suggests that the neutron distribution for ^{122}Sn is bigger than the proton distribution. Assuming this interpretation to be correct, the results yield a value for the neutron-proton rms radius difference of 0.22 ± 0.09 F. This is in good agreement with other estimates of this quantity for Sn isotopes[12] and provides additional evidence for the validity of the present procedures.

In conclusion it has been shown that a nucleon-α interaction obtained from an analysis of low-energy data can be used as the effective interaction for α-nucleus elastic scattering. The resulting fits of α-nucleus data, containing considerable structure, are much improved over those obtained with standard optical models and offer a possible method of obtaining information concerning nuclear-nucleon density distributions.

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Electron Gas in Superstrong Magnetic Fields: Wigner Transition

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It is suggested that in extreme magnetic fields ($H > 10^{12}$ G) a high-density electron gas (in a uniform positive background) undergoes a transition to an ordered structure. This corresponds to the "Wigner" transition of a low-density gas in the absence of a field. It is proposed that the ordered structure is a two-dimensional hexagonal lattice of "charged rods." The lattice spacing is evaluated for densities of astrophysical interest relevant to white dwarfs and pulsars.

In recent years there has been a great deal of interest in the properties of matter in the outer regions of gravitationally collapsed objects, i.e., white dwarfs and neutron stars. These stars are characterized by high density, and in many cases are associated with intense magnetic fields: 10^6 G for white dwarfs and up to 10^{14} G for neutron stars. A plausible model for this situation is that of a relativistic (neutralized) noninteracting electron gas.[1-4] In this note we wish to argue that heretofore neglected effects due to electron-electron interactions may be extremely important and, on the basis of simple considerations we propose a simple picture for the state of matter under these conditions. This view is similar to the picture recently proposed by Ruderman,[5] but is based on quite different considerations.

In the absence of a magnetic field the Hamiltonian for an electron gas has the form $\mathrm{KE}(\sim r_s^{-2}) + \mathrm{PE}(r_s^{-1})$, where r_s is the radius (in Bohr units) of the (spherical) volume per electron. Thus at high density the Coulomb interaction may be treated as a small perturbation on the free-particle behavior.

Our calculation was prompted by the theoretical result that in strong magnetic fields the relative strength of the kinetic energy and exchange (correlation) energy is modified. Indeed, in fields such that $\hbar\omega_c > \epsilon_F$, where ω_c is the cyclotron frequency and ϵ_F is the zero-field Fermi level, the ratio of the exchange energy to the free-electron energy behaves as[6]

$$E_{\mathrm{ex}}/E_{\mathrm{free}} \propto r_s(\hbar\omega_c/\epsilon_F)\ln(\hbar\omega_c/\epsilon_F), \tag{1}$$

i.e., the exchange energy dominates over the independent particle behavior. This is also the case for the correlation energy. We argue that this is to be expected for the following reason. When the Larmor radius r_H for an electron becomes smaller than r_s, the system is essentially a dilute electron gas normal to the magnetic field and should undergo a "Wigner" transition to an ordered state. This should resemble a two-dimensional lattice of charged rods with each rod behaving as a linear electron gas. The magnetic field H_c required for such a state is given by

$$r_H/a_0 = (\hbar c/eH_c)^{1/2} a_0^{-1} = r_s \tag{2}$$

or

$$H_c \cong 2.5\times 10^9 r_s^{-2}\ \mathrm{G}, \tag{3}$$

which is within the range of astrophysical interest ($r_s \sim 0.01-0.1$).

In the following we shall ignore relativistic effects and assume the system is at zero temperature. Unfortunately, we have no workable relativistic theory for interacting electrons so we cannot estimate the effect of the former approximation. Thermal effects should certainly play a role, but here we consider in essence only the infinite-field limit where they are not important. We now calculate the energy and equilibrium lattice spacing of a planar hexagonal lattice of linear electron gases in a uniform positively charged background. In the high-field limit the rods are essentially rigid, and the electron spins are all aligned parallel.

Let there be ν electrons on a line of length L and let $l_0 = L/\nu$. The Fermi energy per rod is

$$E_F = (\hbar^2/2m)(\pi/l_0)^2, \tag{4}$$

so that the total kinetic energy per rod is $E_k = (\hbar^2/6m)\pi^2 L/l_0^3$. Following Wigner, we replace the hexagonal prism (the unit cell in our system) by a cylinder of radius a having the same volume. We also neglect the interactions among the cylinders which we take to be electrically neutral. Thus l_0 and a are connected by the relation $\pi a^2 l_0 n_0 = 1$, where n_0 is the electron density. The kinetic energy per unit volume is

$$\epsilon_0 = \hbar^2\pi/6ml_0^3a^2. \tag{5}$$

To calculate the Coulomb interaction energy per line we adopt the following simple procedure. We assume the density is high enough that the Hartree-Fock approximation is sufficient:

$$\epsilon_1 = \epsilon_d + \epsilon_{\mathrm{ex}}, \tag{6}$$

$$\epsilon_d = e^2\sum_{k,k'}\iint \varphi_k^*(x)\varphi_k(x)\varphi_{k'}^*(x')\varphi_{k'}(x')\frac{dx\,dx'}{|x-x'|}, \quad \epsilon_{\mathrm{ex}} = -e^2\sum_{k,k'}\iint \varphi_k^*(x)\varphi_k(x')\varphi_{k'}^*(x)\varphi_{k'}(x')\frac{dx\,dx'}{|x-x'|}.$$

Next we assume that the φ_k are plane waves and introduce a lower-limit cutoff ϵ on the x integrations to handle the Coulomb singularity. The singular terms in ϵ will cancel and we shall eventually take $\epsilon \to 0$. Without too much difficulty we find

$$\epsilon_d = \frac{2e^2}{\pi a^2 l_0^2}\left[\ln\left(\frac{L}{\epsilon}\right) + \frac{\epsilon}{L} - 1\right], \quad \epsilon_{\mathrm{ex}} = -\frac{e^2}{\pi a^2 l_0^2}\left[\frac{\sin^2\eta}{\eta^2} + \frac{\sin^2\eta}{\eta} - 2\,\mathrm{Ci}(2\eta)\right], \quad \eta = \frac{\pi\epsilon}{l_0}. \tag{7}$$

Next we obtain the Coulomb energy of interaction between the electrons and the uniform background as

$$\epsilon_C = -e^2 l_0 n_0 \iint \frac{d^3r\,dz'}{[\rho^2 + (z-z')^2]^{1/2}}, \tag{8}$$

where ρ and z are cylindrical coordinates and the integration extends over the cylinder. We find

$$\epsilon_C = -\frac{2e^2}{\pi a^2 l_0^2}\left[\ln\left(\frac{2L}{a}\right) - \frac{1}{2}\right]. \tag{9}$$

Finally, the zero-point energy for the motion of the rod about the axis of the cylinder is[7]

$$\epsilon_z = \hbar e/\pi a^3 (2ml_0^{\,3})^{1/2}. \tag{10}$$

We introduce dimensionless parameters $n_0 = \rho/a_0^{\,3}$, $a = \alpha a_0$. (Note that $\rho = 3/4\pi r_s^{\,3}$.) Then the total energy in Rydbergs per cubic Bohr unit is

$$E = \tfrac{1}{3}\pi^5\alpha^4\rho^3 + (2\pi\rho^3)^{1/2} + 4\pi\alpha^2\rho^2[\ln(\alpha^3\rho) + C], \quad C = \ln(\pi^2) + \gamma - 2 \cong +0.867, \tag{11a}$$

or

$$E = \frac{9\pi^2}{64}\alpha^4 r_s^{\,-9} + \left(\frac{27}{32\pi^2}\right)^{1/2} r_s^{\,-9/2} + \frac{27}{4\pi}\alpha^2 r_s^{\,-6}\left[\ln\left(\frac{\alpha}{r_s}\right) + C'\right], \quad C' \cong -0.188. \tag{11b}$$

To determine α, we minimize E with respect to α. The results are shown in Fig. 1. At this minimum, α_0, the value of E is

$$E_0 = -\frac{9\pi^2}{64}\frac{\alpha_0^{\,4}}{r_s^{\,9}} - \frac{27}{8\pi}\frac{\alpha_0^{\,2}}{r_s^{\,6}} + \left(\frac{27}{3\pi^2 r_s^{\,9}}\right)^{1/2}. \tag{12}$$

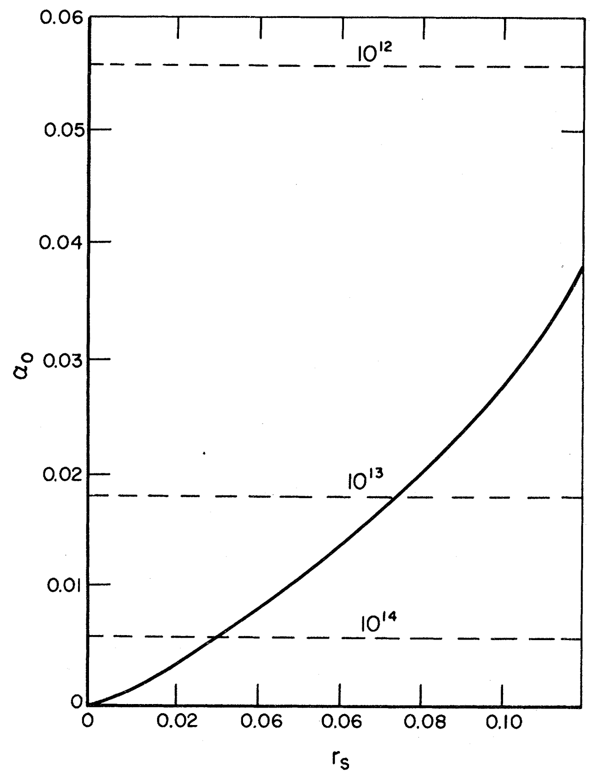

FIG. 1. Lattice spacing versus r_s. Dashed lines, values of r_H in the fields indicated (in Gauss).

As can be seen from Fig. 1, the condition for the validity of our model, i.e., $\alpha < r_H$, is satisfied for $H = 10^{13}$ G by $r_s > 0.07$ and for $H \simeq 10^{14}$ G by $r_s > 0.025$.

In conclusion we note that our model includes the effects of electron-electron interactions neglected in Ref. 5, but presumes the nuclear charge to be represented as a uniform positive background, whereas in Ref. 5 the positive charge is located more realistically as "pearls on a string." A more satisfactory theory would have to amalgamate the two points of view. We intend in the future to calculate the energy of the homogeneous electron gas with a smoothed-out positive charge as a function of applied magnetic field in order to make a comparison with the results described in this note.[6] We also intend to study the effect of a nonuniform positive background such as proposed by Ruderman.

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