Measurements of the Self-Stabilization of a Two–Ion-Beam Instability

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The self-stabilization of the two–ion-beam instability due to microturbulent ion heating has been measured in a plasma with variable temperature ratio ($2.0 < T_e/T_i < 10.0$). When the plasma is unstable, excited ion waves and background microturbulence are observed to grow, saturate, and damp, in conjunction with ion heating. When the plasma is stable, no ion heating is observed. The ion heating e-folding length is $\approx 500\lambda_{de}$. This heating may occur in the formation of collisionless shocks.

We have experimentally observed a two-ion-beam (i-i) instability to be self-stabilized by turbulent ion heating. It has been hypothesized that this phenomenon may play a strong role as a dissipative mechanism in a turbulent collisionless shock[1] or in the collisionless heating of the solar-wind ions.[2] Previous authors[3,4] have presented results of numerical simulations of this phenomenon and have found that the i-i instability can cause significant dissipation in an i-i plasma. Although our experimental parameters are considerably different from the numerical models, we observe the same basic behavior: When the electron-to-ion temperature ratio T_e/T_i is sufficiently large, the system is unstable and a band of electrostatic noise below the ion plasma frequency grows in amplitude, heating the ions. T_e/T_i decreases until the system is stabilized and the noise stops growing. The heating is not a true increase in Maxwellian temperature but more of a filling in of the velocity space between the two beams. When the initial T_e/T_i is not sufficiently large for an instability, there is no growth of noise and no heating of the ion beams.

We have made no attempt to measure the velocity diffusion or heating in the direction perpendicular to the magnetic field since waves with $\kappa_\perp/\kappa_\parallel > 1$ (electrostatic ion-cyclotron waves) should not be unstable for our experimental conditions, $V_D/a_i < 4.0$.[5,6] We also observe no appreciable structure in the frequency spectrum of the noise near the harmonics of the ion-cyclotron frequency, nor any appreciable spread in parallel phase velocities. We have therefore used a one-dimensional assumption in the analysis. Wave-wave interactions have been neglected; these probably play a role in determining the frequency spectrum of the microturbulence, but should not be important in the ion heating.

The experiment was performed in the Cs plasma of a combined Q machine and μ-wave discharge (Fig. 1). The two ion beams are created in a manner similar to that used by Andersen *et al.*[7] The fast ions are created by surface ionization on the 3-cm-diam tantalum hot plate and accelerated into the plasma by the hot-plate sheath.[7,8] The slow ions $[\overline{V}_i \approx a_i \equiv (2kT_i/m_i)^{1/2}]$ are created by a neutral Cs cloud localized in a copper tube located 20 cm downstream from the hot plate. These neutrals are ionized in part by a microwave discharge at the upper-hybrid resonance and in part by charge exchange with the fast ions. This creates two ion beams with parallel temperatures of about 0.1 eV with a difference in drift velocity $V_D = \overline{V}_b - \overline{V}_i \approx 1.5a_i$ to $3.5a_i$, and a density of $\sim 10^8$ cm^{-3}. The electrons created by the hot plate at a temperature of 0.2 eV are too cold for the plasma to be unstable; however, the microwaves can heat the electrons sufficiently to obtain an unstable condition (i.e., $T_e/T_i > 4$).

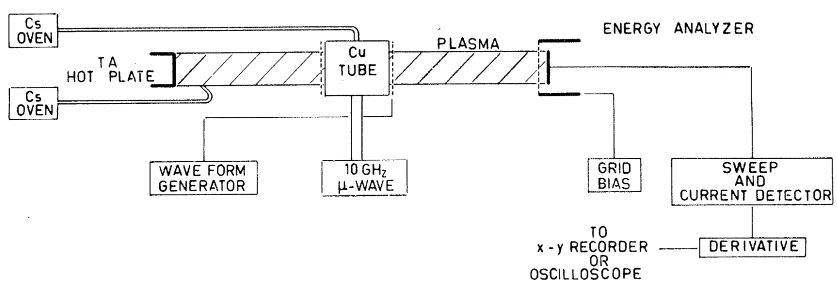

FIG. 1. Schematic of experiment. Q machine plus microwave discharge. Maximum field B is 4700 G; $2.0 < T_e/T_i < 10.0$.

A slightly inhomogeneous magnetic field stabilizes the discharge and allows the local upper-hybrid resonance to extend through the plasma cross section. It also acts as magnetic mirror to transfer the perpendicular energy the electrons obtain from the μ wave into the parallel energy necessary for instability and keeps the high-energy electrons from going upstream and creating an instability in the region inaccessible to our probes. We can thus study a boundary value problem with the i-i instability starting at the discharge region. In the region where the experiment is performed, the change in field B is less than 5%.

The copper tube containing the discharge region has coarse grids (spacing $\gg \lambda_{de}$ and $< r_{ci}$) on either end to help contain the microwave. The downstream grid was also used to excite ion waves by applying a voltage pulse or sine wave.[9] The principal diagnostic tool was an axially movable retarding-potential energy analyzer,[7,8] used to measure the zero-order velocity distribution function and the current fluctuations of either the excited waves or the background noise as a function of position.

The comparison of our experimental results with the linear analysis of the i-i instability by Stringer[10] and Fried and Wong[11] has been presented elsewhere.[12] We have found good quantitative agreement of our measured initial stability limits with those predicted by the linear theory. We also obtained qualitative agreement with the theoretical linear dispersion relation[10,13] in that the unstable waves show little dispersion and have a phase velocity between the two beams. It is difficult to measure the linear dispersion relation accurately since nonlinear effects are important even in the early development of the instability.

In Fig. 2 we plot the amplitude and phase of excited continuous waves as a function of position for an unstable situation (case A). The fact that the higher frequencies grow faster, saturate sooner, and damp faster can be explained by the dependence of the linear dispersion relation on the plasma parameters [$T_i(x)$ and $T_b(x)$, which are changing because of the nonlinear effects]. As T_i and T_b increase, the higher-frequency waves are stabilized first.[12] The frequency spectrum of the background noise behaves the same as the excited waves. Only frequencies less than ω_{pi} are unstable and the higher frequencies grow, saturate, and damp sooner than the lower frequencies. The growth and damping of excited single pulses show essentially the same behavior,[12] eliminating the possibility that the observed behavior of the continuous waves is caused by interference effect or ion-wave echoes.

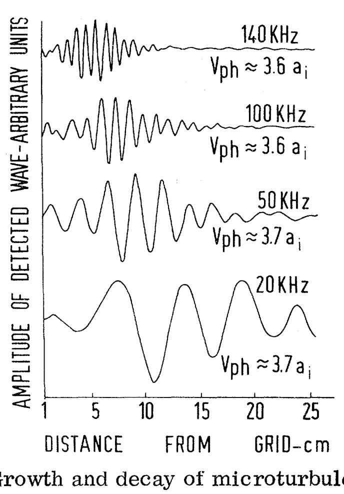

FIG. 2. Growth and decay of microturbulence and excited waves. Plasma parameters: $\eta_b/\eta_i = 0.4$, $T_b/T_i = 0.45$, $V_D/a_i = 4.0$, $T_e/T_i = 10$. The excited waves are detected using an interferometer technique. Phase velocity is measured in laboratory frame.

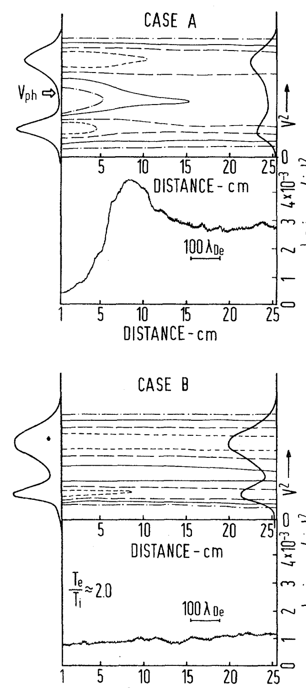

FIG. 3. Phase diagrams showing ion heating in conjunction with the growth of the background noise: Case A, same parameters as in Fig. 2; case B, same parameters as in Fig. 2 except $T_e/T_i = 3.0$. The ordinate V^2 stems from the retarding potential of the energy analyzer and has the effect of making the fast beam seem broader than it is. Case A, unstable situation; case B, stable situation.

Since the experiment is steady state, the development of the ion velocity distribution may be represented as a phase diagram constructed by measuring the velocity distribution at various positions down the plasma column. Figure 3 shows the phase-space diagrams for two cases. Case A (also in Fig. 2) is the initially unstable situation; case B is the initially stable situation. Also shown is the normalized power of noise versus position, and the initial and final distributions. The power of the background noise grows, saturates, and damps in the same way as the excited waves.

For the unstable case (A) there is considerable filling in or heating of the i-i system in concurrence with the growth of the noise. In the stable case (B), there is little change in both velocity distribution and amplitude of the noise. The noise which exists in the stable case consists mainly of drift and Kelvin-Helmholtz-type instabilities which have a parallel phase velocity much higher than the velocity of the ions and should not interact strongly with them. The total density in all four cases is about 10^8 cm^{-3}; the mean free path for energy transfer between the slow and fast ions is more than 2 orders of magnitude longer than the experimental region.

In order to obtain some knowledge of the parallel phase velocity of the noise, a cross correlation was performed on the noise detected on two probes separated by a distance $z_2 - z_1$ along the column. The measured time for maximum correlation varied approximately linearly with probe separation, $\tau_{max} \approx (z_2 - z_1)/V_{cc}$. V_{cc}, the effective parallel phase velocity of the noise, was within 10% of both the measured and the theoretical phase velocity of the excited waves. The arrow in Fig. 3, case A, shows the approximate position of the phase velocities. The amplitude and width of the maximum cross correlation does not change greatly with $z_1 - z_2$ (except when $z_1 \approx z_2$), indicating that there is little dispersion in the noise, as is expected from the linear dispersion relation.

In general, the wave-particle interaction of microturbulence can be described by a diffusion equation.[14,15] Since the experiment was steady state, the form of the theory appropriate to a boundary-value problem is used instead of the usual initial-value problem. In one dimension we have

$$V\frac{\partial}{\partial x}f_j = \frac{\partial}{\partial V}D(V,x)\frac{\partial}{\partial V}f_j, \quad j = i, b. \tag{1}$$

An approximate expression for the diffusion coefficient for ion waves is[16,17]

$$D(x,V) \simeq 2\left(\frac{q_i}{M_i}\right)^2 \int_0^\infty d\omega\, |E_\omega|^2 \int_0^\infty \frac{dx}{V}\exp\left[i\left(k_\omega - \frac{\omega}{V}\right)x - \frac{D\omega^2}{3V^5}x^3\right]. \tag{2}$$

The diffusion coefficient for ion waves has the form of a resonance function in velocity, whose width is determined by nonresonant heating and strong turbulent effects.[1,16-18] This allows the interaction to occur over a region in velocity space comparable to the thermal velocity. The rms value of the fluctuating field can be related to the measured fluctuating current by

$$\int_0^\infty \frac{d\omega}{\omega}|E_\omega(x)|^2 \approx \frac{\langle k\rangle}{V_{ph}}\left(\frac{T}{e}\right)^2\left|\frac{\delta J_{rms}}{J_0}\right|^2. \tag{3}$$

$\langle k\rangle$ is the average wave number of the microturbulence. At the point $V = V_{ph}$ we can obtain a simple expression for the change of the distribution of either species:

$$\frac{1}{f_j}\frac{d}{dx}f_j\bigg|_{V=V_{ph}} \approx \frac{4(V_{ph}-\bar{V}_j)^2}{a_j^2}\frac{a_i}{V_{ph}}\langle k\rangle\left|\frac{\delta J_{rms}}{J_0}\right|^{3/2}. \tag{4}$$

By taking velocity moments of Eq. (1), it is also possible to obtain similar expressions for the rate of change of temperatures.[4] For the parameters in case A we obtain

$$f_b^{-1}\,df_b/dx \equiv L^{-1} \sim (10\langle\lambda\rangle)^{-1} \sim \tfrac{1}{20}\ \mathrm{cm}^{-1}. \tag{5}$$

The e-folding distance is comparable to the measured growth rate, $k_I \approx k_r/10$, and is in approximate agreement with that obtained from the measured distribution function. Examining the linear dispersion relation shows that for our initial parameters the plasma will become stable after about one e-folding in ion or beam temperature.

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Magnon Localization in Antiferromagnets*

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A criterion is developed for the Anderson localization of the one-magnon excitation in a substitutionally disordered two-sublayer Heisenberg antiferromagnet. Assuming nearest-neighbor exchange interactions, application is made to $K(Mn_{1-c}Co_c)F_3$ near zero temperature. Taking, for simplicity, $J^{Mn-Co}=(J^{Mn}J^{Co})^{1/2}$, a critical condition for localization is found for all impurity concentrations, yielding good agreement with recent experiments.

Recently, Buyers *et al.*[1] presented evidence for localized spin excitation modes in the substitutionally disordered antiferromagnets, $K(Mn_{1-c}Co_c)F_3$ and $(Mn_{1-c}Co_c)F_2$. They presented an empirical formula for the relation between the critical energy for localization, and the impurity concentration. They remarked that, "Although the general method of Anderson should in principle be applicable to any disordered system, it is not clear how to extend the formalism ⋯ to spin waves in an antiferromagnet." This Letter extends Anderson's formalism[2] to that problem. We find a different form for the localization criterion than that written down by Buyers *et al.* Our result is in agreement with their data.

We begin with a Hamiltonian appropriate to a substitutionally disordered two-layer sublattice antiferromagnet:

$$H=2\sum_{mn}J_{mn}\vec{S}_m\cdot\vec{S}_n-2\mu_BH_A\sum_m S_{mz}+2\mu_BH_A\sum_n S_{nz}. \tag{1}$$

Here, m and n represent the lattice vectors on sublayers A and B, respectively; and J_{mn}, $\vec{S}_m$, and H_A are the nearest-neighbor exchange interaction, spin vector, and anisotropy field, respectively[3] (the g factor is taken equal to 2). The Hamiltonian is second quantized using the linearized Holstein-Primakoff transformation. Apart from a constant term,

$$H=\sum_m E_mA_m^\dagger A_m+\sum_n E_nB_n^\dagger B_n+\sum_{mn}V_{mn}(B_mA_m+A_m^\dagger B_m^\dagger), \tag{2}$$

where the boson operator $A_m^\dagger$ $(B_n^\dagger)$ or A_m (B_n) creates or destroys a spin deviation at site m (n), re-