

Effect of Nuclear Mass on the Bound-Electron g Factor*

Frederick G. Walther, William D. Phillips,† and Daniel Kleppner

*Research Laboratory of Electronics and Department of Physics,
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*

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The effect of nuclear mass on the g factor of the bound electron has been determined by measuring $G = g_j(\text{H})/g_j(\text{D})$ to a precision of 3×10^{-11} using a pulsed double-mode hydrogen maser. The result, $G = 1 + (7.22 \pm 0.03) \times 10^{-9}$, is in excellent agreement with the theory of Grotch and Hegstrom.

The past several years have witnessed lively theoretical and experimental activity on the question of the interaction of atomic hydrogen with a magnetic field.¹ The problem is important not only because of the intrinsic interest in understanding the nature of electromagnetic interactions of composite systems, but because it bears on the validity of the Zeeman theory of hydrogen which plays an important role in understanding the hydrogen fine structure.

The magnetic moment of the bound electron provides the most sensitive test for the theory of hydrogenlike atoms in external fields. The following result has been derived by Grotch and Hegstrom² for the g factor of hydrogenic atoms in the 1S state:

$$g_j(1S) = g_e \left[1 - \frac{(Z\alpha)^2}{3} \left(1 - \frac{3}{2} \frac{m}{M} + \frac{3}{2} (1+Z) \frac{m^2}{M^2} \right) + \frac{\alpha}{4\pi} (Z\alpha)^2 \left(1 - \frac{5}{3} \frac{m}{M} + \frac{6+Z}{3} \frac{m^2}{M^2} \right) \right], \quad (1)$$

where g_e is the free-electron g factor and M is the nuclear mass. This result has been derived independently by Faustov³ and Close and Osborn⁴ using different calculational techniques. The leading correction term $(Z\alpha)^2/3$, the well-known Breit correction,⁵ has so far not been observed directly because of the technical problem of comparing free and bound electrons. Consequently, interest has centered on the mass-dependent terms which can be observed by measuring the isotope shift of the bound-electron g factor. For hydrogen and deuterium we have, with $G = g_j(\text{H}, 1S)/g_j(\text{D}, 1S)$,

$$(G - 1)_{\text{th}} = \frac{\alpha^2}{2} \frac{m_e}{M_p} \left(1 - \frac{M_p}{M_D} \right) - \alpha^2 \frac{m_e}{M_p} \left(1 - \frac{M_p}{M_D} \right) \left[\frac{m_e}{M_p} \left(1 + \frac{M_p}{M_D} \right) + \frac{5\alpha}{12\pi} \right] = 7.221 \times 10^{-9}. \quad (2)$$

The leading term contributes 7.247×10^{-9} , and the second term contributes -0.026×10^{-9} .

The first g -factor measurements with sufficient accuracy to resolve $G - 1$ from zero were reported in 1969 by two groups. Larson, Valberg, and Ramsey,⁶ using a hydrogen maser operating in a field of 7.6 G, reported $(G - 1)_{\text{LVR}} = (9.4 \pm 1.4) \times 10^{-9}$, while Hughes and Robinson,⁷ using optical pumping techniques to determine $g_j(\text{H})/g_j(\text{Rb})$ and $g_j(\text{D})/g_j(\text{Rb})$, obtained $(G - 1)_{\text{HR}} = (7.2 \pm 1.2) \times 10^{-9}$. The slight disagreement between these results is discomfiting, and in any case a more precise test of the theory is desirable.

We have determined G by a new technique which overcomes the most important experimental limitations of the previous work. Electron spin-flip transitions are observed simultaneously in hydrogen and deuterium using a pulsed double maser operating in a field of 3500 G. Both species are contained within a single-quartz storage bulb permitting an instantaneous determination of G , in contrast to earlier work in which transition fre-

quencies in each species were separately observed and then compared. Since G is independent of the magnetic field, effects of field fluctuations and drift, as well as a number of other systematic effects, are eliminated.

Main features of the apparatus, a high-field hydrogen maser, are described elsewhere.⁸ For this experiment H and D were mixed in the atomic-beam source, and a double-moded cavity was used. The H transition was $(\frac{1}{2}, -\frac{1}{2}) \rightarrow (-\frac{1}{2}, -\frac{1}{2})$ at 9.20 GHz, and the D transition was $(\frac{1}{2}, -1) \rightarrow (-\frac{1}{2}, -1)$ at 9.65 GHz, where the notation indicates (m_j, m_I) . The maser operated below oscillation threshold with free precession signals observed by applying $\frac{1}{2}\pi$ prestimulating pulses simultaneously to both species. Lifetimes were typically 4 msec, corresponding to a resonance linewidth of 80 Hz. The transition frequencies were determined by timing a preselected number of cycles of the two resonance signals, with care that the measurements were made simultaneously. G was determined from the two frequencies by inverting

TABLE I. Typical corrections for systematic errors.

| Systematic effect | Magnitude of correction ($\times 10^9$) | Uncertainty in correction ($\times 10^9$) |
|----------------------------|--|--|
| Instrumental | 0.050 | 0.005 |
| Inhomogeneity and shimming | 0.020 | 0.010 |
| Spin exchange | 0.010 | 0.003 |
| Second-order Doppler | 0.019 | < 0.001 |
| Wall shift | 0.010 | 0.003 |
| Cavity pulling | < 0.010 | < 0.003 |

the Breit-Rabi equation. The linewidths and signal-to-noise ratios were sufficient to obtain a statistical resolution of 1×10^{-11} in G with about 20 min of running time. Consequently, the accuracy of the experiment was limited by systematic effects, and the main burden of the work lay in their investigation and analysis.

The major source of systematic error was the shift in electron transition frequency associated with the motional averaging of the magnetic field inhomogeneities as the atoms moved randomly in the storage bulb. This effect, first pointed out by Brenner,⁹ led to shifts in the electron transition frequency of several parts in 10^{10} in previous work with our apparatus. The shift resulted from the slight difference in magnetic field between the hydrogen storage bulb proper and the collimated stem region arising from diamagnetic effects and residual field gradients. Detailed analysis of the effect indicated that the error in G should vary with the cube of the collimator length, and this was verified experimentally. The frequency shift was reduced by a factor of 10 by confining the atoms to the bulb proper with a short glass collimator, approximately 0.1 cm in length. We were able to correct for the residual shift by using different length collimators and extrapolating the data to zero collimator length. However, the field-shift corrections were no greater than 2×10^{-11} , and the uncertainty in the correction was estimated to be no more than 1×10^{-11} .

Other sources of systematic error are listed in Table I along with their estimated uncertainties. Although instrumental errors in the frequency-measuring circuits could shift G by 5×10^{-11} , these effects were consistent and easily identified; their correction introduced negligible error. None of the remaining corrections was over 2×10^{-11} , and the estimated uncertainties were small. The wall shift in the hyperfine frequency was calculated assuming a hydrogen phase shift per collision of $-5.5 \mu\text{rad}$ for FEP-120 Tef-

lon¹⁰ and $-13 \mu\text{rad}$ for fluorinated Dri-Film $[(\text{CF}_3\text{CH}_2\text{CH}_2)_2\text{SiCl}_2]$.¹¹ As indicated in Table I, the total correction for the wall shift is 1×10^{-11} . The associated uncertainty, 3×10^{-12} , is based on the range of reported phase-shift values. The spin-exchange correction is based on an exchange cross section of $25 \times 10^{-16} \text{ cm}^2$ and a shift parameter of 0.15.¹¹ An estimate of the shift in g_j due to wall collisions, based on the measured shift in solids,¹² indicates that the effect can be neglected.

Since errors in the important systematic effects were uncorrelated, we treat them as statistically independent when estimating the accuracy of a single run. Combining the estimated uncertainty with the statistical resolution gives an overall estimated uncertainty of about 1.5×10^{-11} for separate measurements of G . The auxiliary constants used in determining G are shown in Table II. Their uncertainty introduces negligible error.

The final results are based on three independent series of experiments using different storage bulbs and wall coatings. Each series included measurements obtained with several different length collimators to remove the inhomogeneity

TABLE II. Auxiliary constants.

| |
|--|
| $\Delta\nu(\text{H}) = 1\,420\,405\,751.768(2) \text{ Hz}^a$ |
| $\Delta\nu(\text{D}) = 327\,384\,352.51(5) \text{ Hz}^b$ |
| $-g_j(\text{H})/g_p(\text{H}) = 658.210\,706(6)^c$ |
| $-g_j(\text{D})/g_p(\text{D}) = 4287.842(2)^d$ |

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^bD. J. Larson, P. A. Valberg, and N. F. Ramsey, Ref. 6.

^cP. F. Winkler, D. Kleppner, T. Myint, and F. G. Walther, Ref. 8.

^dFrom data by N. F. Ramsey, *Molecular Beams* (Oxford Univ. Press, London, 1956) and value of $g_j(\text{H})/g_p(\text{H})$ in Ref. 8.

TABLE III. Experimental results.

| Series | (G - 1) ($\times 10^9$) | Standard deviation ($\times 10^9$) |
|---------------|------------------------------|---|
| (1) | 7.210 | 0.017 |
| (2) | 7.250 | 0.015 |
| (3) | 7.211 | 0.016 |
| Average | 7.224 | |
| rms deviation | 0.019 | |

shift. Measurements for each configuration were repeated for several nights to ensure consistency and reproducibility. The spread of results obtained for each configuration was consistent with the estimated error given above.

The results of the three series and their standard deviations are shown in Table III. The three determinations of G have an rms deviation of 1.9×10^{-11} in reasonable agreement with the estimated error for separate measurements.

On several occasions, the measurements showed aging effects leading to shifts in G as large as 1×10^{-10} . These anomalous results were correlated with saturation of the titanium-gettering pump and with poor collimator coatings. When such aging effects were detected, all the data for that configuration were rejected. Although these effects appear to be surface related, they are considerably greater than the normal wall shift for hydrogen accounted for in Table I. There is no evidence that any such surface effects were present in the final data. Nevertheless, the fact that some data had to be rejected raises questions as to whether the final results are entirely free from such effects. We estimate that any residual shifts larger than 2×10^{-11} would have been identified, and we combine this with the estimated uncertainty to give an overall uncertainty of 3×10^{-11} .

The final result is taken to be the mean of the

data shown in Table III,

$$G - 1 = (7.22 \pm 0.03) \times 10^{-9},$$

where the limit of error should be interpreted as our best estimate of a 60% confidence interval. This result provides strong confirmation of the leading term in Eq. (2), and lends some support to the higher-order corrections. Further details are given in a thesis by one of the authors,¹³ and will be presented elsewhere.

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