known to ± 100 keV. As in the previous case this spectrum resembles the data obtained with the (^{7}Li, t) reaction.[15,16] Our results are consistent with the assumption that the (^{12}C, ^{8}Be) reaction mainly populates two rotational bands (see Ref. 16, and references given therein): (i) the positive-parity band based on the ground state and containing the 1.63- (2^+), 4.25- (4^+), and 8.79-MeV (6^+) states. It has $(sd)^4$ structure. (ii) The negative-parity band containing the 5.80- (1^-), 7.17- (3^-), and 10.30-MeV (5^-) states. It has $(sd)^3(fp)^1$ structure. Both bands are expected to be strongly populated by an α-transfer reaction.[16]

In summary, the (^{12}C, ^{8}Be) reaction appears to offer potential as an additional tool for the study of α clustering in nuclei—along with the (^{6}Li, d), (^{7}Li, t), and (^{16}O, ^{12}C) reactions. Complications due to mutual excitation processes are severely reduced by a detection method that discriminates in favor of observing ^{8}Be in its ground state. Through the use of wide-area detectors, the detection efficiency of ^{8}Be's could be easily increased by a factor of 10. This technique plus the availability of ^{12}C beams of sufficient energy and intensity at many tandem and cyclotron laboratories should permit the study of the (^{12}C, ^{8}Be) reaction on a wide variety of targets.

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Possible Validity of the Relativistic Hartree-Fock Approximation in Nuclear Physics*

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Although nonrelativistic estimates of single-particle kinetic-energy expectation values appear to have invalidated the Hartree-Fock relation between total binding energy, single-particle eigenvalues, and kinetic-energy expectation values, a recent relativistic Hartree calculation has been successful at reproducing finite nuclear properties for closed-shell nuclei. Using the ^{16}O nucleus, it is demonstrated that this success is due to a reduction in the expectation values of the relativistic analog of the kinetic-energy operator.

The difficulty of finding Hartree-Fock (HF) or Brueckner-Hartree-Fock models which reproduce the experimental total binding energies and charge distributions of finite nuclei has delayed the establishment of a fundamental basis for the shell model and optical-model theories. The first important breakthroughs in this area came with the exploitation of density-dependent (rearrangement) effects.[1] These works reproduce saturation properties of finite nuclei, but the interactions are essentially phenomenological, and the density dependence induces significant deviations

from a pure HF formalism.

The inadequacy of nonrelativistic HF theory for nuclei was demonstrated by Kohler,[2] who showed that with nonrelativistic estimates of the kinetic energies ($\langle i|T|i\rangle$), the HF total-binding-energy relation,

$$E=\tfrac{1}{2}\textstyle\sum_i(E_i+\langle i|T|i\rangle), \tag{1}$$

could not correlate the experimental total binding energies (E) and single-particle removal energies (E_i) as measured by ($p, 2p$) and ($e, e'p$) experiments.[3] If Koopmans's theorem[4] holds, the needed equivalence between knockout energies and single-particle HF eigenvalues is established. The inability of Eq. (1) to correlate the experimental data is a strong argument for the importance of rearrangement corrections of the types used in Ref. 1. More recent experiments[5] have served only to enhance this conclusion.[6]

A recent relativistic Hartree calculation,[7] using a nucleon-nucleon interaction of the one-boson exchange-potential[8] (OBEP) form, has indeed reproduced not only the saturation properties of closed-shell nuclei (^{16}O–^{208}Pb) but also obtained single-particle eigenvalues in agreement with the knockout energies. This is an apparent contradiction of the arguments of the last paragraph, for the interaction is density independent. In this Letter it is shown that relativistic properties of the OBEP interactions significantly reduce the expectation values ($\langle i|T|i\rangle$) when the relativistic analog of the kinetic-energy operator is used. The interaction used in Ref. 7 must also be considered phenomenological; however, the absence of explicit rearrangement terms represents a simplification of possible importance.

The basic relativistic Hartree-Fock (RHF) formalism is presented in Ref. 7 and will be treated with brevity in this section. The single-particle wave functions $\{\varphi_k\}$ which form the Slater determinant of the nuclear ground state are

$$\varphi_k(\vec{r})=\frac{1}{r}\begin{pmatrix}F_k(r) & \mathcal{Y}_{l_k m_k}^{J_k}(\Omega)\\ iG_k(r) & \mathcal{Y}_{l_k' m_k}^{J_k}(\Omega)\end{pmatrix}. \tag{2}$$

The functions $\mathcal{Y}_{lm}^{J}$ are Pauli central-field spinors. The F_k and G_k are the large and small components, respectively, which obey the radial Dirac equations

$$\begin{aligned}\frac{dF}{dr}&=-\left[\frac{i(U_v^{r}-U_1^{r})}{\hbar c}+\tilde{\omega}\frac{(J+\frac{1}{2})}{r}\right]F+\left[\frac{2Mc^2+U_s-U_v^{0}+E_k}{\hbar c}\right]G,\\ \frac{dG}{dr}&=\left[\frac{U_s+U_v^{0}-E_k}{\hbar c}\right]F-\left[\frac{i(U_v^{r}+U_1^{r})}{\hbar c}-\tilde{\omega}\frac{(J+\frac{1}{2})}{r}\right]G,\end{aligned} \tag{3}$$

where the single-particle labels are dropped, except for the RHF eigenvalue E_k.

The orbital-angular-momentum values differ between the large and small components in Eq. (2) as follows:

$$J=l-\tilde{\omega}/2,\quad J=l'+\tilde{\omega}/2, \tag{4}$$

where $\tilde{\omega}=\pm 1$ is a quantum number related to parity,

$$P=(-)^{J+\tilde{\omega}/2}. \tag{5}$$

Note that $\tilde{\omega}$ appears in the radial equations. The U's in Eqs. (3) are Dirac single-particle potentials obtained by suitable averages over matrix elements of the OBEP interaction. As discussed in Ref. 7, the terms U_s and U_v^{0} are the most important single-particle potentials, coming mainly from exchange of scalar and vector mesons, respectively. The terms U_v^{r} and U_1^{r} come from HF exchange matrix elements (neglected in Ref. 7) and will not be considered here.

With this short introduction one can proceed to investigate the form of the relativistic expectation value ($\langle i|T|i\rangle$) occurring in the RHF total-binding-energy equation. The relativistic kinetic-energy operator is defined as the total single-particle Hamiltonian minus rest energy (Mc^2) in the large-distance region where interactions with other particles have vanished:

$$T=c\vec{\alpha}\cdot\vec{P}+(\beta-1)Mc^2, \tag{6}$$

where the $\vec{\alpha}$ and β are Dirac matrices. Using Eqs. (3) to eliminate derivatives, the relativistic expectation value ($\langle\varphi_k|T|\varphi_k\rangle$) may be shown to obey the following relation:

$$\begin{aligned}\langle\varphi_k|T|\varphi_k\rangle=&\int_0^\infty[E_k-U_s(r)-U_v^{0}(r)]F^2(r)\,dr+\int_0^\infty[U_s(r)-U_v^{0}(r)+E_k]G^2(r)\,dr\\ &+i\int_0^\infty 2U_1^{r}(r)F(r)G(r)\,dr.\end{aligned} \tag{7}$$

The lowest-order approximation to Eqs. (3) is a Schrödinger equation with the potential

$$V(r) = U_s(r) + U_v^0(r). \tag{8}$$

The first term of Eq. (7) thus has the appearance of a nonrelativistic kinetic-energy expectation value. The remaining terms can be viewed as relativistic corrections.

The second term of Eq. (7) provides the major relativistic effects. Ordinarily this term would be small, because the small component (G) is on the order of one tenth of the large component (F); however, the OBEP models have a distinctive property that invalidates this consideration. The OBEP model central potentials result from large cancelations between the contributions of scalar and vector mesons. This cancelation is necessary if one is to fit NN data without hard cores. In the RHF formalism this property results in very large magnitudes for the single-particle potentials $U_s(r)$ and $U_v^0(r)$ (~500 MeV). The scalar potential is attractive and the vector potential repulsive so that their appearance with the same signs in Eq. (8) leads to a nonrelativistic potential well of reasonable depth (~50 MeV). In the second term of Eq. (7), these potentials occur with opposite signs, making them additive rather than cancelling, and making the integrand significant despite the smallness of G^2. The contribution is negative, thus reducing $\langle\varphi|T|\varphi\rangle$.

Adherence to Dirac hole theory compels one to exclude from consideration the negative-energy eigenfunctions of the single-particle potentials; however, vacuum polarization causes the positive-energy states in the single-particle potential to have small admixtures of negative-energy plane-wave solutions. Using $A_+(W)$ and $A_-(W)$ as the energy-dependent positive- and negative-energy plane-wave expansion coefficients,

$$\varphi(r) = \int_0^\infty dW\,[A_+(W)\psi(W+Mc^2, r) + A_-(W)\psi(-W-Mc^2, r)], \tag{9}$$

where ψ are the plane-wave solutions, one finds the following relation for the expectation value:

$$\langle\varphi|T|\varphi\rangle = \int_0^\infty \{A_+^*(W)A_+(W)W - A_-^*(W)A_-(W)[2Mc^2+W]\}\,dW. \tag{10}$$

The negative-energy contributions to Eq. (10) can be significant because of the factor $(2Mc^2+W)$. In fact, Eq. (10) shows that $\langle\varphi|T|\varphi\rangle$ is not even positive definite.

The results of calculations of kinetic-energy expectation values of single-particle states in ^{16}O are shown in Table I. The experimental average kinetic energy ($\bar{T}$) is defined through a rearrangement of Eq. (1),

$$\bar{T} = (2E - \textstyle\sum_i E_i)/A. \tag{11}$$

The $(p, 2p)$ peak separation energies quoted by Becker[6] are used for E_i in Eq. (11). A Coulomb correction of 4 MeV per particle[9] is applied to neutron states, and a center-of-mass correction of 0.7 MeV per nucleon[10] is also applied.

The first row of Table I shows the results of nonrelativistic calculations with the HO model, using Bethe's formula,

$$\hbar\omega = 1.85 + 35.5A^{-1/3}, \tag{12}$$

for the oscillator parameter. The second row shows the results for a relativistic calculation with a Woods-Saxon scalar well (U_s) with depth, radius, and diffuseness of 60, 3, and 0.66, respectively. For this case relativistic effects are negligible as is shown by comparison with the HO result. The third row shows the relativistic calculation with the OBEP model used in Ref. 7. Relativistic effects are very strong here as in the fourth row which represents a relativistic calculation with the same model as row 3, but with exchange matrix elements included. The average

TABLE I. Kinetic-energy expectation values (MeV) of ^{16}O.

	$\langle 1S_{1/2}\|T\|1S_{1/2}\rangle$	$\langle 1P_{3/2}\|T\|1P_{3/2}\rangle$	$\langle 1P_{1/2}\|T\|1P_{1/2}\rangle$	$\bar{T}$
HO	12	20	20	18
Relativistic WS Well	11.8	19.2	19.2	17.4
RH	8.2	16.1	5.3	11.4
RHF	8.8	16.8	2.5	11.2
Expt.	...	...	...	9.2±1.3

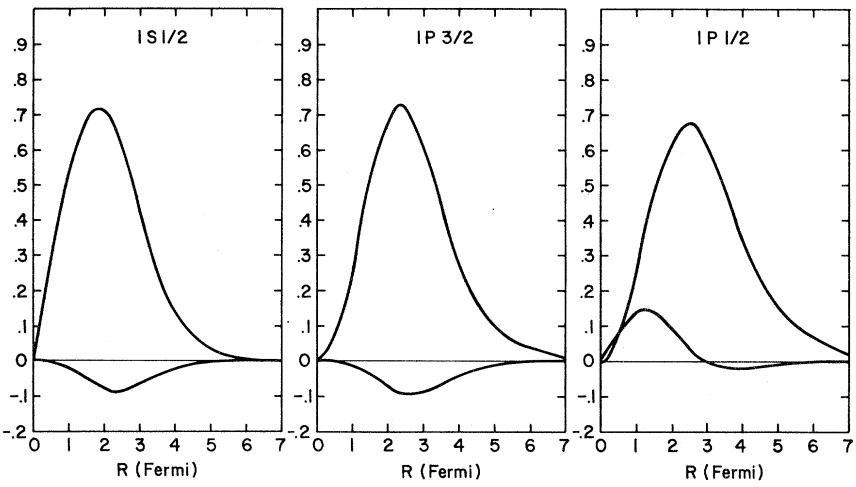

FIG. 1. Large- and small-component radial wave functions for single-particle states in ^{16}O.

matrix elements ($\bar{T}$) in rows 3 and 4 are almost within the error bars of the experimental quantity (row 5).

Note in rows 3 and 4 of Table I that the relativistic effects for the $1P_{1/2}$ matrix element are especially large. This is because of a breakdown of the order relation $F > G$ at small distances for this state as is shown in Fig. 1 where the single-particle radial wave functions are plotted (Coulomb effects are negligible on the graph). The reason for the breakdown is nondynamical, being caused by the boundary conditions

$$F(r) \propto r^{l+1}, \quad G(r) \propto r^{l'+1}, \tag{13}$$

and the fact that l' is less than l for the spin-orbit member of lower J. The effect seems to be significant only if large relativistic effects are already present.

It must be stressed that this work does not represent an attempt to prove that the HF approximation is sufficient for a quantitative understanding of nuclear structure when relativistic interactions are used. It is indeed likely that higher-order corrections to HF including rearrangement corrections play a significant role in nuclear structure. The OBEP model of Ref. 7 does not provide a satisfactory fit to *NN* phase shifts, although it is closely related to models which do.[8] Even if the *NN* data were reproduced it would still be necessary to prove that two-body and higher-order correlations are small before the HF approximation could be fully trusted. The lessons that have been learned from the RHF work so far are: (1) Rearrangement and correlation effects may be smaller than nonrelativistic calculations have indicated; (2) they are not required to explain $(p, 2p)$ experiments as had been previously supposed; and (3) relativistic effects may be larger than expected.

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