Determination of the Basic Parameters of the 3-3 Resonance

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The recent precise measurements of π-N scattering in the 3-3 energy range have been fitted by Rittenberg *et al.*, using four different phenomenological resonance formulas. These fits yield masses in the range 1234 to 1231 MeV, with widths ranging from 112 to 120 MeV. We show that these fits, as well as an ND^{-1} fit, all yield a nearly unique value for the position of the 3-3 pole in the S matrix at $1211-50i$ MeV.

There has always been considerable controversy about the precise definition of the position of a resonance. The recent extremely accurate determination of the π-N phase shifts in the 3-3 resonance region[1] have led to renewed discussion of the phenomenology of resonances.[2] In particular, Rittenberg *et al.* obtained four different four-parameter fits to these new data which shifted the position of the 3-3 mass from 1236 MeV to the range 1234–1231 MeV, depending on the particular formula used. The width was also a function of the resonance formula, varying between 120 and 112 MeV.

The analytic properties shared by all resonance formulas are that they have a branch cut which defines the sheets of the scattering amplitude and a nearby pole on the unphysical sheet whose location is the basic theoretical definition of the mass of the resonance. From an S-matrix–theory point of view, this pole is the definition of a resonance, and its location and residue determine all the physical parameters of this resonance. The differences between various resonance formulas are that they correspond to rather different distant singularities, physical properties which have nothing to do with the resonance. In this paper we examine the location of the resonance pole in various fits to the data. We find that all four fits with acceptable χ^2 given in Ref. 2, as well as a four-parameter ND^{-1} to the data of Carter *et al.*[1] of the type used by Ball, Garg, and Shaw,[3] yield essentially identical values for the pole position,

$$M_{33}=M-\tfrac{1}{2}i\Gamma=1211-50i\ \text{MeV}.$$

We would like to suggest that the 3-3 phase-shift data are now of sufficient accuracy to allow a direct analytic continuation of the scattering amplitude to determine the position of the resonance pole. If this is the case, then *any* resonance formula with reasonable analytic properties that fits the data can be used to determine the mass and width of the 3-3 resonance. It should be noted that these are not the parameters that enter into the formula but the location of the pole.

To investigate the effects of distant singularities, we have used the two-channel, four-parameter ND^{-1} model of Ref. 3 to fit the P_{33} phase shift. This provides an excellent fit to the data, comparable to those of Ref. 2 for the data of Carter *et al.*, but superior to them as can be seen in Fig. 1 for the higher-energy CERN phase shift.[4] This fit, we believe, has more physical analytic properties than the other resonance formulas, and the results seem to support this view. We then used our form to determine the position of the S-matrix pole and obtained

$$M_{33}=M-\tfrac{1}{2}i\Gamma=1211.1-50i\ \text{MeV}.$$

The continuation of the four fits of Ref. 2 agreed with the above result to within 0.5 MeV for both M and Γ.

We conclude that the apparent uncertainty in the 3-3 mass determination is not really present, and the data predict fairly accurately the position of the resonance pole independent of the particular resonance formula used. Hence one need not fear that the value of accurate experiments will

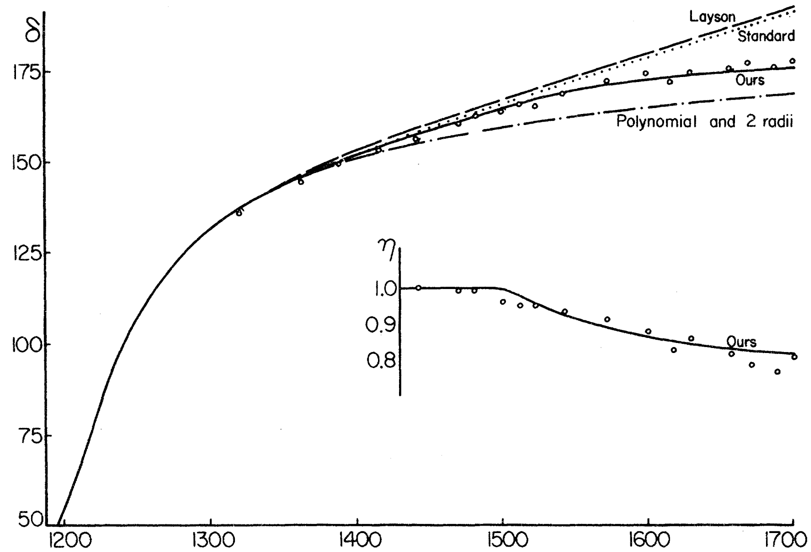

FIG. 1. Plots of δ for the πN, P_{33} phase shift δ and inelastic factor η versus center-of-mass energy W (in MeV). The curves marked "Layson" (with background), "Standard" (with background), and "Polynomial and 2 radii" are the good χ^2 fits (<1 per point) of Rittenberg *et al.* (Ref. 2 and T. Lasinski, private communication) to the fourteen data points of Carter *et al.* (Ref. 1). The curve marked "Ours" is the ND^{-1} fit (with comparable χ^2) to these data [with

$$N=\begin{pmatrix} 76.768 & 258.239 \\ 258.239 & 968.650 \end{pmatrix}(W-6.085)^{-1},$$

$l_1=l_2=1$, $\mu_2=4$, all with $\mu_1 \equiv m_\pi = 1$] using Eqs. (4)–(6) of Ref. 3. Within the scale used here, all five fits go through the points of Carter *et al.* for δ. The curves are extended to higher W and compared with the CERN "experimental" phase-shift solutions of δ and η, denoted by circles.

be negated by the uncertainties in resonance formulas. The need for more accurate measurement is now clear.

The parameters of a resonance also include the residue of the pole, being somewhat less significant than the pole position in that the latter is independent of channel. We have not done this, but we hope that a particle data group will attempt to determine all of these fundamental parameters for resonances whenever the data permit.

Finally, we note that the real part of M_{33}, 1211 MeV, is sufficiently different from the quoted value of 1236 to raise several important questions. For example, what mass should one use in the Gell-Mann–Okubo mass formula, or in a Lagrangian formalism, for the 3-3 resonance?

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[1]A. A. Carter, J. R. Williams, D. V. Bugg, P. J. Bussey, and D. R. Dance, Nucl. Phys. B26, 445 (1971).

[2]A. Rittenberg *et al.* (Particle Data Group), Rev. Mod. Phys. 43, 1 (1971).

[3]J. S. Ball, R. C. Garg, and G. L. Shaw, Phys. Rev. 177, 2258 (1969).

[4]C. Lovelace, CERN Report No. Th-837, 1967 (unpublished).