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Neutron-Proton and Neutron-Carbon Scattering Amplitudes

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Measurements of the n-p and n-C coherent atomic scattering amplitudes are described. The results $a_H = -3.740 \pm 0.003$ fm and $a_C = 6.648 \pm 0.003$ fm were obtained by reflection measurements on seventeen various liquid mirrors with the neutron gravity refractometer, for which a more accurate method of evaluation is given. The mirrors were composed of the elements hydrogen, carbon, and chlorine. The atomic scattering amplitude of chlorine was found to be $a_{Cl} = 9.580 \pm 0.002$ fm. The new a_H is of importance for describing the n-p system.

The coherent neutron-proton scattering amplitude a_H is important for determining the low-energy n-p interaction. Therefore, many experiments for measuring this amplitude have been carried out.[1] Additional precise values for the n-p free cross section at various neutron energies are necessary to fix the constants. Recently such measurements have been reported at a few electron volts.[2] The n-p interaction at such low energies is completely described by the singlet (a_s) and the triplet (a_t) scattering lengths. They are related to the "bound" a_H and to the free cross section σ_H at "zero energy" by $a_H = \frac{1}{2}a_s + \frac{3}{2}a_t$ (a_s and a_t are "free" values) and $\sigma_H/\pi = (a_s^2 + 3a_t^2)$. In Ref. 2 an old and a new set of values for a_s and a_t are given. The old, widely accepted set 1, $a_s = -23.680 \pm 0.028$ fm and $a_t = 5.399 \pm 0.011 \pm 0.011$ fm, is based on $\sigma_H = 20.36 \pm 0.05$ b[3] and $a_H = -3.741 \pm 0.011$ fm, tabulated in Ref. 1, where a nuclear scattering amplitude for the neutron-carbon scattering $a_C = 6.622 \pm 0.017$ fm is given, which was obtained from a free carbon cross section $\sigma_C = 4.704 \pm 0.019$ b. These quantities serve as standard values.

The experiments described in Ref. 2 yielded for σ_H a new value 20.436 ± 0.023 b which is more accurate than the former one and, in addition, $\sigma_C = 4.7461 \pm 0.0045$ b, leading to $a_C = 6.650 \pm 0.009$ fm.

With an accepted but unpublished[4] value for $a_H = -3.721 \pm 0.004$ fm (standard value $a_C = 6.632 \pm 0.002$ fm,[5] a new set 2 was calculated as $a_s = -23.712 \pm 0.013$ fm and $a_t = 5.423 \pm 0.005$ fm. In Ref. 2 this set has been proposed for future use. In the following, however, it is shown that the new set is incorrect. The unpublished data for a_H and a_C corresponded to the status of running experiments in this laboratory for new determinations of scattering amplitudes with the neutron gravity refractometer.[6,7]

These experiments were immediately started when the results of the first measurements were finished and published[7] because it was found that the method of evaluation was incomplete. The final results of this investigation, recently completed, do not agree with the unpublished preliminary values given above. In the neutron gravity refractometer, (slow) neutrons fall in the gravitational field. When they hit a horizontal (liquid) mirror after having fallen a distance h between the culmination point and the mirror surface, they will be reflected with a reflectivity

$$r(h) = \left| \frac{1-(1-h_0/h+iA/h)^{1/2}}{1+(1-h_0/h+iA/h)^{1/2}} \right|^2,$$

wherein A is an effective absorption factor including neutron absorption and inelastic and incoherent scattering. The critical height h_0 for total reflection is determined by the bound coherent scattering amplitude Na_{coh} for N molecules (per cm^3)

$$h_0 = (2\pi\hbar^2/gm)Na_{\text{coh}},$$

where $\hbar$ is Planck's constant, m is the neutron mass, and g stands for that acceleration which acts on the free-moving neutron and which produces a momentum perpendicular to the horizontal surface of the mirror. For calculating this acceleration the following data must be taken into account: (1) the local value g_0 for the gravitational acceleration, (2) the position of the neutron beam on the globe given by the local radius R and the latitude φ, (3) the direction of the beam relative to east direction (angle ψ), (4) the rotation of the globe with angular velocity ω, and (5) the neutron velocity v. It follows that

$$g = g_0 - v^2/R - 2v\omega\cos\varphi\cos\psi.$$

The additional terms besides g_0 are the centrifugal and the Coriolis terms. They yield contributions between -0.9% at 20 cm and -0.2% at 150 cm height of fall which were neglected in the former evaluations.[7]

In addition, further small ($\approx 0.1\%$) corrections became necessary for impurities in the mirrors and for uncertainties (0.5°C) in the temperature of the mirrors. In order to keep the uncertainty caused by impurities not detectable by an analysis described below as small as possible, we carried out reflection experiments on seventeen various liquids from eight different substances composed of carbon, hydrogen, and/or chlorine. The purity of each mirror was investigated by means of gas chromatography and by measuring the content of water in the liquids. The results of the reflectivity measurements with the neutron gravity refractometer are given in Table I in

TABLE I. Results of reflectivity measurements. The errors correspond to 1σ statistical and to an uncertainty $\pm 20\%$ of the applied correction. Data with an asterisk give the corrected results of the first experiments (Ref. 7).

No.	Mirror substance	Composition C	H	Cl	a_{coh} per molecule (fm)	applied correction (fm)
1	Diphenyl-	13	12	-	41.580 ± 0.021	+ 0.098
2	methane				41.502 ∓ 0.018	+ 0.074
3					41.538 ∓ 0.017 *	+ 0.047
	weighted mean				41.536 ± 0.015	
4	m-Xylene	8	10	-	15.792 ± 0.007	-
5	p-Xylene	8	10	-	15.788 ∓ 0.011 *	+ 0.003
	weighted mean				15.791 ± 0.006	
6	Toluene	7	8	-	16.608 ± 0.005	+ 0.005
7	"				16.616 ∓ 0.005	+ 0.012
	weighted mean				16.612 ± 0.004	
8	Benzene	6	6	-	17.451 ± 0.005	+ 0.005
9	Chlorobenzene	6	5	1	30.766 ∓ 0.005	+ 0.005
10	"				30.788 ∓ 0.007	+ 0.015
	weighted mean				30.773 ± 0.006	
11	Hexachloro-butadiene	4	-	6	84.066 ± 0.010	+ 0.011
12	Tetra-	2	-	4	51.614 ± 0.009	+ 0.002
13	chloro-				51.619 ∓ 0.010	+ 0.002
14	ethylene				51.609 ∓ 0.009	+ 0.006
15					51.613 ∓ 0.015 *	+ 0.008
	weighted mean				51.614 ± 0.006	
16	Carbontetra-	1	-	4	44.965 ± 0.010	+ 0.010
17	chloride				44.964 ∓ 0.012 *	+ 0.005
	weighted mean				44.965 ± 0.008	

which the coherent scattering amplitudes per molecule and the applied corrections due to the impurities are quoted.

The measurements represent an overdetermined system of equations for computing the scattering amplitudes of hydrogen, carbon, and chlorine. The solution leads to the following weighted means for the bound atoms and—in brackets—for the bound nuclei:

$$a_H = -3.740 \pm 0.003 \text{ fm} \quad (-3.739 \pm 0.003 \text{ fm}),$$

$$a_C = 6.648 \pm 0.003 \text{ fm} \quad (6.657 \pm 0.003 \text{ fm}),$$

$$a_{Cl} = 9.580 \pm 0.002 \text{ fm} \quad (9.605 \pm 0.003 \text{ fm}).$$

The values for the nuclei were calculated with a neutron-electron scattering amplitude $a_{ne} = -0.00146$ fm/electron (only Foldy interaction).

The new a_H agrees with the old value -3.741 ± 0.011 fm tabulated in Ref. 3, but is much more accurate. The most recent precise experiments led, by means of different methods, to the same results for a_C (nucleus), within the errors: 6.650 ± 0.009 fm,[2] 6.660 ± 0.006 fm,[8] and 6.657 ± 0.003 fm (present work). This fact is quite encouraging and strengthens the belief in the presented value for a_H.

With the new results the following set of parameters for the low-energy n-p system is calculated:

$$a_s = -23.719 \pm 0.013 \text{ fm and } a_t = 5.414 \pm 0.005 \text{ fm}.$$

The change of the old set 1 is mainly due to the new σ_H measured in Ref. 2, where as the result of the present work leads to a further improvement of the accuracy for the low-energy n-p parameters.

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Superheavy Elements and an Upper Limit to the Electric Field Strength*

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An upper limit to the electric field strength, such as that of the nonlinear electrodynamics of Born and Infeld, leads to dramatic differences in the energy eigenvalues and wave functions of atomic electrons bound to superheavy nuclei. For example, the $1s_{1/2}$ energy level joins the lower continuum at $Z = 215$ instead of $Z = 174$, the value obtained when Maxwell's equations are used to determine the electric field.

The spectra of atomic electrons offer one means for the identification of superheavy elements.[1-3] Most of the previous calculations of the atomic structure of these elements have used the relativistic Hartree-Fock-Slater method[1] and have either used perturbation theory to calculate field corrections[2] or ignored the question altogether. Accurate calculations of these corrections have not been made, although a method of calculating them, which does not depend upon an expansion in powers of the external field, has been proposed.[4]

Our search for a method of calculating field corrections in superheavy elements has led us to consider the extent to which these corrections are equivalent to an upper limit for the electric field strength. Such an upper limit emerged quite naturally from the considerations of Born and Infeld, who formulated a nonlinear theory of electrodynamics with the express intention of making the self-energy of the electron finite. We show below that the spectra of atomic electrons bound to superheavy elements with nuclear charges $Z \gtrsim 150$ provide a stringent test of the theory of Born and Infeld and, more generally, of similar theories which lead to an upper limit to the electric field strength. Even if superheavy elements cannot be readily produced, enough information could possibly be gathered in the collisions of heavy ions, such as Pb on Pb or Cf on Cf, to decide if this limit exists. In these collisions the adiabatic approximation should have some validity since the velocity of the electrons in the $1s$ and $2p$ atomic orbitals is much faster