[8]J. D. Litster and T. W. Stinson, III, J. Appl. Phys. 41, 996 (1970).
[9]T. W. Stinson, III, and J. D. Litster, Phys. Rev. Lett. 25, 503 (1970).
[10]T. W. Stinson, J. D. Litster, and N. A. Clark, to be published.
[11]P. G. de Gennes, Phys. Lett. 30A, 454 (1969), and Mol. Cryst. Liquid Cryst. 12, 193 (1971).
[12]W. Maier and A. Saupe, Z. Naturforsch. A15, 287 (1960).
[13]L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Addison-Wesley, Reading, Mass., 1958), Chap. XIV.
[14]C. P. Fan and M. J. Stephen, Phys. Rev. Lett. 25, 500 (1970).
[15]See P. Debye, in *Non-Crystalline Solids*, edited by V. A. Frechette (Wiley, New York, 1960), pp. 1–25.
[16]C. C. Gravatt and G. W. Brady, Mol. Cryst. Liquid Cryst. 7, 355 (1969).

Nature of the Instability Caused by Electrons Trapped by an Electron Plasma Wave

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The sidebands which accompany a large-amplitude electron wave on a plasma column are shown to arise from selectively amplified noise; the amplification mechanism is a parametric coupling between the sideband frequencies and the oscillations of the electrons trapped in the large wave.

It is now well established[1-4] that a finite-amplitude, monochromatic electron wave (frequency ω_0, wave number k_0) propagating in a one-dimensional collisionless plasma may be accompanied by other frequencies ω which appear as one or more "sidebands" to ω_0. These fluctuations increase in intensity along the direction of propagation of the wave, causing it to be damped, and have frequency displacements from that of the original wave, $\Delta\omega \equiv |\omega_0 - \omega|$, which increase with the initial wave amplitude φ_0, and which, at least in the earliest observation,[1] obeyed the approximate relation $\Delta\omega \propto \varphi_0^{1/2}$.

The observation stimulated considerable theoretical work on the stability of plasma supporting a large-amplitude monochromatic wave, all of which invokes the existence of resonant electrons which, by virtue of their having velocity components parallel to the direction of propagation closely equal to the phase velocity of the wave $v_\varphi = \omega_0/k_0$, can become trapped in the electrostatic potential well of the wave. These oscillate in the well with a characteristic frequency $\omega_B = k_0 \times (e\varphi_0/m)^{1/2}$, where e and m are the electronic charge and mass, respectively, and can therefore couple to other oscillations in the system provided certain resonance conditions are satisfied. The trapped electrons also cause periodic amplitude variations of the original wave with wavelength[1] $2\pi v_\varphi/\omega_B$.

Because the original experiment[1] showed $\Delta\omega \simeq \omega_B$, this has tended to be an important criterion (as it turns out, misleading) for testing the various theories. In addition, the relative importance of upper and lower sidebands, or, indeed, the existence of the former, is in some dispute.

Detailed results of various theoretical approaches depend largely on the assumptions made. Manheimer[6] ascribed the unstable sidebands to propagating plasma waves (ω, k) resonant with the (Doppler shifted) oscillation frequency of the trapped electrons, leading to

$$\omega - kv_\varphi = \pm\,\omega_B\,, \tag{1}$$

where (ω, k) satisfies the dispersion relation $\epsilon(\omega, k) = 0$. This relation is implicit in the earlier work of Kruer, Dawson, and Sudan.[6] Mima and Nishikawa[7] assumed the trapped electrons oscillate in a parabolic potential well as in the work of Al'tshul and Karpman[8] and found also higher-order resonances, analogous to the energy levels of a harmonic oscillator,

$$\omega - kv_\varphi = \pm\,(2N+1)^{1/2}\omega_B\,, \tag{2}$$

where $N = 0, 1, 2\cdots$. Bud'ko, Karpman, and Shklyar[9] have recently extended the earlier treatment of O'Neil[10] assuming a sinusoidal potential well. The lowest-order sidebands than become

$$\omega - kv_\varphi \approx \pm\,0.9\omega_B\,. \tag{3}$$

One can deduce that higher-order resonances occur between integer multiples of ω_B. These last results[9] are valid if

$$e\varphi_0/k_BT < (v_e/v_\varphi)^2\,, \tag{4}$$

where $v_e^2 = 2k_B T/m$ (k_B is Boltzmann's constant).

Experimental results are presented here which show that the approach used in most of the above work [i.e., of the stability of a test wave (ω, k) such that $\epsilon(\omega, k) = 0$] does offer a good description of the effect, and that higher-order resonances, leading to several possible growing sidebands, do occur, in broad agreement with Eqs. (2) and (3), and that the sidebands are simply selectively amplified plasma noise.

Our experiments were conducted in a single-ended, thermally ionized Na plasma column, diameter 2.5 cm, temperature 2500°K, density $\sim 3\times 10^7$ cm^{-3} ($\omega_{pe}/2\pi \sim 30$–60 MHz) and uniform over its 80-cm length to within $\sim 1\%$, radially confined by a strong (~ 2 kG), uniform, axial magnetic field. The waves were excited using either fine wire probes or the cold end plate, and their amplitude φ measured using a high-input-impedance probe.[11] Allowing for uncertainties regarding the probe-plasma impedance and its frequency variation, the absolute accuracy of φ was estimated to be $\sim \pm 20\%$, corresponding to an uncertainty in $\omega_B \sim \pm 10\%$.

In general it was found that for waves which in the linear regime are lightly damped ($\omega \lesssim \omega_{pe}$) two lower sidebands and one upper appear, but for $\omega \gtrsim \omega_{pe}$ only the first lower sideband, with an appreciable frequency spread, is strongly excited [Fig. 1, trace (a)]. By directly recording the signals received from two probes with small but variable separation, the phase velocity of individual frequency components within one sideband was measured, confirming that they are electron waves on the same dispersion curve as (ω_0, k_0).

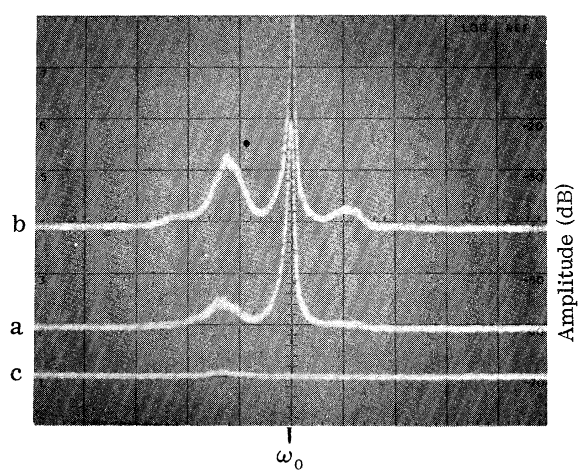

FIG. 1. Spectra measured 43 cm from launcher showing (a) spontaneous sidebands and (b) sidebands enhanced by (c) noise at 41.5 MHz. Injected wave $\omega_0/2\pi = 44$ MHz, $\omega_{pe}/2\pi = 43$ MHz.

By injecting into the plasma a narrow band of frequencies (derived from a filtered noise source) close to any one sideband, its amplitude could be enhanced. At the same time we found other sidebands enhanced, but to a lesser extent, enabling the frequencies of several to be located with greater accuracy. This latter effect suggests some form of parametric coupling between sidebands of different order. Figure 1, trace (b) demonstrates the effect of introducing the noise signal (c) centered on 41.5 MHz.

By accurately measuring the real part of the dispersion of small-amplitude waves[12] in the presence of (ω_0, k_0) (which causes small but significant changes from the linear dispersion particularly when $\omega \gtrsim \omega_{pe}$) the measured sideband frequencies were compared with the theoretical predictions. For example, Eq. (2) suggests that the quantities $\omega' = \omega - kv_\varphi$ for the $N = 1, 0$ lower sidebands and the $N = 0$ upper should be related in the ratio $\sqrt{3}:1:1$, respectively. For $e\varphi_0 \simeq 0.25 k_B T$, and a range of frequencies between 42 and 46 MHz ($\omega_{pe}/2\pi = 43.5$ MHz), the values of ω' were found to be in the ratio $(1.7\pm 0.1):1:(1.0 \pm 0.1)$ thus tending to support (2), which is to be expected since inequality (4) was not satisfied $[(v_e/v_\varphi)^2 \approx 0.06]$.

More quantitative data have been obtained for the first lower sideband ($N = 0$) by (a) varying φ_0 at constant ω_0 (i.e., constant v_φ), (b) varying ω_0 (i.e., v_φ) at constant φ_0. The experimental results are summarized in Fig. 2, where the frequency difference $\Delta\omega$ is plotted against the experimentally varied parameter. The solid lines represent the prediction of Eq. (2) for $N = 0$; the shaded area corresponds to the experimental uncertainties in both φ_0 and the plasma dispersion. The dashed line in each case is for $\Delta\omega = \omega_B$, which clearly offers a far poorer description of the data than Eq. (2).

Notice that the actual form of the variation $\Delta\omega(\omega_B)$ depends on the shape of the dispersion curve. It is simple to show that for $\Delta\omega \ll \omega_0$, $N = 0$, Eq. (2) becomes

$$\Delta\omega \simeq \omega_B (v_\varphi\, dk/d\omega - 1)^{-1}. \qquad (5)$$

If, as in the experimental condition of Ref. 1, $d\omega/dk \approx 0.5 v_\varphi$, then $\Delta\omega \approx \omega_B$.

The amplification process was demonstrated by introducing from a second launching antenna a small-amplitude, coherent test wave, propagating in the same direction as (ω_0, k_0) and whose

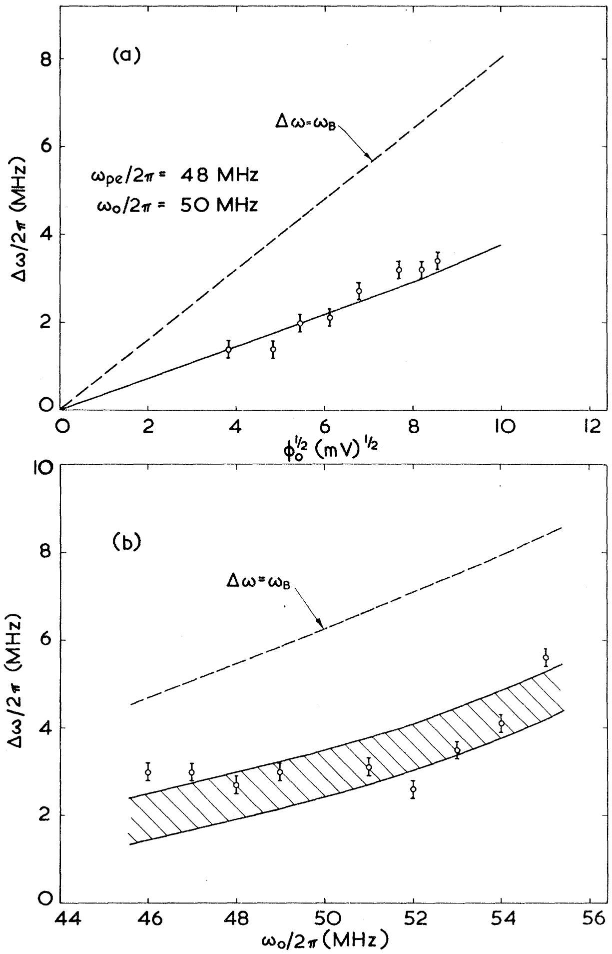

FIG. 2. Observed frequency difference $\Delta\omega$ for the first lower sideband 43 cm from launcher versus (a) wave amplitude φ_0, ω_0 constant; (b) wave frequency ω_0, φ_0 constant. The solid curve in (a) and the shaded area in (b) are theoretical predictions of Eq. (2) with $N=0$, and are based on the measured dispersion.

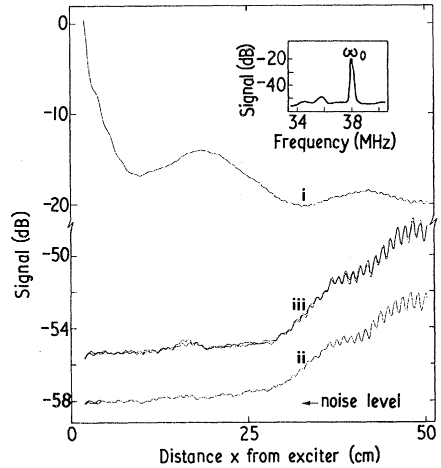

FIG. 3. Spatial variation of (i) input wave at 38 MHz undergoing periodic Landau damping and generating sidebands as shown in the inset spectrum taken at $x=50$ cm; (ii) a 1-kHz-wide band at the center of a spontaneously growing sideband (35.8 MHz); (iii) a test wave injected at $x=-17$ cm, with amplitude 3 dB above noise, at the same frequency as (ii). $\omega_{pe}/2\pi=36$ MHz, $\lambda_D=0.086$ cm, $k_0=3.3$ cm^{-1}.

frequency was varied. This wave grew along the direction of propagation of the large wave (ω_0, k_0) with a maximum growth rate when at the same frequency as a sideband maximum. Measured values of the amplification coefficient (for values of $\varphi_0 \lesssim 60$ mV) correspond typically to $k_i/k_r \sim (1-2)\times 10^{-2}$, and vary with φ_0 like $k_i/k_r \propto \omega_B \propto \varphi_0^{1/2}$. A similar proportionality has been found in computer simulation[3] for the *temporal* growth rate γ/ω_{pe}. Further, the process is clearly distinguishable from nonlinear growth arising from either decay instabilities[13,14] or nonlinear Landau damping,[15] for which the growth is proportional to φ_0^2. A test wave propagating in the opposite direction to (ω_0, k_0) did not grow.

Typical data for spatial amplitude variations are seen in Fig. 3: It shows (i) the periodic amplitude variation of the injected wave of amplitude sufficient to exhibit sidebands; (ii) the growth at the peak of the sideband, and (iii) two superimposed traces of a test wave at the same frequency as (ii) which clearly has the same growth as (ii). The value of $\omega_B/2\pi$ from the inset sideband spectrum and by using Eq. (1) is 3.7 ± 0.4 MHz; the measured bounce length gives 3.3 ± 0.3 MHz.

Further evidence that the sidebands originate from a linear amplification of initial noise was provided by deliberately increasing the plasma noise level (by connecting the test-wave launcher to a noise source) until it was just observable (in the absence of the large wave) on the spectrum analyzer. For sufficiently low noise levels, the sideband amplitudes increased proportionately to the rms noise level.

To demonstrate that the above effects are associated with trapped electrons, the amplitude of the sideband was measured in the presence of a second perturbing wave (ω_1, k_1) whose frequency was well removed from the unstable regions. The sideband amplitude (and hence the amplification) decreased with increasing amplitude of the second large wave. When both waves had comparable amplitudes, thus effectively displacing the time invariant potential well to a phase velocity[16] $(\omega_1-\omega_0)/(k_1-k_0)$ and thus destroying the necessary resonance, the sidebands completely disappeared. Simultaneously, the amplitude oscillations of (ω_0, k_0) decreased, and its attenuation approached the value for linear Landau damping. The sidebands also disappeared when the electron collision frequency was increased by adding neutral gas.

The effects described here, which will be described in greater detail elsewhere, can and usually do occur concurrently with other nonlinear effects which lead to enhanced decay, e.g., induced decay into ion waves[13] and into other electron modes.[14]

Finally, notice that these effects suggest that stable electron wave equilibria of the Bernstein, Greene, and Kruskal (BGK)[17] type cannot occur if the system can support other (electrostatic) waves which satisfy resonance conditions such as (2). Instability of BGK modes by decay into ion waves was noted by Sagdeev and Galeev.[18]

Results closely related to these have recently been obtained in Nagoya University[19] for ion waves propagating in a collisionless plasma with $T_e \sim 20T_i$.

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[1]C. B. Wharton, J. H. Malmberg, and T. M. O'Neil, Phys. Fluids 11, 1761 (1968).

[2]R. N. Franklin, G. J. Smith, S. M. Hamberger, and G. Lampis, in Proceedings of the Fourth European Conference on Fusion and Plasma Physics, Rome, Italy, 1970 (unpublished), p. 158.

[3]W. L. Kruer and J. M. Dawson, Phys. Fluids 13, 2747 (1970).

[4]J. Denavit and W. L. Kruer, Phys. Fluids 14, 1782 (1971).

[5]W. M. Manheimer, Phys. Rev. A 3, 1402 (1971).

[6]W. L. Kruer, J. M. Dawson, and R. N. Sudan, Phys. Rev. Lett. 23, 838 (1969).

[7]K. Mima and K. Nishikawa, J. Phys. Soc. Jap. 30, 1722 (1971).

[8]L. M. Al'tshul and V. I. Karpman, Zh. Eksp. Teor. Fiz. 49, 515 (1965) [Sov. Phys. JETP 22, 361 (1966)].

[9]N. I. Bud'ko, V. I. Karpman, and D. R. Shklyar, Zh. Eksp. Teor. Fiz. 61, 1463 (1971).

[10]T. M. O'Neil, Phys. Fluids 8, 2255 (1965).

[11]H. Ikezi, P. J. Barrett, R. White, and A. Y. Wong, Phys. Fluids 14, 1997 (1971).

[12]R. N. Franklin, S. M. Hamberger, G. Lampis, and G. J. Smith, in Proceedings of the Third International Conference on Quiescent Plasmas, Elsinore, Denmark, 1971 (to be published), p. 144.

[13]R. N. Franklin, S. M. Hamberger, G. Lampis, and G. J. Smith, Phys. Rev. Lett. 27, 1119 (1971).

[14]R. N. Franklin, S. M. Hamberger, G. Lampis, and G. J. Smith, Phys. Lett. 36A, 473 (1971).

[15]K. Gentle and A. Malein, Phys. Rev. Lett. 26, 625 (1971).

[16]E. Ott and C. T. Dum, Phys. Fluids 14, 959 (1971).

[17]I. B. Bernstein, J. Greene, and M. Kruskal, Phys. Rev. 108, 546 (1957).

[18]R. Z. Sagdeev and A. A. Galeev, *Nonlinear Plasma Theory* (Benjamin, New York, 1969), p. 43.

[19]H. Ikezi, Y. Kowamoto, K. Mima, and K. Nishikawa, in Proceedings of the Third International Conference on Quiescent Plasmas, Elsinore, Denmark, 1971 (to be published), p. 334.

Electric Potentials near a Superconducting-Normal Boundary*

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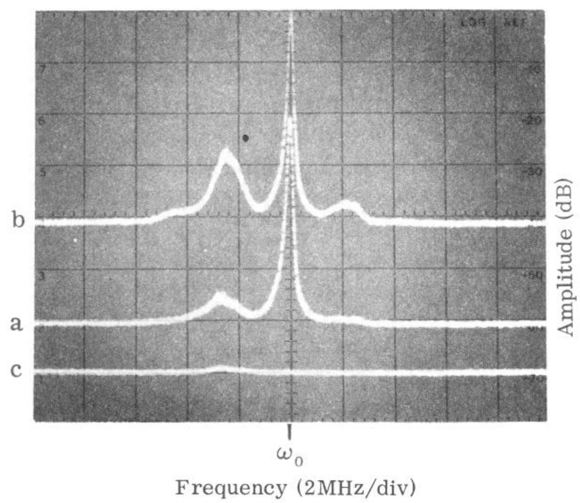
Measurements have been made within a few micrometers of a current-carrying superconducting-normal boundary, which indicate that an electric potential exists in the superconductor in this region. Near the transition temperature the experimental results are consistent in magnitude and temperature dependence with a theory recently proposed by Rieger, Scalapino, and Mercereau.

In a recent paper by Rieger, Scalapino, and Mercereau[1] a time-independent solution was proposed for a superconductor in a nonequilibrium state in which the chemical potentials of the paired electrons and the quasiparticles, μ_p and μ, respectively, are not equal to each other as they are in case of a superconductor in thermodynamic equilibrium. They noted that by defining the pair potential $2\mu_p$ as

$$2\mu_p = \frac{i\hbar}{2|\psi|^2}\left(\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi^*}{\partial t}\right)$$

and using a phenomenological time-dependent Ginzburg-Landau equation to determine $\partial\psi/\partial t$, it



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