Electron-Beam Filamentation in Strong Magnetic Fields*

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Intense electron beams which are partially current neutralized can be unstable to transverse modes which form small pinch structures. Beams guided by strong B_z fields experience a circular particle drift at frequency ω_β^2/ω_c, which proves highly destabilizing for filamentary modes. Growth rates considerably exceed the resistive diffusion rates normal for a resistive instability. The mode may be difficult to distinguish experimentally from interchange, but our computed growth rates are in qualitative agreement with present experiments.

Weibel[1] first noted that transverse filamentary instabilities occur in plasma streams. The instability can be suppressed by transverse temperature if the thermal transit time through an instability wavelength exceeds the growth rate, $k_\perp v_\perp > \omega_i$. Transverse temperature rises as a current pinches radially, so relatively unpinched beams prove most unstable. This propensity to form local pinch structures should thus occur in intense electron beams which have their current I_b partially neutralized[2-4] by counterflowing background plasma currents I_p. The signature of filamentation is a rippling and bunching in the beam damage pattern downstream, or even streamers of ejected matter when the beam is viewed from the side. Such behavior makes beam transport and focusing more difficult.

Many applications of intense electron beams utilize a strong external magnetic field B_z for guidance, and it is then generally assumed that transverse instabilities are suppressed; a recent calculation[5] gives a growth rate $\omega_i \approx (\omega_b^2 v_z^2/c^2 - \omega_c^2)^{1/2}$, where v_z is the beam velocity, ω_b (ω_c) the beam plasma (cyclotron) frequency, and the beam density n_b equals the plasma density n_p. If $\omega_c > \omega_b v_z/c$, there is no instability. Yet recently filamentary instabilities have been observed in experiments which employ guide fields B_z—a toroidal configuration,[6] a simple mirror device,[7] and a straight uniform system.[8] While some filamentary effects may arise from curved field lines in some of these cases, it seems opportune to point out that for strong B_z fields the circular electron drift due to a weak beam self-field is severely destabilizing. Rapid growth results from this motion, because the drift frequency is only slightly dependent on energy spread in the beam.

Present heating experiments[6,7] use a preionized background gas for which conductivity σ is high, so we shall treat the (purely growing) instability reisitively, $\sigma > \omega$. When the rising beam current I_b enters the plasma, an opposite induction current I_p flows. I_p decays in a resistive diffusion time $\tau = 4\pi\sigma a^2/c^2$, where a is the beam radius; usually τ exceeds the beam pulse time.

In addition to streaming along B_z field lines, beam electrons undergo betatron oscillations in their net self-field with frequency

$$\omega_\beta(I_p/I_b) = \omega_b(v_z/c)[\tfrac{1}{2}(1 - I_p/I_b)]^{1/2}. \qquad (1)$$

If the beam could pinch across B_z, the full betatron frequency $\omega_\beta(0)$ would apply after a time τ and transverse Landau damping would shut off the instability. However, B_z constrains electrons to stream along field lines and the only transverse motion is a circular drift[2] about the beam center at a frequency $\overline{\omega} = \omega_\beta^2/\omega_c$, assuming $\omega_\beta \ll \omega_c$. Thus electrons move in helices with large pitch angles. The velocity dependence of $\overline{\omega}$ is weak for relativistic beams, $\overline{\omega} \sim v_z^2/c^2 \sim 1$. This sharpness of $\overline{\omega}$ indicates a strongly destabilizing feature if the energy spread in the beam is not too large. Electron motions transverse and along B_z are decoupled, and we can write the distribution function f_0 in terms of the transverse and longitudinal constants of the motion. A simple Vlasov equation analysis follows, with f_0 of the form $g(M)S(\overline{\omega})\delta(v_z - v_0)$, where M is the transverse angular momentum and $S(\overline{\omega})$ is the distribution of drift frequencies in the beam. It is only necessary for our purposes to specify that $g(M)$ represent a net beam rotation, and that the beam have some radial profile, $p(r)$. We take an infinitely long cylindrical beam and an electromagnetic perturbation, the vector potential $A_z \sim \exp[i(kz + m\theta - \omega t)]$. Calculating the perturbed current, Maxwell's equations become

$$\left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} - \frac{m^2}{r^2} + \frac{4\pi i\sigma\omega}{c^2}\right)A_z = -2\frac{\partial p(r)}{\partial r}GA_z, \qquad (2)$$

with

$$G = m\omega_\beta^2 \int_0^\infty \frac{\bar{\omega}^{-1} S(\bar{\omega})\, d\bar{\omega}}{m\bar{\omega} - (\omega - kv_0)}. \qquad (3)$$

G is the expected resonant factor due to the resonance between drift orbits and the Doppler-shifted perturbing fields. Resistive instabilities typically grow in a time τ, but the drift resonance makes possible great enhancement of this growth. For an average frequency spread $\delta\bar{\omega}$ over the distribution, we can approximate $\omega - kv_0 \approx m(\bar{\omega} \pm \delta\bar{\omega})$ if $S(\bar{\omega})$ is not too rapidly varying as the integral G passes through the resonance. Then $G \approx \omega_\beta^2/\bar{\omega}\,\delta\bar{\omega}$. With beam energy spread $\delta\gamma m_b c^2$ and an ambient field inhomogeniety δB_z, $\delta\bar{\omega}/\bar{\omega} \approx [(\delta B_z/B_z)^2 + (2\delta\gamma/\gamma^3)^2]^{1/2}$. One may expect $\delta B_z/B_z < 0.1$ and $\delta\gamma \ll \gamma$, if $\gamma > 3$.

We may solve Eq. (2) readily for a uniform beam profile, $dp(r)/dr = \delta(r-a)/a$, where a is the beam radius. The usual Bessel-function dispersion relation results, and here we present specific solutions for varying boundary conditions. If the beam is immersed in a conducting channel of radius R, for $R \gg a$ we find (if $\omega_i\tau \gg m^2$), $\omega_i\tau \approx G^2[1 - (3m/4G)^2]^2$, so growth is enhanced over the rate τ^{-1} by G^2 and falls with higher mode number m. If $R \approx a$ (as may be appropriate to reduce radial convection for intense plasma heating), so that $R/a = 1 + \epsilon$, $\epsilon \ll 1$, ω_i falls until $m = m_0$, where $m_0 \approx \frac{4}{3}G[1 + \ln(2\epsilon G)/2\epsilon G]^{1/2}$, and then growth rises monotonically for higher m. When $m \ll m_0$, the growth for this case takes the same form as when $R \gg a$. However, in experiment if $m_0 \gg 1$ probably only the low values of m could be observed; fine-grain structure is easily obliterated by deviations from perfect transport conditions.

Similarly, we find that an annular beam of thickness d and radius a will experience growth $\omega_i\tau \approx 2d(G^2 + m^2)/a$ with angular mode numbers $m \gtrsim a/d$ growing most rapidly. Such patterns have been observed recently in annular-beam experiments,[7] and our growth rate and mode number estimates above agree qualitatively with them. (More precise comparison is difficult since the plasma conductivity was only roughly known.)

The above rates are quite rapid (one may find $\omega_i\tau > 100$ for present beams), in part because we used a square beam current profile. The decay time τ is an average over the beam profile, but small current fluctuations near the edge can form field perturbations which diffuse across the square beam edge in very short times. Our instability analysis averages the rapid decay at the edge with slower-decaying perturbations toward the origin. As the profile is smoothed, the growth rate drops quickly. If $p(r)$ is Gaussian with rms radius $a\sqrt{2}$, for the case $R \gg a$ we find $\omega_i\tau \approx G[1 - (3m/4G)^2]$. Similarly, the other cases above are changed by replacing $(\omega_i\tau)^2$ with $\omega_i\tau$. An initial sharp profile will be smeared radially by the instability, reducing ω_i, but in a strong B_z this will require substantial development of filamentation. Thus a square profile in a strong field will still experience the faster growth rates.

The above results apply also to a beam model without imposed B_z, if the helical orbits represent betatron motion and the beam quality factor $(\nu/\gamma)^{-1}$ is high. The same forms for growth result, but the resonant factor in the integrand of G becomes $m[m^2\omega_\beta^2 - (\omega - kv_0)^2]^{-1}$, so the resonant character is sharper. However, energy spread in the beam distribution will smear ω_β more and growth will not be as rapid as in the strong-field case.

In practice, filamentation in strong fields may be difficult to distinguish from the interchange instability, which leaves a similar rippled pattern at the beam surface, but does not perturb the inner beam area until well advanced. Indeed, the preionized gas will interchange with B_z even *before* beam injection, so that a fluted field pattern then guides and ripples the beam. Experiments should be designed to avoid retention of such preinjection fossils.

However, if the gas is not preionized there is also a price to pay: Both interchange and nonresistive filamentation ($\sigma < \omega$) have rapid growth rates before gas breakdown occurs. Interchange[9] has growth $\omega_i \approx kv_0\omega_b a(m\gamma m_e/Mc^2)^{1/2}$ and for nonresistive filamentation $\omega_i \approx \omega_b(mm_e/4\gamma M)^{1/2}$. Here the electron-ion mass ratio m_e/M appears because ions must be moved by the perturbation to avoid strong electric fields which would suppress the instability. Both instabilities grow in a few nanoseconds in intense electron beams. After gas breakdown, interchange growth is decreased by $(n_b/n_p)^{1/2}$, and filamentary modes become the resistive type we have studied above.

Thus it may be difficult in practice to avoid some filamentary or fluting behavior without careful attention to the injection phase. Also, it is not yet clear whether recent highly asymmetric beam patterns in toroidal transport experiments[6] derive from the toroidal field curvature or filamentation effects we have described above. Similar patterns seen in a uniform-B_z experiment[8] may be due to fringing fields of the magnets used, which can induce several sorts of instability. A z-pinch confinement technique[10] seems to avoid these difficulties altogether by substituting a guide field B_θ for a uniform B_z. Any instabilities present are retarded by the inertia of the strong B_θ field.

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Ionic Mobilities in Solid Helium*

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Measurements of charge-carrier mobilities in solid helium have been made which are the first to be sufficiently extensive to allow conclusions to be drawn concerning the fundamental nature of the transport processes involved. Positive and negative charge carriers are quite different physical entities but in each case our data are able to provide quantitative support for a particular model to describe the carrier and its motion.

The first ionic current measurements in solid helium were reported by Shal'nikov and co-workers,[1-3] who measured the current-voltage (I-V) characteristics of a diode filled with solid helium. Ions of both signs were produced near a cathode coated with a low-energy β source, and the results obtained are indicated schematically in Fig. 1(a). For the plane-parallel geometry employed, the current I in the quadratic space-charge-limited region is given by[4]

$$I=9\mu V^2 A/32\pi L^3, \tag{1}$$

where μ is the mobility, V is the applied voltage, A is the collector area, and L is the electrode spacing. Although Shal'nikov and co-workers[5] have also reported mobilities obtained using a time-of-flight technique, no measurements to date have been sufficiently extensive to elucidate the nature of the carriers or the mechanisms which control their motion.

To provide such data we have measured the mobilities of positive and negative carriers in the hcp phase of solid ^{4}He over an extensive temperature range, viz., $1.1 < T < 3.6$ K. We measured the space-charge-limited current in a diode similar to the one described by Shal'nikov[1] and obtained the mobility with the use of Eq. (1). The I-V characteristics are measured at each temperature, and a typical plot of $I^{1/2}$ versus V is shown in Fig. 1(b). This technique enables one to make measurements on carriers with transit times up to 10 h. Diode spacings of 0.121 and 0.458 cm are used, and crystals are grown from a copper cold finger located directly below the diode. Our criteria for a good crystal are that the steady-state current be established in a time τ_s not more than 50% greater than the transit time and that the plots of $I^{1/2}$ versus V are linear. In crystals strained by rapid cooling the mobility becomes field dependent, and τ_s is increased. Many precautions are taken, particularly in temperature regulation, to insure accurate results, and the experimental details will be presented elsewhere.

The temperature dependencies of the mobilities to within 10 mK of the melting curve are presented in Fig. 2, together with the few data points available from Ref. 4. Relative errors for a given crystal are less than the width of the symbols on the graph except where error bars are shown.