^2$ of about 2.1 seems quite reasonable. For the deuteron, the Reid potentials[4] give $|C|^2 = 1.7$. In both cases, the fact that $|C|^2$ is greater than unity indicates that there is a hole in the wave function. The size h of the hole is roughly given by

$$h = R \ln |C|$$

with values of 0.9 and 1.1 for ^{3}He and the deuteron, respectively.

In summary, we have shown that the use of peripheral phase shifts in higher partial waves both improves the quality of phase-shift analysis of the data and determines some useful normalization constants for tails of light nuclear wave functions.

The idea that this sort of phase-shift analysis might be useful is due to P. Shanley, and one of us (M.B.) thanks him for interesting and useful discussions.

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Observation of the Decay $K^+ \to \pi^0 \pi^0 e^+ \nu$†

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We have observed two decays analyzed as examples of the decay $K^+ \to \pi^0 \pi^0 e^+ \nu$ in a heavy-liquid bubble chamber. All four converted γ's are seen. We report a branching ratio of $1.8^{+2.4}_{-0.6} \times 10^{-5}$ for this decay. The form factor f_1 for the decay is reported as $|f_1| = 0.97^{+0.50}_{-0.19}$. These results are in good agreement with $\Delta I = \frac{1}{2}$ predictions.

This paper reports on the first experimental observation of the rare decay $K^+ \to \pi^0 \pi^0 e^+ \nu$ (K_{e4}' decay). The results of this experiment along with the results of $K^+ \to \pi^+ \pi^- e^+ \nu$ experiments allow a new, albeit crude, test of the validity of the $\Delta I = \frac{1}{2}$ rule for weak interactions.

The K_{e4}' decay mode was searched for in film from the heavy-liquid 40-in. bubble chamber at the Argonne zero gradient synchroton. The chamber liquid was chosen to be heavy Freon (CF_3Br) because of its short radiation length. The decay mode was searched for by having scanners look for electron secondaries from stopped K^+ decays with four converted γ's pointing to the decay origin. Stopping K^+ decays were identified by the darkening of the K^+ ionization near the decay vertex, and electron secondaries were identified by curvature, sometimes by bremsstrahlung off of the secondary and by the lack of any decay of the secondary. For normalization purposes the scanners record all electron secondaries and noted the number of pointing γ's. γ's that could reasonably be interpreted as bremsstrahlung were discarded by scanners. Film containing 674 200 stopped K^+'s in an appropriate fiducial volume was scanned, and 148 candidates for K_{e4}' were found and measured. These events were fitted by the three-vertex, three-constraint fit for K_{e4}' and only events with a χ^2 probability of greater than 0.001 were accepted (cut number 1).

A cut was also applied to the data to eliminate events where one γ could have come from bremsstrahlung of the electron secondary. This cut amounted to eliminating any event where any of the four γ's had a measured angle relative to the electron direction of less than 15° (cut number 2).

48 events survived these two cuts and were edited by a physicist to see if the events were really 4γ electron events. In this edit the secondary was checked very carefully to make certain it was an electron. Only secondaries in which there was no possible confusion among electron, pion, or muon secondaries were kept. Nine events were left after editing. The discarded events were made up of 4γ pion events and electron events in which the scanner had included bremsstrahlung or a γ from another alternate origin as one of the four γ's. Seven of these nine events have a measured electron momentum less than 60 MeV/c. These events could be examples of colinear τ' decays. This is the ordinary decay $K^+ \to \pi^+\pi^0\pi^0$ in which the π^+ carries off only a small amount of momentum. Then, if the π^+ and μ^+ from the π-μ-e decay chain are not observed, one has an event that appears to have an electron secondary. However, the maximum electron momentum from a π-μ-e chain is 53 MeV/c. Thus, the events with small electron momentum were fitted by the three-vertex, three-constraint τ' hypothesis assuming no knowledge of the π^+ secondary. Most of these events had a low-χ^2 fit with this colinear τ' hypothesis with a reconstructed π^+ of momentum less than 40 MeV/c. Thus, these events were most likely examples of colinear τ' decays that faked K_{e4}'. There was one event with a measured electron momentum of 63 MeV/c, and this is just above the τ' region. This event was measured four times and each time the colinear τ' fit reconstructed a π^+ with a momentum of 94 ± 8 MeV/c. A pion with this momentum has a range of 3.5 cm and would definitely be seen on the scanning table, and hence the colinear τ' background is ruled out for this event. Thus, the cutoff between colinear τ' background and real candidates was set at 60 MeV/c. We, therefore, eliminated colinear τ' events by discarding any event with $P_{\text{meas}}(e^+) < 60$ MeV/c (cut number 3).

Two candidates are left after this cut. A τ' decay with an energetic π^+ can be background if the entire π^+-μ^+-e^+ decay chain fakes the appearance of an electron. This can happen if the ionization information is poor and both the μ^+ and the e^+ decay forward. Hopefully, all of this background was eliminated at the editing stage, but to check for it, the two remaining candidates, along with the initial events that satisfield cuts 1 through 3, but were discarded earlier because they had a secondary that could be interpreted either as an electron or as a pion on the scan table, were all remeasured several times assuming the secondary to be a pion. These events were fitted by the three-vertex, six-constraint τ' hypothesis. In general, one would not expect a real K_{e4}' decay to fit τ' as K_{e4}' is a four-body decay while τ' is a coplanar three-body decay. The results of this τ' fit were that both remaining K_{e4}' candidates failed to reconstruct as a τ' while all the events with ambiguous secondaries had reasonable τ' fits. Thus, the necessity of accepting only good electron secondaries is established.

τ' and K_{e4}' decays are the only expected K^+ decays with four converted γ, and τ' background has been eliminated as stated above. Thus, any other background must involve an accidental γ or a bremsstrahlung γ. The two remaining candidates were carefully edited, and it was found that all γ's point well and have no alternate origins anywhere in the chamber. Furthermore, for both events all the γ's convert within 15 cm of the decay origin. From a qualitative study of the probability that random γ's in the chamber point to K^+ decay origins but do not originate in the chamber, it is concluded that the accidental background is negligible. Note that since the bremsstrahlung background has been removed, two γ's must accidentally point to a false K^+ origin without actually originating in the bubble chamber.

Thus, all known background has been ruled out and two K_{e4}' events remain. For one of the events only one possible combination of γ's making π^0's had a K_{e4}' fit, while the other event had two combinations. In the ambiguous case, information on the opening angles for the π^0 decays did not favor one combination over the other;

Table I. Fitted variables for two K_{e4}' decays.

Event number	χ^2 for K_{e4}' fit	Particle	Azimuth (deg)	Dip (deg)	Momentum (MeV/c)
1	1.3	e^+	184 ± 3	-52 ± 3	137 ± 13
		π^0	40 ± 25	29 ± 23	54 ± 21
		π^0	55 ± 9	72 ± 17	77 ± 6
		ν	309 ± 25	9 ± 21	57 ± 20
2	5.3	e^+	283 ± 5	26 ± 8	61 ± 15
		π^0	255 ± 8	-41 ± 9	74 ± 13
		π^0	116 ± 1	-2 ± 3	149 ± 14
		ν	339 ± 13	21 ± 18	78 ± 13

however, the γ energies as estimated on the scan table matched up best with the fit energies for the lower-χ^2 fit. Thus, the combination with the lowest χ^2 was chosen. The fitted variables for the two K_{e4}' events are given in Table I, and Fig. 1 shows one of the events.

The K_{e4}' rate as compared with that for K_{e3}' is calculated as follows:

$$\frac{\Gamma(K^+ \to \pi^0\pi^0 e^+\nu)}{\Gamma(K^+ \to e^+\pi^0\nu)} = \frac{\text{total } K_{e4}' \text{ found } (4\gamma)}{\text{total } K_{e3} \text{ found } (0, 1, 2\gamma)} \frac{1}{E_{\text{tot}}},$$

where $E_{\text{tot}} = N_1 N_2$ (efficiency for finding 4γ K_{e4}' events relative to finding K_{e3} events), N_1 being the 4γ detection probability and N_2 being the efficiency for real events surviving cuts and editing.

3γ electron events were also picked up in the

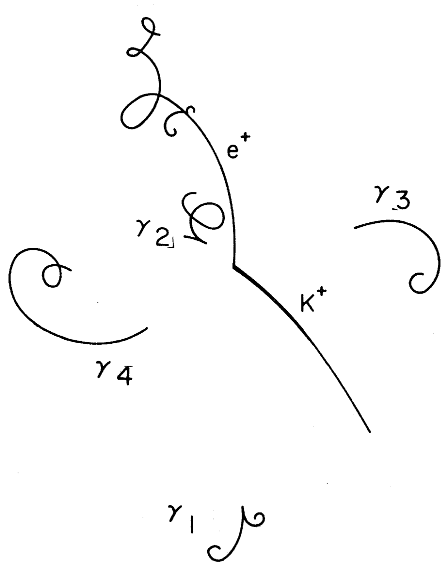

FIG. 1. Sketch of K_{e4}' event number 1. γ_1 and γ_2 are from the decay of one π^0 and γ_3 and γ_4 from the other π^0. Note the δ rays or bremstrahlung attached to the electron secondary.

K_{e4}' scan. These events were analyzed for radiative K_{e3} and during the editing for that process the physicist looked for any fourth γ's missed by the original scan. One of the above two K_{e4}' candidates was indeed found this way. The probability that all four γ's would convert in the chamber was found to be $(43 \pm 5)\%$ from a study of the γ's from $K_{\pi 2}$ and τ' decays. N_2 includes an estimated 5% loss due to cuts 1 and 2, a 6% loss due to the editing criteria for good electrons, and a $(40 \pm 5)\%$ loss due to cut 3. The loss due to discarding low-energy electrons was calculated from the electron spectrum obtained in a K_{e4} experiment.[1] Thus, E_{tot} is 23%. The scanners recorded 27 506 electron secondaries in this experiment. From an edit of these events it was determined that 17% of these events were not really electrons and hence the total number of valid K_{e3} decays found was 22 952. Thus, the rate from the K_{e4}' events is

$$\Gamma(K^+ \to \pi^0\pi^0 e^+\nu)/\Gamma(K^+ \to e^+\pi^0\nu) = 3.8 \times 10^{-4}.$$

This implies a branching ratio of[2]

$$\Gamma(K^+ \to \pi^0\pi^0 e^+\nu)/\Gamma(K^+ \to \text{all}) = 1.8^{+2.4}_{-0.6} \times 10^{-5}.$$

The errors were calculated using the Poisson distribution.

From the $\Delta I = \frac{1}{2}$ rule one obtains the following relationship among the rates for the various K_{e4} decays[3]:

$$\Gamma(K^+ \to \pi^0\pi^0 e^+\nu) = \tfrac{1}{2}\Gamma(K^+ \to \pi^+\pi^- e^+\nu) - \tfrac{1}{4}\Gamma(K^0 \to \pi^0\pi^- e^+\nu). \quad (1)$$

Also, the K_{e4} rates can be expressed in terms of the four form factors used to express the matrix element. In the case of K_{e4}' decay, two of the form factors are zero due to Bose statistics and the $\Delta I = \frac{1}{2}$ rule,[4] and a third is negligible because it is multiplied by the electron mass. Thus, one obtains[4]

$$\Gamma(K^+ \to \pi^0\pi^0 e^+\nu) \approx (1.59\times 10^3\ \mathrm{sec}^{-1})|f_1|^2, \qquad (2)$$

where f_1 is the axial-vector form factor multiplying the sum of the pion momenta in the matrix element. Another result of the $\Delta I = \frac{1}{2}$ rule is that f_1 for the decay $K^+ \to \pi^0\pi^0 e^+\nu$ is equal to the f_1 for the decay $K^+ \to \pi^+\pi^- e^+\nu$.

From Eq. (2), the result of this experiment can be expressed as

$$|f_1| = 0.97^{+0.50}_{-0.19}.$$

The results of this experiment are in good agreement with the predictions of the $\Delta I = \frac{1}{2}$ rule. Our value for the magnitude of f_1 agrees with the value of 1.19 ± 0.13 obtained by Berends, Donnachie, and Oades,[5] from an analysis of $K^+ \to \pi^+\pi^- e^+\nu$ data. The experimental $K^+ \to \pi^+\pi^- e^+\nu$ branching ratio is reported[6] as $(3.3 \pm 0.3)\times 10^{-5}$ and in a recent experiment[7] as $(4.11 \pm 0.38)\times 10^{-5}$. Since a small rate for $K^0 \to \pi^0\pi^- e^+\nu$ is predicted by several theoretical models for K_{e4} decays,[8] the experimental results are compatible with the $\Delta I = \frac{1}{2}$ prediction given by Eq. (1).

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Multiplicity Fluctuation and Multiparticle Distribution Functions in High-Energy Collisions*

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Speculations are made concerning the fluctuation in the multiplicity in the fragmentation of hadrons in high-energy collisions. We analyze implications for the two-particle distribution function ρ_2 and higher-order distributions ρ_ν.

The aim of this Letter is to stress the importance of the possible phenomena of wide multiplicity fluctuation in high-energy collisions and to speculate on some possible characteristic behavior of the two-particle distribution function. The subject is of immediate interest because a basic general feature of a number of current models in multiparticle production (statistical models, multiperipheral models) is the rough independence of the outgoing particles. This basic feature does not allow for a wide fluctuation of the multiplicity. On the other hand, the hypothesis of limiting fragmentation *does,* since fragmentation into very few fragments is envisaged as having a finite probability at very high energies, while the average multiplicity continues to increase with incoming energy.

We shall concentrate on discussing the charged-pion multiplicity n_{ch} since that is the simplest quantity to measure experimentally. Define

$$n_{\mathrm{ch}} = n_{\mathrm{ch}}^{R} + n_{\mathrm{ch}}^{L}, \qquad (1)$$

where R and L refer to the right and left hemispheres in the center-of-mass system for the outgoing momenta. We shall use the variable x defined by

$$x = p_\parallel^* / (p_\parallel^*)_{\mathrm{incoming}}, \qquad (2)$$

where $p_\parallel^*$ denotes the longitudinal momentum in the c.m. system. The hemisphere R is defined to be the one for which x is positive. For simplicity we shall integrate over all transverse momenta and concentrate on $d\sigma/dx_1$, $d^2\sigma/dx_1dx_2$, etc. and their limits at very high incoming energies:

$$(d\sigma/dx_1)\,dx_1 \to \rho_1(x_1)\,dx_1, \qquad (3)$$

$$(d^2\sigma/dx_1dx_2)dx_1dx_2 \to \rho_2(x_1,x_2)\,dx_1dx_2. \qquad (4)$$