rived by Kirkwood and Buff.[7]]

The relation of our theory to those of Fisk and Widom and Felderhof can be seen easily as follows. Let us make a local Ornstein-Zernike approximation on the direct correlation function,

$$\hat{C}(\vec{q};z_1,z_2)\cong C_0(z_1)\delta(z_1-z_2)-\tfrac{1}{6}l^2(q^2-d^2/dz_1{}^2)\delta(z_1-z_2). \qquad (21)$$

The first term $C_0(z)$ is related to the local compressibility of the fluid at density $n_0(z)$, or to the local Helmholtz free energy per particle, $a(n)$, at z,

$$kT(n^{-1}-C_0)=n^{-1}\partial p/\partial n=\partial^2 na(n)/\partial n^2. \qquad (22)$$

The quantity l is a characteristic (Debye) length, of the order of the range of the direct correlation function. In this approximation, the density profile is determined by the differential equation

$$[\partial^2 n_0 a(n_0)/\partial n_0{}^2]\, dn_0/dz+\tfrac{1}{6}kTl^2 d^3n_0/dz^3=0. \qquad (23)$$

One integration leads to an equation equivalent to the one used by Fisk and Widom,

$$\partial n_0 a(n_0)/\partial n_0+\tfrac{1}{6}kTl^2 d^2n_0/dz^2=\text{const.} \qquad (24)$$

In the same approximation, $K_2(z_1,z_2)$ becomes $\frac{1}{6}l^2\delta(z_1-z_2)$, and the surface tension is given by

$$\alpha=\tfrac{1}{6}kTl^2\int dz\,[dn_0(z)/dz]^2. \qquad (25)$$

This is equivalent to the formula used by Fisk and Widom in their theory of surface tension.

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[4]It appears in an unpublished report of a talk given by J. Yvon at the International Union of Pure and Applied Physics Symposium on Thermodynamics, Brussels, 1948; we can find no reference to this in abstract journals or in the Science Citation Index. Also, it has been obtained independently (but not published) by F. P. Buff. (We are indebted to Professor Buff for calling attention to this prior work.)

[5]J. K. Percus, in *The Equilibrium Theory of Classical Fluids,* edited by H. L. Frisch and J. L. Lebowitz (Benjamin, New York, 1964), pp. II-57 to II-61.

[6]This is the formula derived by Yvon. An apparently different expression has been given by F. P. Buff and R. A. Lovett, in *Simple Dense Fluids,* edited by H. L. Frisch and Z. W. Salsburg (Academic, New York, 1968). Professor Buff and Professor Lovett have both informed us that their formula can be transformed exactly into the one given in Eq. (20).

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Early Departure from Free Acceleration and Turbulent Heating of Electrons in a Toroidal Experiment*

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Shortly after initiation of a high-voltage toroidal discharge in Ar, departure from free-electron acceleration is observed, together with electron heating such that the total energy input is accounted for. This early behavior is attributed to a high-frequency instability ($\omega_{pi}<\omega<\omega_{pe}$) due to interaction between relatively few trapped electrons and the streaming electrons.

In this paper we report an investigation of the early time behavior of a high-voltage, toroidal discharge. The transition from free-electron acceleration to acceleration at a reduced rate is observed, as well as the concurrent onset of turbulence and electron heating.

The experimental system has been described previously[1] in a report on the behavior of the dis-

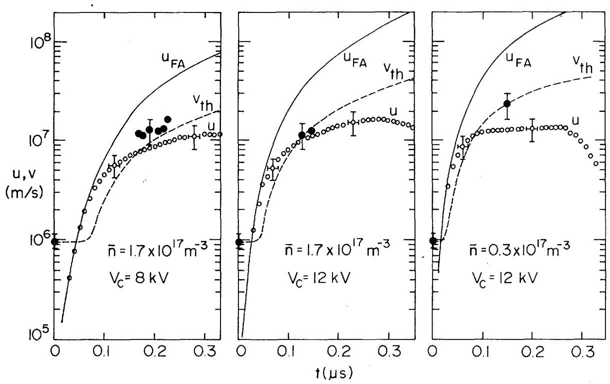

FIG. 1. Electron drift and thermal velocities. Open circles, measured average drift velocity u; solid lines, velocity for free acceleration, u_{FA}; closed circles, measured thermal velocity $(\kappa T_e/m)^{1/2}$; dashed lines, thermal velocity calculated from energy balance, v_{th}. For all three cases the argon gas pressure $p=0.3$ mTorr and $B=1.0$ kG.

charge at later times (after ~1 μsec) when ion heating is observed. A betatron-type electric field ($E \leqslant 6$ kV/m) is applied to a preformed argon plasma confined by a toroidal magnetic field ($B \leqslant 2$ kG). Measurement of the current I and the mean plasma density $\bar{n}$, as in Ref. 1, gives the mean electron drift velocity u. This is compared in Fig. 1 with the velocity u_{FA} for free acceleration in the field E, whose measured value is corrected for plasma self-inductance.[1,2]

Also shown is the electron thermal velocity $(\kappa T_e/m)^{1/2}$ determined at time $t=0$ from diamagnetism measurements,[3] at later times with an orbit-analyzer probe,[4] and calculated (v_{th}) by integrating the energy balance relation

$$eEu=(d/dt)(\tfrac{1}{2}mu^2)+(d/dt)(\tfrac{3}{2}mv_{th}^2),$$

with the use of measured values of E and u. Whereas the measured thermal velocities correspond only to transverse motion, equal heating in the three degrees of freedom is assumed in the energy-balance calculation.

Thermal velocities calculated from the energy-balance relation agree closely with those measured with the orbit-analyzer probe (at $t\sim0.1$–0.2 μsec), which have also been checked in some cases by diagmagnetism measurements. Figure 1 shows that although the electrons initially accelerate freely, departure from free acceleration occurs soon after u exceeds v_{th}. At the same time, rapid electron heating takes place, such that the total input energy is accounted for. This same behavior has been observed over a range of initial plasma densities $\bar{n}=(0.17$–$2.5)$ $\times10^{17}$ m^{-3} (density increase due to additional ionization $\lesssim 15\%$), electric fields $E_{max}\simeq 3$–6 kV/m (determined by the capacitor voltage V_c), and magnetic fields $B=0.5$–2.0 kG.

The observed initial free acceleration is in contrast to an expected reduction in u (up to 40% at large $\bar{n}$) due to the classical skin effect. The magnetic field in the plasma column has been measured at times as early as $t=0.05$ μsec using a small movable coil. The results confirm the absence of a skin effect. Although this is not understood, it is possible that a gradient in the electron drift velocity—required for a skin effect—is inhibited by some form of velocity shear instability,[5] which can operate without a noticeable initial departure from free acceleration.

Departure from free acceleration occurs in only ~0.1–1 ion plasma periods, whereas low-frequency fluctuations ($\omega\simeq\omega_{pi}$) become prominent only after several ion plasma periods, and no significant ion heating is observed[1] until $t\sim1.0$ μsec. Thus electron-ion two-stream instability is evidently relatively important initially. Instead, the early behavior, including observed large-amplitude, high-frequency fluctuations, is apparently due to a two-stream interaction between streaming and relatively few trapped electrons.

Electron trapping may occur[6] in electrostatic potential wells created when a current begins to flow along the bumpy toroidal magnetic field. ($\Delta B/B\sim0.03$ at the center of the chamber and ~0.10 at the plasma boundary.) Numerical simulations have been carried out[7] (linear interpolation in cells, one-dimensional, infinite ion mass) in which electron trapping is induced by imposing, in place of a bumpy magnetic field, an initial bumpy plasma density ($\Delta n/n=0.125$) along the direction of a constant applied electric field. The electrons indeed split into two groups, one freely accelerating and the other trapped near zero velocity, and the ensuing electron-electron two-stream instability leads to early reduction in acceleration as well as to heating of the electrons. The only part played by the ions in this instability is in setting up the trapping potentials and in conservation of momentum; the forward momentum lost by the electrons is transferred through the self-consistent fields to the ions. Although close quantitative agreement with experiment cannot be claimed, the simulation work does demonstrate the possibility of a two-stream interaction between trapped and streaming electrons and the role of the ions in such an instability.

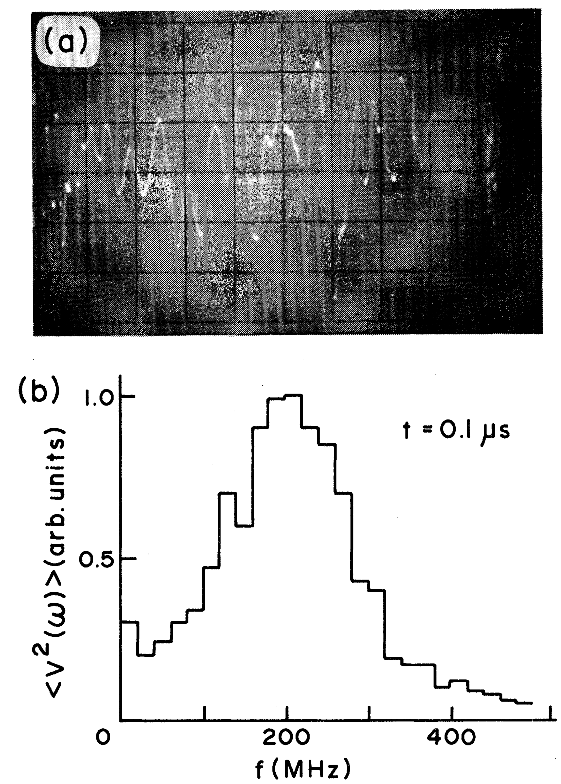

FIG. 2. (a) Example of observed fluctuations. Scale, 5 nsec/cm. (b) Power spectrum of the fluctuation averaged over eight oscillograms taken under the same conditions: $\bar{n}=2\times10^{17}$ m^{-3}, $V_c=8.5$ kV, $B=1.5$ kG.

Fluctuations in the plasma have been investigated by using two independent, movable probes (tungsten wires of 0.3 mm radius and 1.5 mm length). Figure 2(a) shows a sample oscillogram taken with one of the probes 0.1 μsec after the application of the electric field. The fluctuations grow rapidly ($\gamma\simeq10^8$ sec^{-1}) until $t\sim0.05$ μsec, and slowly thereafter. In Fig. 2(b), a power-spectrum analysis is shown. The dominant frequency component ($\simeq$ 200 MHz) is much higher than the ion plasma frequency ($\simeq$ 17 MHz) and the so-called Buneman frequency[8] ($\simeq$ 30 MHz), but much lower than the electron plasma frequency ($\simeq$ 4.5 GHz). It is also observed that (1) the waves propagate nearly parallel to the toroidal magnetic field in the direction of the electron drift velocity, and (2) the wave number $k_\parallel$ of the dominant modes satisfies $k_\parallel\simeq\omega_{pe}/u$.

Because of the high frequency ($\omega\gg\omega_{pi}$) of the instability, the plasma ions can hardly participate in it. Let us assume, as stated earlier, two electron groups, one trapped with zero average drift velocity and the other drifting. Such a double-humped distribution function is subject to electrostatic instabilities with frequency ω satisfying $\omega_{pi}\ll\omega<\omega_{pe}$. In the frame of the trapped electrons, the dispersion relation for one-dimensional electrostatic waves can be written as

$$2k^2=k_T{}^2Z'\,\omega/k\beta_T+2k^2\omega_{pe}{}^2/(\omega-ku)^2,$$

where k_T is the Debye wave number of the trapped electrons, $\beta_T=(2\kappa T_T/m)^{1/2}$ is their thermal velocity, and $Z'(\zeta)$ is the derivative of the plasma dispersion function.[9] We have neglected the thermal effects of the drifting electrons and assumed, for simplicity, that the trapped group has a Maxwellian distribution function. The adiabaticity requirement is $u^{-1}\,\partial u/\partial t\ll|\omega|$.

If $|\omega/k\beta_T|\gg1$ (cold trapped electrons), then for $\omega\equiv\omega_r+i\gamma$ the dispersion equation yields $\omega_r\simeq\gamma\simeq(n_T/n_0)^{1/3}\omega_{pe}$ at $k\simeq\omega_{pe}/u$, where n_T is the trapped electron density and n_0 is the density of drifting electrons. If $|\omega/k\beta_T|\ll1$, we obtain

$$\omega_r=ku-\omega_{pe}(1+k_T{}^2/k^2)^{-1/2},$$

$$\gamma=-\frac{\sqrt{\pi}\,\omega_r}{2k\beta_T}\frac{k_T{}^2}{k^2}\frac{(\omega_r-ku)^3}{\omega_{pe}{}^2}\quad(>0).$$

For both cases, the growth rates peak when the resonance condition ($k\simeq\omega_{pe}/u$) is satisfied, and the wave frequency ω_r is much less than the electron plasma frequency.

Figure 3 shows the time variations of the fluctuation, electron drift, and thermal energies. It can be seen that when the drift velocity exceeds the initial thermal velocity, a rapid increase in the fluctuation energy occurs, and the electrons no longer accelerate freely. The thermal energy starts increasing somewhat later but rapidly catches up with the drift energy and finally exceeds it.

The absolute magnitude of the fluctuation energy shown in Fig. 3 is subject to considerable uncertainty due to a poor knowledge of the probe impedance. An independent method of estimating the fluctuation level is to use the observed momentum-transfer collision frequency $\nu_{\rm eff}$, which is related to the fluctuation energy density through[10] $\nu_{\rm eff}\simeq2\tau^{-1}\epsilon_0|\vec{E}|^2/nmu^2$, where τ is the correlation time of the fluctuations. A typical value of τ (at $t\sim0.1$–0.2 μsec) is about 5 nsec, and the experimentally observed collision frequency $\nu_{\rm eff}=(eE-m\,du/dt)/mu$ is about 5×10^7 sec^{-1}. Thus, the actual fluctuation energy is estimated to be only about $\frac{1}{4}$ of the drift energy, and the fluctuation potential $V(\omega)$ determined with the probes is consequently too large by a factor of 2. The

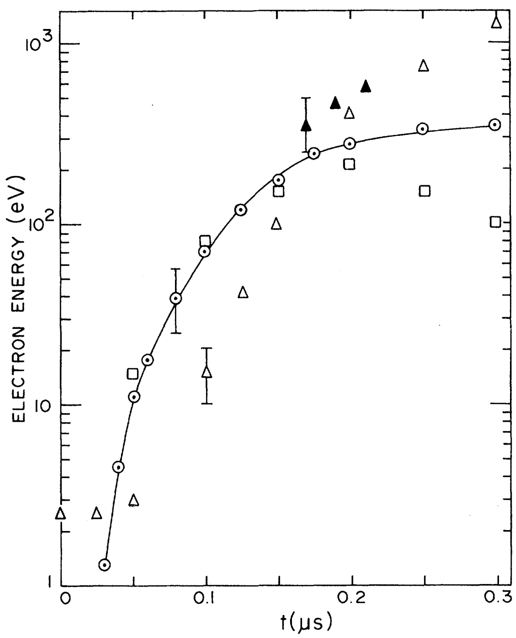

FIG. 3. Turbulence energy (squares), electron drift energy (circles) and one-dimensional thermal energy $\frac{1}{2}\kappa T_e$ (open triangles, from diamagnetic measurements; closed triangles, from orbit-analyzer probe) as functions of time. The fluctuation energy is calculated from $\epsilon_0 \sum_\omega k^2(\omega) V^2(\omega)/n$, taking $|\omega| \ll ku \simeq \omega_{pe}$ into account. $V(\omega)$ is the observed potential fluctuation.

short correlation time of the instability also explains the observed rapid electron heating.

Although electron trapping in the present experiment is attributed to magnetic field bumpiness, it could have other origins, for example, a small density inhomogeneity initially present in the plasma. Thus the instability may occur widely and could explain the early departure[11] from, or apparent absence[12] of, free acceleration in other toroidal experiments. It may also provide an initial boost in the level of turbulence, from which ion wave fluctuations can grow to noticeable levels surprisingly soon[13] after the application of an electric field.

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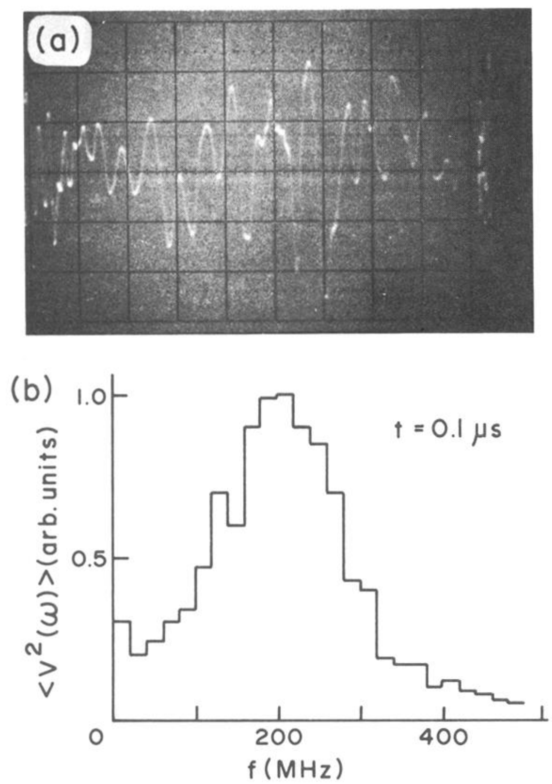

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