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Second-Sound Velocity, Scaling, and Universality in He II under Pressure near the Superfluid Transition

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The velocity of second sound in He II was measured for $\epsilon \equiv 1-T/T_\lambda \geq 2\times10^{-5}$ between saturated vapor pressure (SVP) and the melting curve. For $P>$SVP, the corresponding superfluid fraction ρ_s/ρ' cannot be described by a simple power law in ϵ. When reasonable general assumptions are made about higher-order singular contributions, the asymptotic behavior of ρ_s/ρ may be deduced and agrees well with scaling and universality.

Near the superfluid transition in He^4 there exists extremely convincing experimental evidence for departures[1-3] from scaling[4-6] and universality.[7] We wish to provide additional results which have a strong bearing upon this question, and present high-resolution measurements of the second-sound velocity u_2 in He II under pressure P. These data, in conjunction with known thermodynamic parameters and a relation based upon linear two-fluid hydrodynamics,[8] yield the superfluid density ρ_s. Our results demonstrate the existence of higher-order contributions to ρ_s which are singular at T_λ. When such terms are permitted in the data analysis, we obtain agreement with scaling and universality.

For P greater than saturation vapor pressure (SVP) we *cannot* represent ρ_s by a pure power law

$$\rho_s/\rho = k(P)\epsilon^\zeta, \quad \epsilon\equiv 1-T/T_\lambda, \tag{1}$$

for $\epsilon \gtrsim 2\times10^{-5}$ over any appreciable range of ϵ. Previous measurements of ρ_s at vapor pressure generally have been interpreted in terms of Eq. (1). The departures of ρ_s/ρ from Eq. (1), even for small ϵ, prevent us from determining the asymptotic exponent of ρ_s as accurately as we had hoped on the basis of the high precision and temperature resolution of the data. Although our results clearly establish the existence of singular higher-order contributions to ρ_s, it seems virtually impossible to establish from experiment their functional form. Therefore, we have *assumed* that the relation

$$\rho_s/\rho = k(P)\epsilon^\zeta[1+a(P)\epsilon^x], \tag{2}$$

with ζ equal to the asympototic exponent, may be used to represent the data for $\epsilon \leq 10^{-2}$. All of our current conclusions regarding the asymptotic behavior of ρ_s/ρ (i.e., ζ) are based upon the validity and adequacy of Eq. (2) and linear two-fluid hydrodynamics for $2\times10^{-5}\lesssim\epsilon\lesssim10^{-2}$. We find that within permitted systematic and random errors ζ and x are independent of P. In addition, $k(P)$ and $a(P)$ can be described within their permitted errors by the polynomials

$$k(P) = k_0 + k_1 P, \tag{3}$$

$$a(P) = a_0 + a_1 P + a_2 P^2. \tag{4}$$

A single least-squares fit of the measurements at all pressures with $2\times10^{-5}\lesssim\epsilon\lesssim10^{-2}$ to Eqs. (2) to (4) yields

$$\zeta = 0.677\pm0.02, \tag{5}$$

$$x = 0.36\pm0.1, \tag{6}$$

where the uncertainties include our best estimate of permitted systematic errors.

The velocity measurements with a precision of at least 0.1% were made by determining the frequency of the plane-wave resonant modes of a stainless-steel cylindrical resonator with diameter and height both equal to 1 cm. The cylinder was terminated at each end by superleak condenser transducers.[9,10] The diaphragm components of these transducers were 1-μm-Nuclepore membrane filters[11] made electrically conductive by evaporating a thin film of gold on one side. The resonator was situated in a pressure cell which was in turn suspended inside a vacuum can placed in a pumped liquid-helium bath. Except for the acoustics, the apparatus was similar to one described elsewhere.[1]

Measurements were made with frequencies between 1 and 10 kHz along isochores following two different procedures. For $t\gtrsim0.4$ mK [$t\equiv T_\lambda(V)$

$-T]$ the resonant frequencies were determined by holding the temperature constant and sweeping the frequency. For $t \lesssim 0.4$ mK, it was more convenient to fix the frequency and to permit the temperature to drift slowly ($\approx 5\ \mu$K/min) through the resonances up to $T_\lambda(V)$. Occasional comparison of the two methods showed that the drift method did not introduce unsuspected errors. Within our resolution, no dispersion was detected upon changing the resonance frequency by a factor of 6. We also looked for possible nonlinear effects by varying the amplitude of the driving voltage by a factor of 10. The largest driving voltages tended to produce some distortion; however, no shift in the peak frequency was detected. The driving voltage used in collecting data provided a strong signal with no measurable distortion. Although the attenuation increases as the transition is approached, the quality factor Q, which was of the order of 2000 away from the transition, did not fall below 100 even for the smallest values of t. Thus the relative shift in the resonant frequency due to the attenuation, which goes as Q^{-2}, is much smaller than our resolution.

Most of our measurements were made along isochores. Before examining them for their asymptotic behavior, they were first corrected to isobars which meet the λ line at the same point as the isochores. This correction primarily consists of an adjustment of the "distance" from T_λ along the temperature axis with $T_\lambda - T$ along isobars given by

$$\theta \equiv T_\lambda(P) - T = t + \left[\int_0^t \left(\frac{\partial P}{\partial T}\right)_v dt'\right]\left(\frac{\partial T}{\partial P}\right)_\lambda. \qquad (7)$$

One set of measurements was made along an isobar and was in good agreement with the corrected isochore data.

Measurements of u_2 at vapor pressure usually have been presented graphically as a function of ϵ on logarithmic scales. For our present purpose, such a graph has insufficient resolution. Instead, we multiplied u_2^2 by $\epsilon^{-0.772}$, and show this product as a function of ϵ on logarithmic scales in Fig. 1. The multiplicative factor $\epsilon^{-0.772}$ is unlikely to have any fundamental significance and was chosen only because previous measurements at vapor pressure[12] indicate that the corresponding product is nearly independent of ϵ and thus more readily displayed with high resolution. At vapor pressure, our results are compared with the best fit to the data of Pearce, Lipa, and Buckingham.[12] Although there is fine agreement

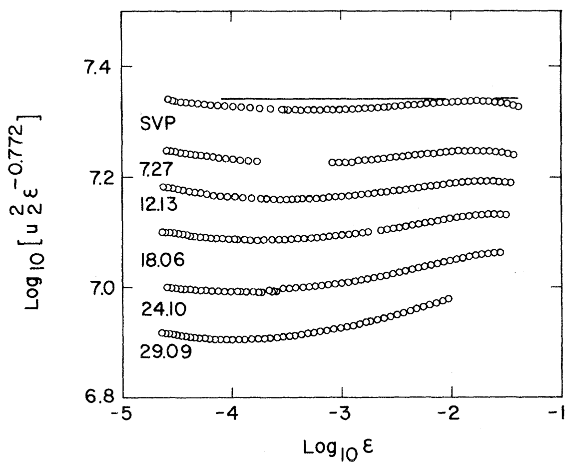

FIG. 1. High-resolution plot of the second-sound velocity u_2 (in cm/sec) along isobars. The numbers give the pressure in bars. Solid line, fitted results of Pearce, Lipa, and Buckingham (Ref. 12).

in magnitude, the precision of the present results clearly reveals curvature. This curvature is, in part, due to the deviations from a simple power law which are expected, even if ρ_s is given by Eq. (1), because of the contribution from C_p to u_2. If we nonetheless fit our SVP results for $\epsilon < 10^{-2}$ to a power law, we obtain $u_2^2 \cong 2142\epsilon^{0.7740}$ m^2/sec^2, in good agreement with Pearce, Lipa, and Buckingham. Although not indicated in the figure, the present data are also consistent with the vapor-pressure results of Williams *et al.*,[9] and Johnson and Crooks,[13] but differ slightly from the measurements of Tyson and Douglass.[14] There appear to be no other measurements of u_2 near the transition under pressure; but over the complete pressure range and for temperatures between 1.6 and 2.0 K with $\epsilon > 10^{-2}$, our velocities agree with those of Peshkov and Zinov'eva[15] to within 2%.

We expect that ρ_s/ρ may be obtained from u_2 through the result[8]

$$\rho_s/\rho = u_2^2(u_2^2 + S^2T/C_p)^{-1} \qquad (8)$$

of linear two-fluid hydrodynamics. Here C_p is the heat capacity at constant pressure and S is the entropy.[16] As we did with u_2, we would also like to display the results for ρ_s/ρ graphically with high resolution. The usual plot of ρ_s/ρ versus ϵ on logarithmic scales does not serve this purpose well. Instead, we have again used previous results at vapor pressure as a guide and show in Fig. 2 $\log_{10}[(\rho_s/\rho)\epsilon^{-2/3}]$ as a function of

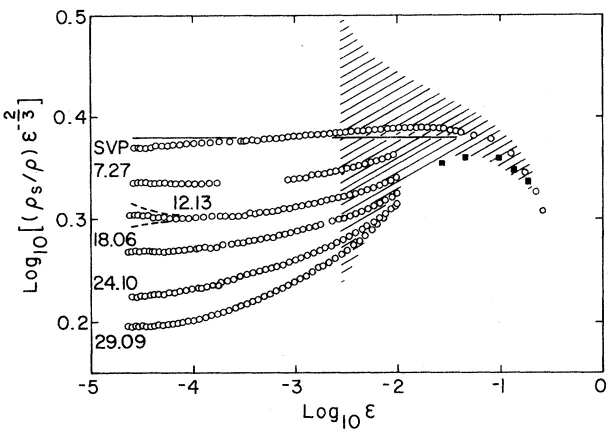

FIG. 2. High-resolution plot of the superfluid fraction ρ_s/ρ along isobars derived from measurements of u_2. The numbers give the pressure in bars. The solid squares correspond to u_2 at 25.3 bars measured by Peshkov and Zinov'eva (Ref. 15). The effect of a shift in T_λ of ± 2 μK upon the results is demonstrated by the dashed curves in the figure. The solid line represents directly measured values ρ_s/ρ at vapor pressure by Tyson (Ref. 17). The shaded area corresponds to the range of values of $(\rho_s/\rho)\epsilon^{-2/3}$ under pressure permitted by the results of Romer and Duffy (Ref. 20).

$\log_{10}\epsilon$. We also show in Fig. 2 as a solid line the best fit to the SVP data of Tyson[17] and note that it never differs by more than 2% from our results. Likewise, our measurements at SVP agree well with those by Clow and Reppy,[18] and by Kriss and Rudnick.[19] There have also been measurements of ρ_s/ρ under pressure by Romer and Duffy.[20] These results, all at $P \lesssim 20$ bars, fall on a universal curve which nearly coincides with the present vapor-pressure measurements. Therefore, there appears to be a disagreement under pressure between the more direct measurements and our results which use a prediction, Eq. (8), based on linear two-fluid hydrodynamics. However, permitted systematic errors in the Romer and Duffy results increase rapidly with decreasing ϵ, the range of permitted values of $(\rho_s/\rho)\epsilon^{-2/3}$ being shown by the shaded area in Fig. 2. It overlaps with the present high-pressure results. Values of ρ_s/ρ for $\epsilon > 10^{-2}$ were computed from our u_2 at 20.3 bars. Although not shown in the figure, they fall close to the results obtained from the u_2 measurements by Peshkov and Zinov'eva at 25.3 bars which are shown in Fig. 2 as solid squares.

Let us emphasize that if ρ_s/ρ follows a pure power law, then the data in Fig. 2 should yield a straight line. If, in addition, the exponent ζ is $\frac{2}{3}$, then the slope of this line should be zero. It is evident from the curvature of the data in Fig. 2 that the results cannot be fitted to a pure power law over any reasonable and accessible range of ϵ except at vapor pressure. If we assume, as is permitted by the results, that $a(\mathrm{SVP}) \equiv 0$ in Eq. (2) [i.e., that Eq. (1) is valid], then we obtain

$$\zeta_{\mathrm{SVP}} = 0.674 \pm 0.001 \tag{9}$$

from data with $2\times10^{-5} \leq \epsilon \leq 10^{-2}$. The quoted error includes possible systematic errors. The result, Eq. (9), also agrees with ζ_{SVP} obtained from any subset of our data which spans any one decade in ϵ, provided always that $\epsilon \leq 10^{-2}$. Even an analysis which uses only data with $\epsilon \leq 10^{-3}$ and which treats T_λ as an adjustable parameter yields the result Eq. (9). The best T_λ obtained by this last analysis differs from our measured T_λ by $(2\pm2)\times10^{-7}$ K. Within combined errors Eq. (9) also agrees reasonably well with the independent measurements of ρ_s/ρ,[17] which yielded $\zeta_{\mathrm{SVP}} = 0.666 \pm 0.006$. It is also consistent with the scaling prediction

$$\zeta = \tfrac{1}{3}(2-\alpha'), \tag{10}$$

which with the experimentally permitted[1] $\alpha' = -0.02$ at SVP yields 0.673.

In view of the large nonasymptotic but singular contributions which are revealed by the curvature of the higher-pressure data in Fig. 2, it seems unreasonable to attach much significance to the value and probable error for ζ_{SVP} given by Eq. (9), in spite of the apparently nearly perfect fit by a power law which is revealed by our analysis. When we permit $a \neq 0$ in Eq. (2), the permitted errors in ζ become much larger, and ζ and x at a given pressure are uncertain by ± 0.02 and ± 0.1, respectively. Within these limits, ζ and x are independent of pressure, with permitted values of $\frac{2}{3}$ and $\frac{1}{3}$, respectively. When measurements at all P are analyzed jointly, we obtain for $\epsilon \leq 10^{-2}$ the best values given in Eqs. (5) and (6). As a good analytic representation of ρ_s/ρ, we quote the parameters

$$\begin{gathered} \zeta = \tfrac{2}{3}, \quad x = \tfrac{1}{3}, \quad k_0 = 2.329, \quad k_1 = -0.02983, \\ a_0 = 0.3384, \quad a_1 = -0.01833, \quad a_2 = 0.002424. \end{gathered} \tag{11}$$

The asymptotic exponent ζ is in good agreement with the scaling law Eq. (10) and the experimental result[21] $\alpha' = 0.000 \pm 0.005$ for the exponent of C_p.[2,3] The absence of a pressure dependence for ζ, x, and α' would be expected from universality arguments, for a change in the pressure P does not affect the symmetry of the transition. The observation that within our errors universality and scaling are obeyed in He II may be compared with the previously observed *departures* from scaling and universality in the ratio A_0/A_0' of the amplitudes of

$$C_p = -A_0 \ln|\epsilon| + B_0 \qquad (12)$$

above and below T_λ. Experiments yield $A_0/A_0' > 1$ and dependent upon P. Scaling requires $A_0/A_0' = 1$, and from universality we expect A_0/A_0' to be independent of P. Our present results strengthen our previous belief[2,3] that the source of departures from universality and scaling near T_λ must be found by examining the high-temperature phase.

We have demonstrated in this paper that the superfluid density cannot be described by a pure power law. When we invoke higher-order singular contributions of a reasonable functional form to ρ_s/ρ, we can obtain agreement with scaling and universality within our permitted errors.

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[16]The entropy may be obtained from the calorimetric entropy $S_{\lambda 0} = 6.24$ J mole^{-1} K^{-1} at vapor pressure and T_λ by first integrating (see Ref. 3) $(\partial S/\partial T)_\lambda = 1.97 + 0.0938(1 - T_\lambda/2.260)^{-1.380}$ J mole^{-1} K^{-2} along the λ line to yield $S(P,0) - S_{\lambda 0}$, and by then integrating C_p/T with C_p given by Eq. (12) to yield $S(P,\epsilon) - S(P,0)$. The coefficients of C_p in J mole^{-1} K^{-1} can be represented by (see Refs. 1, 3) $A_0' = 5.100 - 0.0403P_\lambda$ and $B_0' = 15.57 - 0.488\,P_\lambda + 0.0129\,P_\lambda^2$. At vapor pressure (see Ref. 1), it is necessary to add to Eq. (12) the terms $D_0'\epsilon \times \ln|\epsilon| + E_0'$, but for our other isobars Eq. (12) is adequate for $\epsilon \lesssim 10^{-2}$. For large ϵ, appropriate thermodynamic data in numerical form were used.

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Energy Loss of a Low-Energy Ion Beam in Passage through an Equilibrium Cesium Plasma*

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The energy loss of a beam of cesium ions traversing a near-thermal equilibrium cesium plasma has been measured as a function of plasma density at ion-beam energies of 35 to 150 eV. The plasma electron/ion temperature was 2100°K, and the charged-particle density was varied from $(0.1 \text{ to } 3.7)\times 10^{11}$ cm^{-3}. The measured energy loss is found to agree very well with theoretical predictions.

Despite the fact that a large amount of theoretiwork[1-10] has been devoted to the problem of the energy interchange of a charged test particle with a plasma, there has been little experimental work performed. This is, of course, largely attributable to the experimental difficulties involved, including the relatively small amount of energy loss, complications produced by the con-