Nuclear-Coulomb Interference in π^+ - ^{12}C Scattering*

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The small-angle scattering of π^+ from ^{12}C was measured at 115, 167, and 242 MeV. The measurements covered the Coulomb-nuclear interference region from 3° to 10°. The real part of the forward nuclear amplitude was extracted from the data.

The small-angle scattering cross section of positive pions from carbon 12 has been measured for laboratory scattering angles from 3° to 10° at pion kinetic energies of 115, 167, and 242 MeV. These data can be directly compared with the recent CERN[1] π^--^{12}C data from 5° to 20° at 120, 180, and 260 MeV as the nuclear amplitudes for the two reactions are equal under the assumption of charge independence. In this angular region the nuclear-Coulomb interference is most pronounced, and the known Coulomb scattering allows one to extract the magnitude and phase of the forward nuclear scattering amplitude. This information provides a check on nulcear models such as the Kisslinger optical model[2] and the Glauber model.[3]

The experimental arrangement for the π^+-^{12}C measurements is indicated in Fig. 1. The 600-MeV proton beam from the synchrocyclotron at the Space Radiation Effects Laboratory (SREL) is used to generate the pion beam. The pion channel at SREL consists of quadrupole doublets Q_1, Q_2, Q_3 and bending magnets M_1 and M_2. The pions from the CH_2 production target T_1 are focused onto the scattering target T_2. A ±2% momentum bite is provided by slits between M_1 and M_2. The graphite scattering target is 1.07 g/cm^2 thick and 10 cm × 10 cm in area. The scattered beam is not magnetically analyzed; therefore, inelastic events are not rejected. The smallest scattering angle for which the cross section was analyzed is determined by Coulomb multiple scattering of the beam in the target.

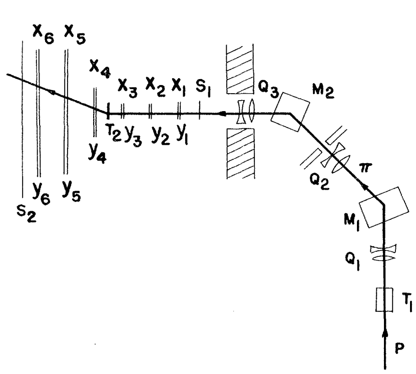

FIG. 1. Schematic of experimental apparatus. The π's produced at the CH_2 production target T_1 by 600-MeV protons are momentum analyzed and focused on the scattering target T_2 by quadrupole doublets Q_1, Q_2, Q_3, and bending magnets M_1 and M_2. S_1 and S_2 are thin plastic scintillators and X_1–X_6, Y_1–Y_6 are multiwire proportional counters. A typical π trajectory is indicated.

The (x, y) coordinates of the π^+ trajectory are measured using six two-dimensional multiwire proportional counters (MWPC). When the system is triggered by a fast coincidence between the scintillators S_1 and S_2, placed before and after the counters, the wire number of each coordinate is read out digitally and fed into an IBM 360-44 on-line computer. The rejection of protons in the π^+ beam is achieved by placing a Lucite absorber at the momentum slit between M_1 and M_2. Any remaining protons are rejected on the basis of time-of-flight and dE/dx information from S_1 and S_2. Details of the MWPC system are published elsewhere.[4]

Straight-line fits to the three (x, y) coordinates of the incident and scattered pions are performed on line. The scattering angle θ is calculated from $\cos\theta = \hat{r}_1 \cdot \hat{r}_2$, where $\hat{r}_1$ and $\hat{r}_2$ are unit vectors in the directions of the incoming and outgoing trajectories, respectively. The intersections of these lines with the target plane are required to be less than 5 mm apart. This criterion allows a tenfold reduction in the decay muons accepted by the system by limiting the effective decay length to ±10 cm from the target.

The muons from the decay $\pi \to \mu + \nu$ in the target-out data provide a convenient method of measuring the beam momentum and purity. The maximum opening angle of the muons in the laborato-

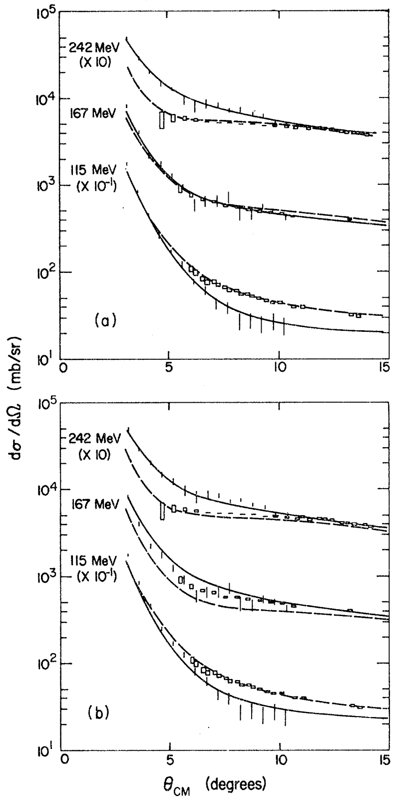

FIG. 2. π^+-^{12}C differential cross sections at 115, 167, and 242 MeV (vertical lines), and CERN π^--^{12}C differential cross sections at 120, 180, and 260 MeV (see Ref. 1) (rectangles). The heights of the symbols indicate the error bars. Solid curves, π^+ data; dashed curves, π^- data. (a) The curves are a fit to the data with a parametrized nulcear amplitude model. K was allowed to vary (see text). (b) The curves were calculated using the Kisslinger optical model with the best-fit parameters from Ref. 2. The optical model was evaluated at 115, 167, and 242 MeV for the π^+, and at 120, 180, and 260 MeV for the π^-.

ry frame is a measure of the π^+ momentum, and the integral of the decay-muon distribution is proportional to the pion flux. Techniques for using the decay muons to obtain beam momentum and purity will be published elsewhere.[5]

Figure 2 shows the π^+-^{12}C differential cross sections. The π^--^{12}C differential cross sections of Ref. 1 at 120, 180, and 260 MeV are given for comparison. The π^+-^{12}C data have been corrected for solid-angle effects due to the finite size of the MWPC's. The Coulomb scattering due to μ^+ has been subtracted. It is clear from Fig. 2 that the π^+ scattering exhibits destructive interference at 115 MeV, and constructive interference at 242 MeV, while the π^- scattering exhibits interference of the opposite sign.

As mentioned, inelastic events are not rejected in these measurements. Any inelastic scattering must be subtracted from the measured cross sections to obtain the elastic scattering. The total inelastic differential cross sections are typically 5–10% of the elastic in the π^--^{12}C forward scattering.[6] The inelastic events in the data make the true destructive interference larger and the true constructive interference smaller than that seen in Fig. 2.

The solid curves in Fig. 2(a) are the results of fits using the following formalism.[1] The differential cross section is obtained from the nuclear scattering amplitude f_N, and the Coulomb scattering amplitude f_C:

$$d\sigma/d\Omega = |f_N + |f_C| e^{2i\delta}|^2, \tag{1}$$

$$f_N(t) = (P/4\pi)\sigma_{\rm tot}[i + \alpha(0)] \times \exp[-(R_S^2 + iK)|t|/6], \tag{2}$$

$$\alpha(0) = \mathrm{Re} f_N(0)/\mathrm{Im} f_N(0), \tag{3}$$

$$2\delta = -2\eta \ln\left(\sin\frac{\theta^*}{2}\right) - \eta\int_{-4P^2}^{0} \frac{dt}{|t'-t|}\left[1 - \frac{f_N(t')}{f_N(t)}\right], \tag{4}$$

where P and θ^* are the respective center-of-mass momentum and scattering angle, η is the effective Coulomb coupling constant, and the four-momentum transfer $t = -2P^2(1-\cos\theta^*)$. The nuclear phase-variation parameter K and $\alpha(0)$ are parameters that are varied to fit the data. The present data are insensitive to the strong-interaction radius R_S, because the range of momentum transfer covered is small compared to the width of the diffraction peak. For this reason the value of R_S was interpolated from the values given in Ref. 1. The total cross section was interpolated from the measured π^--^{12}C total cross sections of Ref. 6. For $K=0$, the above form for the nuclear amplitude does not allow the phase of f_N to vary as a function of t.

Table I gives the values of $\alpha(0)$ and the real part of the forward scattering amplitude obtained from the analysis of the π^+-^{12}C data with $K=0$, along with the input parameters. A better fit can be obtained only at 115 MeV by allowing K to vary. It is interesting to note that the destructive interference in both the π^+ and π^- scattering is difficult to fit with this simple model unless this additional parameter is varied. The present π^+ values of $\mathrm{Re} f_N(0)$ are in excellent agreement with the CERN π^- results, if K is allowed to vary for

TABLE I. Forward nuclear scattering-amplitude parameters from Eq. (2).

E (MeV)	$\alpha(0)$	R_S [a] (fm)	K (fm^2)	χ^2/n [b]	σ_T [c] (mb)	Im$f_N(0)$ [d] (fm)	Re$f_N(0)$ (fm)
115 [e]	0.23 ± 0.06	3.49	27 ± 11	0.68	675 ± 8	5.83 ± 0.07	1.34 ± 0.35
115	0.24 ± 0.06	3.49	0	1.58	675 ± 8	5.83 ± 0.07	1.40 ± 0.35
167	$0.03^{+0.09}_{-0.06}$	3.32	0	0.88	685 ± 8	7.50 ± 0.09	$0.23^{+0.68}_{-0.45}$
242	-0.28 ± 0.04	3.18	0	1.30	565 ± 7	8.10 ± 0.10	-2.27 ± 0.33

[a]Values interpolated from π^--^{12}C amplitude fits from Binon *et al.* (Ref. 1).
[b]$n = 12$ is the number of experimental points fitted.
[c]Values interpolated from π^--^{12}C total cross sections from Binon *et al.* (Ref. 6).
[d]Im$f_N(0) = P\sigma_T/4\pi$.
[e]K allowed to vary.

the latter. Since Re$f_N(0)$ is consistent with zero at 167 MeV, the $(\frac{3}{2}, \frac{3}{2})$ resonance energy for pion-carbon scattering must be near this energy. This result is in disagreement with a Glauber-model treatment, which predicts Re$f_N(0) = 0$ at the π-nucleon resonance energy.

Calculations of the Kisslinger optical model[2] using interpolated parameters from fits to π^--^{12}C large-angle data[6] are indicated in Fig. 2(b) by the solid curves. The optical model gives qualitative agreement with these small-angle cross sections except near the $(\frac{3}{2}, \frac{3}{2})\pi^{\pm}$-^{12}C resonance energy. Near the resonance energy, it is difficult to interpolate the rapidly varying $l = 1$ parameters in the model, and this is the probable cause of the disagreement.

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Quantized Longitudinal and Transverse Shifts Associated with Total Internal Reflection

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A recent experiment has shown us that the Goos-Hänchen shift is quantized, the eigenfunctions being the transverse electric and magnetic modes. We give here a straightforward theory of this phenomenon, together with the prediction that our new transverse shift is also quantized, with the circularly polarized modes inside the evanescent wave as eigenfunctions. An experimental test of the latter point is being considered.

First we make it clear that the now well-known Goos-Hänchen[1] longitudinal shift, and the new transverse shift one of us[2] has calculated and proved experimentally, are not simultaneously observable. The existence of the Goos-Hänchen shift entails that total reflection at a plane interface is *not* optically stigmatic, so that the image of a point source S as produced by a pencil of