Evidence for Rapidly Rising p-p Total Cross Section from Cosmic-Ray Data

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We give estimates of lower bounds on the p-p total cross section based on cosmic-ray data. We find that σ_{pp} rises at least as rapidly as $38.8+0.4\ln^2(s/s_0)$ from $E\sim 10^3$ to 3×10^4 GeV incident energy, corresponding to $\sigma_{pp}\geqslant 60$ mb at the highest energy available. In view of large experimental errors at the highest energies, a value of $\sigma_{pp}\geqslant 48$ mb can be estimated at 95% confidence level at $E\sim 10^4$ GeV.

Total particle-proton cross sections are of immense theoretical interest. Much of the excitement about the early Serphukov data arose because of the unexpected behavior of these cross sections, and a variety of models and explanations for the behavior of the cross sections have been advanced and await experimental test.[1] Unfortunately, all of these models seem to suggest that the dependence of the total cross sections on energy will be logarithmic, so that it becomes difficult to test them at accelerator energies.

One way of overcoming this problem is to go to cosmic-ray energies, where even logarithmic variations become appreciable. The difficulty with using high-energy cosmic rays as the incident beam for p-p interactions has always been that it is very hard to measure anything except the inelastic cross section because of the extreme experimental difficulty of detecting small-angle elastic scattering. In this note, we point out that this difficulty can be overcome by using known theoretical techniques to extract the p-p total cross section from experimental data on proton-air *inelastic* cross sections. Since data of this type already exist up to 3×10^4-GeV incident proton energies, we are able to make statements about the p-p total cross sections at these energies. We find that the most reasonable value of σ_{pp} up to that energy is

$$\sigma_{pp}=38.8+0.4\ln^2(s/s_0),$$

where the scale factor s_0 is the value of the square of total center-of-mass energy, s, corresponding to an incident energy of 70 GeV:

$$\sigma_{pp}\sim 60 \text{ mb at } 10^5 \text{ GeV}.$$

In other words, this value is extracted by (1) analyzing existing cosmic-ray data to derive lower bounds to proton-air cross sections, and then (2) interpreting these cross sections in terms of a nuclear model to estimate σ_{pp}.

It is well known that the interactions of particles and nuclei at high energies are described by the multiple scattering theory of Glauber.[2] We shall see later that it is necessary to be able to calculate only the small-angle proton-nucleus elastic scattering amplitude, so that the nuclei in air can be described by a simple Gaussian ground-state density, for which the elastic amplitude F_{el} is well known.[3] From this amplitude, we can get the total p-nucleus cross section from the optical theorem,

$$\sigma_T(p\text{-air})=(4\pi/P)\operatorname{Im}[F_{el}(0)],$$

and the elastic cross section from

$$\sigma_{el}(p\text{-air})=\int|F_{el}(\Delta)|^2d\Delta,$$

where Δ is the momentum transfer and P is the incident momentum.

Thus, from a knowledge of the elastic particle-nucleus scattering amplitude, the inelastic particle-nucleus cross section (the quantity usually measured in cosmic-ray experiments) can be calculated. It is well known[3] that F_{el} depends strongly on σ_{pp} (essentially because σ_{pp} determines the mean free path of hadrons in nuclear matter), and only weakly on other parameters describing the elementary p-p interaction, provided that the ratio of real to imaginary parts, α, is small. In what follows, we take $\alpha\lesssim 0.1$, so that we do not consider the possibility of $\alpha\to\infty$, which some theories (those which violate the Pomeranchuk theorem) would predict.[4]

As a check that this method of calculating $\sigma_{in}(p\text{-air})$ is valid, we can compare the predicted values of $\sigma_{in}(p\text{-air})$ with the measured values at 20 GeV,[5] where $\sigma_{pp}=40$ mb. The predicted values for ^{9}Be, ^{12}C, and ^{27}Al are 224, 250, and 449 mb, respectively, while the measured values are 227 ± 6, 254 ± 9, and 472 ± 20 mb. This good agreement gives us some confidence that we can use our theory to describe the interactions of cosmic-ray protons, with air, since we can calculate the inelastic cross section for protons on nitrogen and oxygen nuclei.

The idea of obtaining interaction mean free paths from measurements of unaccompanied hadron spectra has been discussed before,[6] and has been carried out in a recent paper by Yodh *et al.*[7] We briefly outline the method to emphasize several important features.

During the last ten years there have been several cosmic-ray experiments to measure the energy spectra of "unaccompanied" hadrons at different depths in the atmosphere. A comparison of these fluxes with the flux of cosmic-ray particles at the top of the atmosphere allows one to examine the behavior of proton-air inelastic cross sections up to 30 000 GeV. In particular, we can deduce lower bounds for the proton-air inelastic cross sections.

Experiments which measure unaccompanied hadron spectra usually consist of a total ionization spectrometer (or calorimeter) to detect the hadron as well as determine its energy, and an extensive air-shower array to establish accompaniment of the hadron. The calorimeter (usually ~3 to 7 interaction mean free paths deep) determines the energy of the hadron to an accuracy which can vary from $\pm100\%$ to $\pm15\%$. The shower anticoincidence array, which is located close to the calorimeter, will not detect showers accompanying the hadron if they have a size less than N_{min} (typically $N_{min}\sim100$ to 500 shower particles). Therefore, the meaning of the adjective "unaccompanied" varies from experiment to experiment. However, the measured "unaccompanied" hadron flux does provide an upper limit to the surviving proton flux. Furthermore, the efficiency of the shower array to detect accompanying showers increases as the energy of the hadron increases and, consequently, at sufficiently high energies this upper limit may approach the true value of the surviving proton flux. We utilize data obtained in experiments done at atmospheric depths of 1000,[8] 730,[9] 698,[10] and 550[11] g/cm^2, respectively.

There have been many measurements of the primary cosmic-ray spectra.[12] Most of these spectra agree very well on the energy dependence and intensity of the spectrum from 10 to 10^4 GeV/c. For protons, Grigorov *et al.*[13] measured a steepening of the spectrum above 1500 GeV. Up to about 3000 GeV, this has not been seen in measurements of Ryan *et al.*[14] For the incoming spectrum of protons we take, therefore, the "conventional" integral spectrum $f(>E, x=0)=7300E^{-1.7}$ particles/m^2 sec sr where E is in GeV (Fig. 1).

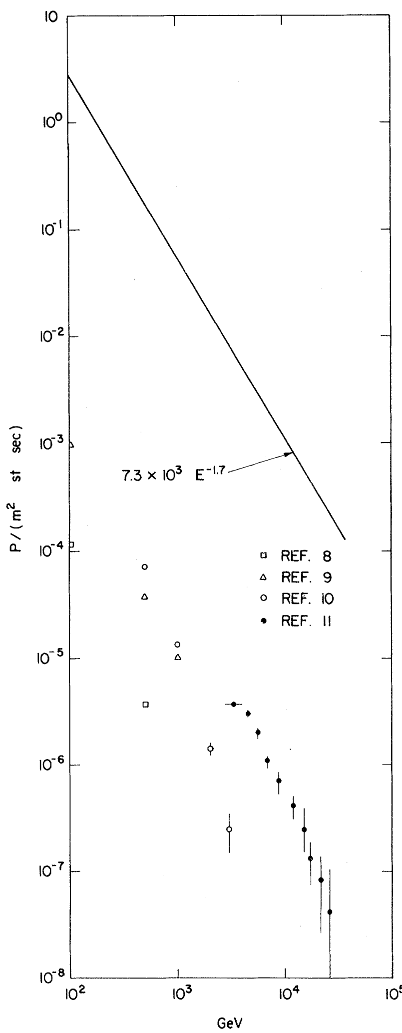

FIG. 1. Integral hadron spectra $f(>E,x)$. The upper points are at $x=0$, and correspond to the primary spectrum of protons. The lower points correspond to unaccompanied hadron spectra as measured at different depths in the atmosphere (see text). The difference between the upper and lower curves is a measure of the absorption of protons in air.

We define an effective inelastic cross section for protons in air by the relations

$$f(>E, x) = \exp(-x\sigma_{\mathrm{eff}} N) f(>E, x=0),$$

where N is the number of air nuclei per gram. As discussed above, the flux of surviving protons *must be less than* or *equal to* the measured flux of unaccompanied protons $f(>E, x)$ at depth x, which implies that

$$\sigma_{\mathrm{in}}^{p\text{-air}}(>E) \geqslant \sigma_{\mathrm{eff}}(>E).$$

The lower bound extracted in this manner is shown in Fig. 2. [It is important to emphasize that fractional error in the cross section depends only on the logarithm of the uncertainty in energy and a systematic error of 200% in absolute energy determination will make only a 14% correction to the value of $\sigma_{\mathrm{eff}}(>E)$.]

Since it is σ_{eff}, and not σ_{in} which is actually measured, we must be able to calculate the inelastic p-nucleus cross section as a function of energy in order to confront experiment. This means, in turn, that we shall have to take some model which gives σ_{pp} as a function of energy (the nuclear parameters are, of course, independent of energy). We shall consider several currently fashionable models.[1]

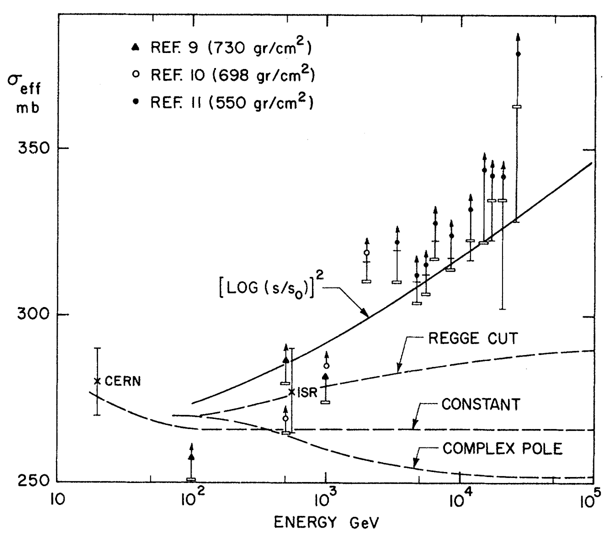

FIG. 2. Comparison of σ_{eff} predicted by various theories with the data. Error bars with single line, statistical error only; error bar with double line, uncertainties introduced by energy resolution (in calculating confidence levels mentioned in the text, the larger of these two was taken as being a representative error). Points without statistical error, those for which no such errors were quoted. The 20-GeV point is an extrapolation of the CERN data (Ref. 5) to air, while the 500-GeV point is the value of $\sigma_{\mathrm{in}}(p\text{-air})$ calculated using the value of σ_{pp} from Ref. 19 (intersecting-storage-ring data) and plotted, for clarity, at 550 GeV.

(i) Constant total p-p cross section at 38.8 mb at all energies above 70 GeV.

(ii) Regge-pole-plus-cut models,[16] which give

$$\sigma_{pp} = P^{-1}\,\mathrm{Im}\{\textstyle\sum_i \gamma_i \exp(-i\pi\alpha_c/2) - \gamma_c \Gamma(\lambda+1) \times \exp(-i\pi\alpha_c/2) E^{\alpha_c}[\ln(E-b-\tfrac{1}{2}i\pi)]^{-\lambda-1}\},$$

where $\sum_i$ represents the sum over the Regge poles, and the various parameters are given in Ref. 16.

(iii) Complex-Regge-pole models,[17] which give

$$\sigma_{pp} = 38.8 - 4.3\cos(0.7\ln s + 2).$$

(iv) Cross sections which grow as the square of the logarithm of the energy, which is the limit imposed by the Froissart bound, and is specifically realized in some field-theory models,[18]

$$\sigma_{pp} = 38.8 + C[\ln(s/s_0)]^2.$$

For each of these models, we can calculate σ_{eff} as a function of energy and compare it to available cosmic-ray data on lower bounds, as shown in Fig. 2. We see that the observed increase σ_{eff} with energy effectively rules out cross sections which do not rise at least as fast as $38.8 + 0.4 \times \ln^2(s/s_0)$ up to $\sim 3\times10^4$ GeV. Our best value for σ_{pp} at this energy is 60 mb. If we take note that the value of σ_{eff} at $\simeq 10^4$ is $\gtrsim 305$ mb at 95% confidence level, then we would also say that at 95% confidence, $\sigma_{pp} \gtrsim 48$ mb at 3×10^4 GeV.

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[1]For a review of the theoretical situation, see V. Barger and R. J. N. Phillips, to be published.

[2]R. J. Glauber, in *High Energy Physics and Nuclear Structure*, edited by S. Devons (Plenum, New York, 1970), and references contained therein.

[3]J. Newmeyer and J. S. Trefil, Nucl. Phys. B32, 315 (1970), and references contained therein.

[4]R. J. Eden, Rev. Mod. Phys. 43, 15 (1971).

[5]G. Belletinni, Nucl. Phys. 79, 609 (1966).

[6]N. L. Grigorov *et al.*, in *Proceedings of the Ninth International Conference on Cosmic Rays, London, 1965*, edited by A. C. Stickland (Institute of Physics and Physical Society, London, England, 1966), p. 860.

[7]G. B. Yodh *et al.*, Fields and Quanta 1, 233 (1971).

[8]E. Cowan and K. Matthews, Phys. Rev. D 4, 37 (1971).

[9]L. W. Jones *et al.*, Phys. Rev. Lett. 25, 1679 (1970).

[10]N. L. Grigorov *et al.*, in *Proceedings of the Ninth International Conference on Cosmic Rays, London, 1965*, edited by A. C. Stickland (Institute of Physics and Physical Society, London, England, 1966), p. 860.

[11]T. Kaneko *et al.*, in Proceedings of the Twelfth International Conference on Cosmic Rays, Hobart, Tas-

mania, Australia, August 1971 (to be published).
[12]W. R. Webber, in *Proceedings of the Eleventh International Conference on Cosmic Rays, Budapest, 1969*, edited by T. Gémesy *et al.* (Akademiai Kiado, Budapest, 1970), p. 275.
[13]N. L. Grigorov *et al.*, Acta Phys., Suppl. 1, 29, 518 (1970), and papers presented at the Twelfth International Conference on Cosmic Rays, Hobart, Tasmania, August 1971 (to be published). The Grigorov *et al.* results on proton, α-particle, and all particle fluxes are unusual and should lead to several inconsistencies with other cosmic-ray experiments. If the proton component has a "cutoff" below 1000 GeV then the charge composition of cosmic rays at 10^6 GeV would be mainly heavy nuclei. The energy fluctuations in the hadronic component of air showers at $E \geqslant 10^6$ GeV would be expected to be much narrower than observed [see, for example, H. V. Bradt and S. A. Rappaport, Phys., Rev. Lett. 22, 960 (1969)], and in addition the extrapolated flux should be considerably less than that observed by air-shower experiments. It would also be difficult to account for deserved muon flux above TeV energies if the composition were mainly heavy nuclei.
[14]M. J. Ryan *et al.*, in Proceedings of the Twelfth International Conference on Cosmic Rays, Hobart, Tasmania, Australia, August 1971 (to be published).
[15]K. Pinkau *et al.*, Acta Phys., Suppl. 3, 29, 291 (1970).
[16]V. A. Barger and R. J. N. Phillips, Phys. Rev. Lett. 24, 291 (1970).
[17]R. R. Amann, Phys. Rev. D 3, 2861 (1971).
[18]H. Cheng and T. T. Wu, Phys. Rev. Lett. 24, 1456 (1970).
[19]M. Holder *et al.*, Phys. Lett. 35B, 361 (1971).

Neutrinos with Mass and the Decay $K_L{}^0 \to \bar{\nu}_l + \bar{\nu}_l$

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One of the few exact results that would be vitiated by the recently discussed possibility of neutrinos with mass, would be the statement of the forbidden nature of the decay $K_L{}^0 \to \bar{\nu}_l + \nu_l$ because of angular-momentum conservation in the usual neutrino theory. We note that the decay $K_L{}^0 \to \bar{\nu}_\mu + \nu_\mu$ could easily exist with a rate comparable to that for $K_L{}^0 \to 2\gamma$ if $m_{\nu_\mu} \approx 100$ eV. The processes $\nu_l + p \to \nu_l + \Sigma^+$, $\nu_l + n \to \nu_l + \Lambda$ and $\Sigma^+ \to p + \bar{\nu}_l + \nu_l$, $\Lambda \to n + \bar{\nu}_l + \nu_l$ would then occur in lowest order, but at minute rates.

It has recently been suggested,[1] in connection with the unexpectedly low counting rate in the solar-neutrino experiment,[2] that neutrinos with a finite mass could be unstable. Since the empirical limit[3] on the mass of ν_μ is only $m_{\nu_\mu} < 1.6$ MeV, whereas that for ν_e is $m_{\nu_e} < 60$ eV, the question of finite mass and possible instability is surely also relevant for ν_μ[1]. Apart from their possible instability, neutrinos with mass would vitiate the exactness of the statement that the decay $K_L{}^0 \to \bar{\nu}_l + \nu_l$ is forbidden by angular-momentum conservation in the usual neutrino theory.[4] This decay could be mediated by an effective Lagrangian density[5,6]

$$L_{K^0}{}^{\mathrm{eff}} = (-\lambda m_{\nu_l}\sqrt{G_{\mathrm{F}}})[K^0\bar{\psi}_{\nu_l}(1-\gamma_5)\psi_{\nu_l} + \bar{K}^0\bar{\psi}_{\nu_l}(1+\gamma_5)\psi_{\nu_l}], \tag{1a}$$

$$\Rightarrow L_{K_L{}^0}{}^{\mathrm{eff}} = (\sqrt{2}\,\lambda m_{\nu_l}\sqrt{G_{\mathrm{F}}})K_L{}^0\bar{\psi}_{\nu_l}\gamma_5\psi_{\nu_l}, \tag{1b}$$

where $G_{\mathrm{F}} \cong 10^{-5} m_N{}^{-2}$ is the Fermi constant in terms of the nucleon mass m_N, and λ is a dimensionless number. In fact an effective interaction of the form (1b) would be generated in perturbation theory by a "standard" (neutral) intermediate-boson Lagrangian[7]

$$L_W = [igm_W W_\mu(\partial_\mu K^0) + gW_\mu\bar{\psi}_{\nu_l}\gamma^\mu(1-\gamma_5)\psi_{\nu_l}] + \mathrm{H.c.}, \tag{2}$$

where m_W denotes the boson mass and g is the dimensionless semiweak coupling, $g^2/m_W{}^2 = G_{\mathrm{F}}/\sqrt{2}$. The two terms in (2) generate (1b) with $\lambda = 2^{3/4}g$.[7] From (1b) we compute a rate for $K_L{}^0 \to \bar{\nu}_l + \nu_l$ in terms of the effective coupling constant $f^2 = \lambda^2(m_{\nu_l}/m_N)^2 \times 10^{-5}$,

$$R(K_L{}^0 \to \bar{\nu}_l + \nu_l) = f^2(6\times10^{22}\ \mathrm{sec}^{-1}) \Rightarrow 6.8\times10^3\ \mathrm{sec}^{-1} \tag{3}$$

for $m_{\nu_l} = m_{\nu_\mu} = 100$ eV and $\lambda \cong 1$.[7] For comparison we note the empirical $R(K_L{}^0 \to 2\gamma) \cong 10^4\ \mathrm{sec}^{-1}$. Thus $K_L{}^0$ could easily be decaying into neutrino pairs at a rate which is a fraction of a percent of $(\tau_{K_L{}^0})^{-1}$!

Together with strong interactions, (1b) generates the following processes in lowest order[8]:

$$\nu_l + p \to \nu_l + \Sigma^+, \tag{4a}$$

$$\nu_l + n \to \nu_l + \Lambda, \tag{4b}$$