$^{12}C(^{12}C, \alpha)^{20}Ne$: Direct Reaction with a Difference*

J. V. Noble†
Physics Department, University of Virginia, Charlottesville, Virginia 22901
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The simplest direct mechanism (pole diagram) for $^{12}C(^{12}C,\alpha)^{20}Ne$ has about as much gross structure as the data. Thus, the hypothesis of a compound or "doorway" mechanism is unnecessary.

Suppose we believe in the existence of nuclear levels resembling α particles moving about some core. Then we would expect to excite such states selectively by transferring the appropriate number of α clusters from nuclei in which they are loosely bound. Such reactions should favor direct mechanisms because the latter can take place at relatively large values of the classical impact parameter.[1] There is by now abundant evidence[2] for the direct excitation of states with a single excited α cluster. Recently Middleton, Garrett, and Fortune (MGF)[3] reported the selective excitation of states of ^{20}Ne, which look like two s-d shell α particles attached to a ^{12}C core, via the reaction $^{12}C(^{12}C,\alpha)^{20}Ne$. The existence of such states had previously been predicted on the basis of shell-model considerations by Arima, Gillet, and Ginocchio,[4] and their existence also follows from the α-cluster model of light $4n$ nuclides. The MGF data consist of angular distributions taken at several energies, corresponding to various excited states of ^{20}Ne, as well as small-angle excitation functions which exhibit pronounced and rapid variations with energy. Observing only the excitation functions, one might conclude that the reactions involve the formation of a succession of ^{24}Mg compound states with typical widths ~ 1–2 MeV. However, the angular distributions belie this interpretation since they appear characteristically direct, that is, they vary smoothly and are forward peaked. (We recall that the identical-boson initial state makes the differential cross sections necessarily symmetric about $\theta_{c.m.} = \pi/2$ so they are also backward peaked.)

The object of this note is to point out that even the simplest direct mechanism for the $(^{12}C,\alpha)$ reaction already represents a complicated amplitude which is capable of reproducing at least the gross structure (i.e., large variations every 2 MeV or so) of the excitation functions. A secondary purpose of this paper is to briefly review some of the reasons why various hypotheses put forward by MGF and others[5] involving elaborate "doorway structures," compound states which mysteriously appear in some channels but not others and which have remarkably large overlaps with rather complicated nuclei, or unspecified "semidirect mechanisms," are unlikely to be correct (except in the sense that the terms are used so vaguely that they could be construed to mean anything).

The lowest-order perturbation-theoretic amplitude for the reaction $^{12}C(^{12}C,\alpha)^{20}Ne$ may be represented by the diagrams of Fig. 1, where, as usual, the blobs (vertex functions) are the amplitudes for the virtual processes

$$^{12}C \rightarrow \alpha + {}^{8}Be(S) \tag{1a}$$

and

$$^{12}C + {}^{8}Be(S) \rightarrow {}^{20}Ne(f), \tag{1b}$$

and the region between the blobs stands for an energy denominator (propagator). The label S stands for all the internal quantum numbers of ^{8}Be which can be emitted, propagated, and absorbed in all internal states consistent with the usual conservation laws. Since we do not observe the ^{8}Be state, we must sum over all *amplitudes* in which ^{8}Be is transferred in all possible states. (Usually a few terms dominate this sum, however.[6]) The strong absorption in incident and emergent channels suppresses contributions from the low partial waves

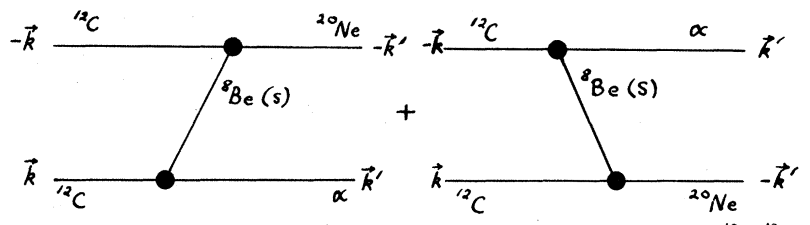

FIG. 1. Diagrammatic representation of the lowest-order contribution to $^{12}C(^{12}C,\alpha)^{20}Ne$.

(i.e., small impact parameters) and, while reducing the magnitude of the cross section, has little effect on the qualitative behavior induced by the structure of the vertex functions. It is just this characteristic of direct amplitudes which permits the relatively clean extraction from them of information about the vertex functions (i.e., wave functions). To see what can happen with the simplest assumptions, we consider the (plane-wave Born approximation) amplitude corresponding to the first term of Fig. 1:

$$A_{JM}^{(S)}(\vec{k},\vec{k}') = [-\textstyle\sum_{M_S}\langle v|\vec{k}'-\frac{1}{3}\vec{k};SM_S\rangle\langle\vec{k}-\frac{3}{5}\vec{k}';SM_S|u_M{}^J\rangle][(3\hbar^2/4m_\alpha)(\vec{k}'-\frac{1}{3}\vec{k})^2+B_S]^{-1}, \tag{2}$$

where B_S is the separation energy for the virtual Reaction (1a). In terms of the function (2), the entire amplitude may be written

$$A(\vec{k},\vec{k}') = \textstyle\sum_S[A_{JM}^{(S)}(\vec{k},\vec{k}')+A_{JM}^{(S)}(\vec{k},-\vec{k}')]. \tag{3}$$

The vertex amplitude describing process (1a) may be further decomposed so as to exhibit its spin and angle dependence explicitly:

$$\langle v|\vec{q};SM_S\rangle = v_S(q)Y_{SM_S}(\hat{q})\langle SM_S;S,-M_S|00\rangle, \tag{4a}$$

and similarly,

$$\langle u_M{}^J|\vec{Q};SM_S\rangle = \sum_{L,M_L} u_{SL}{}^J(Q)Y_{LM_L}(\hat{Q})\langle LM_L;S,-M_S|JM\rangle, \tag{4b}$$

where $v_S(q)$ and $u_{SL}{}^J(Q)$ have the forms

$$v_S(q) = (N/q)[q^2+(\alpha_S)^2]\textstyle\int_0^\infty dr\, rF_S(qr;\eta_q)\psi_S(r), \tag{5a}$$

$$u_{SL}{}^J(Q) = (N/Q)[Q^2+(\beta_{SL}{}^J)^2]\textstyle\int_0^\infty dr\, rF_L(Qr;\eta_Q)\varphi_{SL}{}^J(r). \tag{5b}$$

The constants $(\alpha_S)^2$ and $(\beta_{SL}{}^J)^2$ are related to the binding energies of the nuclei ^{12}C and ^{20}Ne(J,M), and the normalization constants N involve (among other things) the effective numbers of α particles in ^{12}C and ^{20}Ne, as well as fractional-parentage coefficients. The functions $\psi_S(r)$ and $\varphi_{SL}{}^J(r)$ are wave functions representing the relative motion of the clusters $\alpha+{}^8$Be, and ^{8}Be$+{}^{12}$C, respectively, and $F_L(\rho,\eta)$ is the Coulomb wave function which is regular at the origin.[7]

Now the behavior of the vertex functions $\langle v|$ and $|u\rangle$ is relatively independent of the details of the functions ψ_S and $\varphi_{SL}{}^J$, as long as the latter are "reasonable"; in particular, ψ and φ must vanish rapidly as $r\to 0$ (because of the exclusion principle), and exponentially as $r\to\infty$.[8] That is, they are strongly peaked at the surfaces $r=R,R'$ so that for reasonable values of q and Q,

$$v_S(q) \simeq [F_S(qR;\eta_q)/qR](q^2+\alpha_S{}^2)\,\text{const}, \tag{6a}$$

$$u_{SL}{}^J(Q) \simeq [F_L(QR';\eta_Q)/QR'][Q^2+(\beta_{SL}{}^J)^2]\,\text{const}. \tag{6b}$$

For the purpose of the ensuing rough calculation, I have taken $R = 1.5[(4)^{1/3}+(8)^{1/3}]$ fm $\simeq 5.4$ fm and $R' = 1.5[(8)^{1/3}+(12)^{1/3}]$ fm $\simeq 6.4$ fm. Let us now examine the simplest case, in which the ^{20}Ne is left in a 0^+ state. Then for a particular value of S the approximation (6) gives

$$A^{(S)} = \frac{N_i{}^{(S)}N_f{}^{(S)}}{RR'}\left[\frac{F_S(qR;0.74/q)F_S(QR';4.0/Q)}{qQ}(q^2+\alpha_S{}^2)P_S(\hat{q}\cdot\hat{Q})\right.$$
$$\left.+(-1)^S\frac{F_S(pR;0.74/p)F_S(PR';4.0/P)}{pP}(p^2+\alpha_S{}^2)P_S(\hat{p}\cdot\hat{P})\right], \tag{7}$$

where $P_S(\cos\theta)$ is the Legendre polynomial of order S, and $\vec{q}=\vec{k}'-\frac{1}{3}\vec{k}$, $\vec{p}=\vec{k}'+\frac{1}{3}\vec{k}$, $\vec{Q}=\vec{k}-\frac{3}{5}\vec{k}'$, and $\vec{P}=-\vec{k}-\frac{3}{5}\vec{k}'$. We do not know the magnitudes or signs of the products $N_i{}^{(S)}N_f{}^{(S)}$. Our eventual aim is to deduce them from the data in the manner described by Noble and Coelho,[6] for example. However, we do not yet know the coefficients $N_i{}^{(S)}$ (that is, the cluster parentage of ^{12}C) in sufficient detail to then extract from a fit of $N_i{}^{(S)}N_f{}^{(S)}$ new information about ^{20}Ne. Eventually such an analysis should be possible and will no doubt prove enlightening. For the purpose of this article, I have calculated unnormalized excitation functions at $\theta_{\text{c.m.}}=0$, for E_{lab} between 23 and 35 MeV, using the approximation (7), by assuming that the am-

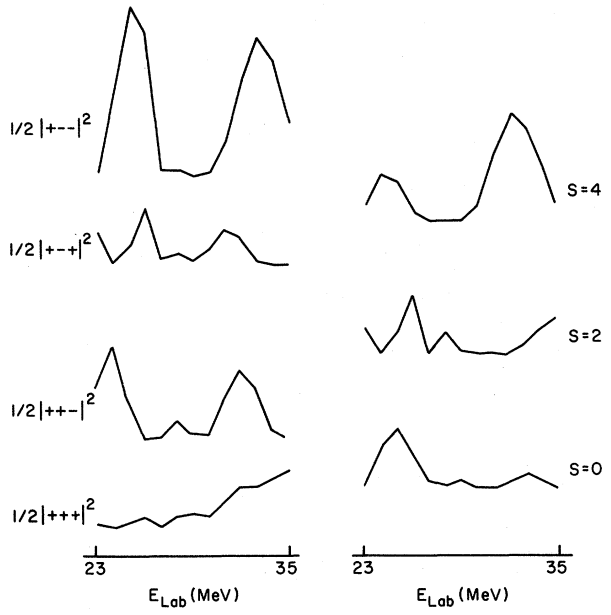

FIG. 2. Plot of the excitation function at $\theta = 0$ for mythical ^{20}Ne states which cause the basic amplitudes $A^{(S)}$ of Eq. (7), with $S=0$, 2, 4 to be combined with equal parentage coefficients and all permutations of relative sign. The squares of the amplitudes $A^{(S)}(\theta = 0)$ are also plotted on the same scale.

plitudes with $S = 0$, 2, and 4 are present with equal strength, and combining them with all possible relative signs. (This is not implausible because ^{12}C is known to have large and comparable components of $S = 0$, 2, and 4.[9]) The results are plotted, together with the squares of the individual amplitudes ($S = 0$, 2, 4) in Fig. 2. The large variations result from constructive and destructive interference between the various terms. Of course in actuality, when the direct amplitude is small, higher-order contributions[1] will interfere somewhat and fill in the minima, so that the actual peak-to-valley ratios will be less extreme and there may be more bumps. The form of Eq. (7) roughly indicates the angular dependence. Depending on the relative phases and magnitudes the amplitudes can oscillate rapidly or vary smoothly with $\cos\theta = \hat{k} \cdot \hat{k}'$. Since the $\cos\theta$ dependence of q, Q, p, and P is unlike their dependence on E, it is entirely possible to find amplitudes smooth in angle and wiggly in energy, or any other combination of behavior. We expect the strong absorption to modify the possible kinds of behavior only quantitatively but not qualitatively.

In conclusion I should like to emphasize two points. First, *any* reaction which proceeds through *compound* nuclear levels, whether "doorway states" or any other kind, will have similar excitation functions in all channels coupled to the compound system. Nothing of the sort appears to be evident in the MGF data. Rather, the excitation functions for each ^{20}Ne state all appear different, as MGF note. Semidirect processes (box diagrams) are not expected to be the dominant contribution to transfer reactions when the pole graphs are allowed as in this case.[1] Moreover, if the semidirect amplitudes *were* dominant, the interpretation of the results would be obscured since in that case, nonquartet states should be excited as easily as quartet states. There is only one second-order process which preserves the interpretation of the final state: inelastic excitation of the 2^+ (4.4-MeV) state of ^{12}C in ^{12}C-^{12}C scattering (expected to be very strong) followed by ^{8}Be transfer. But if this amplitude is important, then that of Fig. 1 must be more so, and we are back where we began.

The second point has already been alluded to above: If we wish to interpret the results of the MGF experiments in terms of simple nuclear states, then we must also regard the reaction mechanism as direct. The reason why even the simplest direct process appears so complicated as compared to (d,p), where we need consider only the $n+A \to B$ vertex function, or even compared to $(^7\mathrm{Li},t)$, where we need to consider both the $^7\mathrm{Li} \to \alpha + t$ and $\alpha + A \to B$ vertex functions, is that in the $(^{12}\mathrm{C},\alpha)$ reaction we transfer ^{8}Be in several states, and thus have to consider several vertex functions for both the projectile and residual nuclei. (Also, the vertex functions themselves are complicated.) There is unfortunately no way to evade this additional complexity, and we must therefore expect that the spectroscopic information we will ultimately extract from the $(^{12}\mathrm{C},\alpha)$ and similar reactions[5] will not be as quantitative as that derived from simpler processes.

Note added in proof.—Since this Letter was written, new data on the reaction $^{12}\mathrm{C}(^{13}\mathrm{C},\alpha)$ have been published by Middleton *et al.*[10] Since this reaction in effect labels one of the incident particles, it makes the direct character of many of the transitions more obvious than did the data on $^{12}\mathrm{C}(^{12}\mathrm{C},\alpha)$ alone.

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†Alfred P. Sloan Foundation Fellow.

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[7]See, e.g., A. Messiah, *Quantum Mechanics* (North-Holland, Amsterdam, 1961), Vol. I, p. 411 ff. Also please note that the use of Coulomb functions in the vertex functions (6) is an approximate method for including the Coulomb interaction in a few-body situation. The qualitative conclusions of the paper are insensitive to this treatment.

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Polarization of Neutrons in n-^{12}C Scattering: A Standard for Polarization Studies in the MeV Region*

R. J. Holt, F. W. K. Firk, R. Nath, and H. L. Schultz
Electron Accelerator Laboratory, Yale University, New Haven, Connecticut 06520
(Received 21 September 1971)

The polarization of neutrons elastically scattered from ^{12}C at 50° has been measured as a continuous function of energy between 2 and 5 MeV using a neutron double-scattering technique. This measurement provides an absolute standard for polarization studies in this energy range. The intense pulsed yield of neutrons obtained from the first scatterer is shown to be a practicable source of polarized neutrons suitable for high-resolution experiments in the MeV region.

Despite the importance of neutron-polarization studies in furthering our knowledge of nuclear-reaction mechanisms and of nuclear structure,[1,2] no measurements of the absolute polarization of neutrons have been reported as continuous functions of energy. In previous experiments, the neutron polarization has been measured relative to the analyzing powers of other nuclei. The most notable of these is ^{4}He in which the polarization has been deduced from phase-shift analyses of the scattering data.[3] Furthermore, in those experiments[4] in which ^{12}C has been used as a neutron polarization analyzer, the analyzing powers were taken from the differing sets of phase shifts of Meier, Scherrer, and Trampy,[5] Wills *et al.*,[6] or Reynolds *et al.*[1] Hence, the need arises for an absolute calibration standard for the ^{12}C analyzing power. In this Letter, we report the results of a neutron double-scattering[7,8] experiment in which the polarization of neutrons in the reaction ^{12}C$(n,n)^{12}$C is measured absolutely throughout the energy range 2 to 5 MeV. As a refinement, the spin-precession method,[9] which we have developed recently for use with a continuous energy spectrum of neutrons,[10] is used to reduce systematic errors to negligible amounts.

The present work exploits the prolific source of neutrons available at the Yale University electron linear accelerator when operating in a mode which produces pulses of 50-MeV electrons at a rate of 300 sec^{-1}. The pulse width is 20 nsec and the peak current at the the target is 7 A. Photoneutrons are generated in a 5-cm cube of natural lead and uranium. A fraction of these neutrons is scattered from a flat plate of graphite 15 cm long × 7.5 cm wide × 1.5 cm thick (equivalent to 0.2 of a neutron mean free path at 2 MeV) placed at a reaction angle of 50° (see Fig. 1). The angular resolution of this arrangement is ± 7°. The neutrons travel along a 27-m flight path and scatter from a second graphite plate (identical to the first) into two scintillation counters which are placed at angles of ± 50° with respect to the beam axis. The counters are located 0.4 m from the center of the scatterer giving angular resolutions of ± 7°. The time-of-flight spectra are stored in an on-line PDP-7 computer; the channel widths are 6 nsec. At 2 MeV the resolution of the spectrometer is 56 keV.

A solenoid, 1.2 m long and 7.5 cm i.d., is located one quarter of the way along the flight path. The maximum axial field is 4.5 kOe, which is sufficient to give precession of the spin of a 4-MeV neutron through 180°. Since neutrons with a wide range of energies are produced in the (γ,n) target, it is necessary to determine the preces-