# Effective Interaction in $\alpha$-Nucleus Scattering*

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Elastic $\alpha$-nucleus differential cross-section data (15°–175°) are analyzed using a potential model. The real potential is calculated using an effective potential obtained from a global analysis of nucleon-$\alpha$ data. The resulting four-parameter model given an excellent representation of the $\alpha$-nucleus data. This representation is significantly better than can be obtained with a six-parameter optical model and is sensitive to the nuclear-nucleon density distribution.

Several studies have been reported in which an effective interaction is used to describe elastic $\alpha$-nucleus scattering data. In some cases a nucleon-nucleon effective force is used[1] and in others a nucleon-$\alpha$ force.[2] The data analyzed have often covered only a limited angular range (<60°), and the parameters of the effective interaction have been adjusted for an optimum fit. Little attention has been paid to how well the interaction represents nucleon-$\alpha$ data. Batty, Friedman, and Jackson[3] examined this latter point and showed that, in general, the interactions had only limited success in reproducing the nucleon-$\alpha$ data. Where large-angle $\alpha$-nucleus data have been considered, difficulty has been experienced in simultaneously fitting both the forward- and backward-angle regions.[4] The present work uses an effective interaction, obtained from a global analysis of nucleon-$\alpha$ data, to fit elastic $\alpha$-nucleus data over a wide angular range (15°–175°) and succeeds in obtaining an excellent fit to the data at all angles. This fit is considerably better than can be obtained with a standard optical model despite the use of fewer parameters.

A 28-MeV $\alpha$-particle beam from the Minnesota MP tandem accelerator was used to obtain elastic differential cross sections from a number of nuclei. The range 15°–175° was covered in 2.5° steps with a detector angular acceptance of ±0.25°. The uncertainty in the differential cross sections was about ±2% forward of 90°; at backward angles the uncertainty was about ±5% except for points near the positions of deep minima. A typical set of data points, obtained with a $^{65}$Cu target, is included in Fig. 1.

The data were analyzed initially using a six-parameter optical model with independent real and imaginary Woods-Saxon potentials. This six-parameter space was explored extensively, resulting in several (six–eight) equivalent parameter sets for each nucleus, each of which gave an equally good fit to the data. Wide variations were present among individual parameter values for these equivalent sets. However, irrespective of the particular parameter values, the tail regions of the potentials were similar. This was particularly true of the tails of the equivalent real potentials which, on a logarithmic plot, were almost parallel and intersected at a point approximately 4 F beyond the nuclear-nucleon half-density point. This suggests that this tail region of the real potential is well determined by the data,[5] irrespective of other details of the representation and hence can be considered independently of such details. The predictions given by one of the equivalent parameter sets yielding an optimum fit to $^{65}$Cu$(\alpha, \alpha)^{65}$Cu data are included in Fig. 1, where it is seen that beyond about 120° the fit is relatively poor.

Since the important region of the real central potential corresponds to a point of low nuclear-nucleon density, it seems reasonable to attempt to construct the potential from a folding of a free-nucleon–$\alpha$ interaction with the nuclear-nucleon density distribution. The $\alpha$ energy, at the crossover point of the equivalent real central potentials, mentioned above, corresponds to a laboratory nucleon energy, in a nucleon-$\alpha$ interaction, of 4–5 MeV. The nucleon-$\alpha$ interaction was therefore determined from an analysis of data spanning this energy. Neutron-$^4$He data[6] up to 10 MeV and proton-$^4$He data[7] up to 12 MeV were analyzed using the "global" program BOM.[8] This program simultaneously analyzes all the data sets considered to yield a best overall parameter set.[9] Only real potentials were used since the energies considered are below the reaction threshold. The best fit was obtained for a real central potential of Woods-Saxon form with $V_R = 42.5$ MeV, $R_R = (1.43 - 0.0009E)A^{1/3}$ F and $a_R = 0.34$ F (with $E$ the lab nucleon energy), and for a Thomas-form real spin-orbit potential with $V_{s.o.} = (2.496 \pm 0.256E)$ MeV (neutrons), $V_{s.o.} = (3.864 \pm 0.032E)$ MeV (protons), $R_{s.o.} = 1.117A^{1/3}$

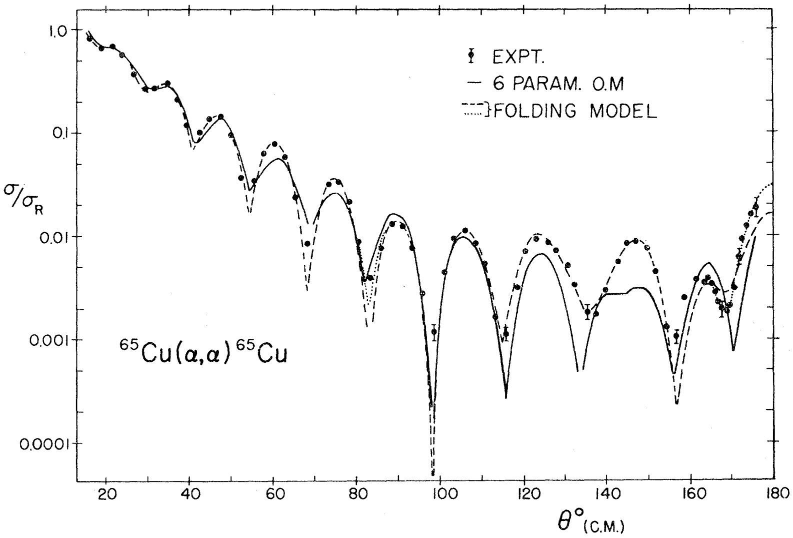


FIG. 1. Angular distribution of 27.18-MeV $\alpha$ particles elastically scattered by $^{65}$Cu. Full line ($\chi^2$ per point = 70), representative of the best fit that can be achieved with a six-parameter optical model. Dashed curve ($\chi^2 = 32$), obtained with the folding procedures, using measured nuclear proton parameters with $r = 1.032$ F, $a = 0.520$ F, $\langle r^2\rangle^{1/2} = 3.76 \pm 0.08$ F. The dotted curve ($\chi^2 = 17$) follows the dashed curve, except where shown, and was obtained with $r = 1.083$ F, $a = 0.473$ F, $\langle r^2\rangle^{1/2} = 3.796$ F.

F and $a_{\text{s.o.}} = 0.34$ F. These potentials gave an excellent representation of all the nucleon-$\alpha$ data.

The radial form for the real potential used in the analysis of the $\alpha$-nucleus data was obtained by folding the central part of this nucleon-$\alpha$ potential with the appropriate nuclear density distribution. The energy $E$ was chosen to correspond to the crossover point of the equivalent potentials found using the six-parameter optical model. The strength of the $\alpha$-nucleus real potential was treated as an adjustable parameter ($V_R$).[10] For medium-weight nuclei the nucleon density distribution was assumed to be equal to the proton density distribution, which was derived from the results of Acker *et al.*[11] and had a Woods-Saxon shape. To this real potential was added a three-parameter ($W_I$, $r_I$, $a_I$) imaginary term, also of Woods-Saxon shape. This resulted in four parameters ($V_R$, $W_I$, $r_I$, and $a_I$) which were adjusted to obtain optimum fits.

As expected, the tail of the folded potential agrees closely with that previously determined using the six-parameter optical-model analysis described earlier. However, the actual fit to the $^{65}$Cu($\alpha$, $\alpha$)$^{65}$Cu data, using this folded shape, is shown in Fig. 1, where it is seen that a significant improvement has been achieved relative to the six-parameter model.

The effect of allowing the nuclear-nucleon density distribution parameters ($r_m$, $a_m$) to depart from the proton values was investigated; this resulted in a further improvement in the representation of the data as can be seen in Fig. 1. The radius parameter was changed from 1.032 to 1.083 F and the diffuseness parameter $a_m$ from 0.520 to 0.473 F.

Experiments such as those of Acker *et al.*, which were used to obtain the proton density parameters, best determine the rms radius of the distribution which, in the case of $^{65}$Cu, was estimated to be accurate to $\pm 2\%$. The change in rms radius, associated with the improvement discussed above, is $+1.0\%$ which is within the error limits of the proton measurement. This suggests that analyses of the present type may be useful in determining the shape of the nuclear-nucleon distribution.

A similar analysis has been performed for $^{122}$Sn($\alpha$, $\alpha$) data, among others. The $^{122}$Sn data show much less structure than the $^{65}$Cu data at 28 MeV. In this case the proton rms radius was $4.581 \pm 0.03$ F with $r = 1.081$ F and $a = 0.52$ F.[11] Using these values for $r_m$ and $a_m$ and the above folding procedures, the $\chi^2$ value obtained was about 3 times greater then could be obtained with the standard six-parameter optical model. However, an improved fit over the standard model was achieved with radius and diffuseness values of 1.108 and 0.535 F corresponding to an rms radius about 3% greater than the proton value. This suggests that the neutron distribution for $^{122}$Sn is bigger than the proton distribution. Assuming this interpretation to be correct, the results yield a value for the neutron-proton rms radius difference of $0.22 \pm 0.09$ F. This is in good agreement with other estimates of this quantity for Sn isotopes[12] and provides additional evidence for the validity of the present procedures.

In conclusion it has been shown that a nucleon-$\alpha$ interaction obtained from an analysis of low-energy data can be used as the effective interaction for $\alpha$-nucleus elastic scattering. The resulting fits of $\alpha$-nucleus data, containing considerable structure, are much improved over those obtained with standard optical models and offer a possible method of obtaining information concerning nuclear-nucleon density distributions.

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# Electron Gas in Superstrong Magnetic Fields: Wigner Transition

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It is suggested that in extreme magnetic fields ($H > 10^{12}$ G) a high-density electron gas (in a uniform positive background) undergoes a transition to an ordered structure. This corresponds to the "Wigner" transition of a low-density gas in the absence of a field. It is proposed that the ordered structure is a two-dimensional hexagonal lattice of "charged rods." The lattice spacing is evaluated for densities of astrophysical interest relevant to white dwarfs and pulsars.

In recent years there has been a great deal of interest in the properties of matter in the outer regions of gravitationally collapsed objects, i.e., white dwarfs and neutron stars. These stars are characterized by high density, and in many cases are associated with intense magnetic fields: $10^6$ G for white dwarfs and up to $10^{14}$ G for neutron stars. A plausible model for this situation is that of a relativistic (neutralized) noninteracting electron gas.[1-4] In this note we wish to argue that heretofore neglected effects due to electron-electron interactions may be extremely important and, on the basis of simple considerations we propose a simple picture for the state of matter under these conditions. This view is similar to the picture recently proposed by Ruderman,[5] but is based on quite different considerations.

In the absence of a magnetic field the Hamilton-