

The direct production of three particles has usually been considered only as the random distribution of events in the available three-body phase space. However, the "spectator-pole" theory used by Donovan¹ for knockout reactions such as $d+d \rightarrow d+p+n$ is similar to the more complicated direct-mode theory proposed here. Compound direct decay has been observed,^{2,3} but the compound sequential decay processes have received most of the theoretical attention.⁴⁻⁹

During the past year data taken¹⁰ from the reaction $\text{Li}^6 + \text{Li}^6 \rightarrow \alpha + \alpha + \alpha$ at a bombarding energy of 6.0 MeV were analyzed in detail. These experiments were performed with two small detectors fixed at $\theta_{F1} = +15^\circ$ and $\theta_{F2} = +30^\circ$ on one side of the scattering chamber, and a movable 30° -wide position-sensitive detector on the other side of the chamber. Data were obtained at all angles in the ranges of $-10^\circ < \theta_P < -170^\circ$ (θ_{F1}) and $-25^\circ < \theta_P < -150^\circ$ (θ_{F2}) for coincident events between the position detector and one or the other of the fixed detectors. A large anomalous peak was found in each of the fixed-detector energy spectra for a narrow angular range ($\sim 40^\circ$) near $\theta_P = -160^\circ$ (θ_{F1}) and $\theta_P = -140^\circ$ (θ_{F2}). It was found that these peaks corresponded to no known levels in the Be^8 nucleus if the assumption of sequential decay was invoked. However, the energies of the peaks were correctly given, assuming a direct interaction for which zero momentum transfer to the beam α particle occurred. Another peak corresponding to zero momentum transfer to the target α particle was also expected, but not seen in this particular set of data because the energy range was not sufficient to include it.

A simple theoretical model was constructed for the purpose of reproducing the energies and shapes of the anomalous peaks. For convenience all computations were performed in the laboratory coordinate system. The three-body differential cross section for particle F , i.e., the energy spectrum, was written as a product of constants, a phase-space factor,¹¹ and the square of a matrix element. The matrix element was calculated using the plane-wave Born approximation where the nuclear potential was taken to be a zero-range interaction between the deuterons. The initial state was approximated by products of incoming plane waves and internal wave functions. The nucleons in the Li^6 nucleus were postulated to have a cluster structure composed of an α particle and a deuteron bound by a square-well potential. The wave function for the 2S state, rather than for the 1S state, was chosen

because it seemed to provide a more realistic approximation to the actual nucleus. The final state was described by outgoing plane waves for the α particles and an internal wave function for the α particle formed from the deuterons. When the appropriate expressions were substituted and the integrations performed over suitable coordinates, the matrix element had the functional form of the Fourier transform of the Li^6 bound-state wave function. The wave functions were explicitly symmetrized since α particles and deuterons are bosons. The detailed derivation of the theoretical model may be found in Gadeken.¹²

The theoretical model was compared in detail with additional experimental data obtained at bombarding energies of 2.0, 4.5, 6.0, 8.0, and 13.0 MeV. The model predicted that for a fixed detector angle of $+30^\circ$, the anomalous peaks in the E_F spectrum would rise and fall as a function of angle only in the range from $-120^\circ < \theta_P < -150^\circ$ for $E_B = 2.0$ MeV to $-70^\circ < \theta_P < -145^\circ$ for $E_B = 13.0$ MeV. The experiments were performed using two detectors: a small fixed detector with circular apertures which subtended a 5° angle relative to the center of the target, and a larger position-sensitive detector with a rectangular aperture which spanned an angle of 31° in the reaction plane. Each coincident event in the detectors corresponding to a three-body nuclear reaction gave three pieces of information: the energy E_F of the particle incident on the fixed detector, and the energy E_P and the angle θ_P of the particle incident on the position detector. This information determined each $\text{Li}^6 + \text{Li}^6 \rightarrow \alpha + \alpha + \alpha$ event completely, except for the polarizations of the Li^6 nuclei.

This paper presents only the experimental data and the theoretical computations for the 6.0-MeV bombarding energy. The data and computations for the other energies, as well as the experimental and theoretical details, are dealt with in a longer article.¹³ The main features in experiment and theory which are present for $E_B = 6.0$ MeV are also present at the other bombarding energies.

The fixed detector was set at $\theta_F = +30^\circ$ and the position detector was centered at the angles -95° , -120° , and -145° for the data obtained at the 6.0-MeV beam energy. The data were normalized using the overlapping angles of the position detector. A theoretical θ_P -vs- E_F matrix was constructed by calculating energy spectra for the θ_P values after correcting for the thickness of the beam-stopping foil in front of the fixed detector.

The experimental and theoretical energy spectra were obtained by summing the appropriate 5° -wide segments of the respective θ_p -vs- E_F matrices. Since the experimental measurements and the theoretical calculations were concerned only with the relative values of the cross section, experiment and theory were normalized to the maximum height of the peak that corresponded to zero momentum transfer to the target α particle (the high-energy direct peak). The only free parameter in the theoretical model was the radius of the square well. The value $R = 3.2$ fm was used in the calculations. The results were essentially the same in the range $2.4 < R < 4.0$ fm for the bombarding energies which were studied. This suggested that the shape of the nuclear potential for the Li^6 nuclei was not a critical factor and that the relatively sharp peaks of the calculated energy spectra were produced by the long tail of the bound-state wave function.

The theoretical computations reproduced with reasonable accuracy the large direct-interaction peaks which were found in the experimental data for $E_B = 6.0$ MeV as well as for the other bombarding energies. The energies of the majority of the peaks were well predicted and their relative heights were fairly well duplicated. In general, however, the widths of the experimental direct peaks rose from background to their maxima and fell again below background extremely rapidly as a function of angle. Typically, this range was 40° for the high-energy direct peak, and 25° for the low-energy direct peak. This variation was followed very well by the theoretical model.

To be sure that the peaks attributed to direct interactions were not due to sequential decay processes, the behavior of the excited states in Be^8 which could possibly produce peaks in the region of the experimental direct peaks was carefully checked as a function of θ_p for each bombarding energy. When θ_p is plotted versus E_F , the allowed loci occurred either as straight lines (constant E_F) or as curved lines where E_F varied rapidly with small changes in θ_p . In contrast, the energies of the direct peaks changed little with angle in the regions where they appeared. The relative cross sections of the observed states in Be^8 were between one tenth and one twentieth of the maximum observed for the direct peaks. The states which were unambiguously identified in the experimental spectra were the 0.0-, 2.9-, 16.6-, 16.9-, and 22.5-MeV levels. Those which could occur in the region of interest were the

2.9-, 11.4-, and 22.5-MeV states. Peaks due to these Be^8 states should have been clearly visible for angles other than those for which the experimental direct peaks were observed. As pointed out above, most of the angular range $0 < \theta_p < 180^\circ$ was studied for $E_B = 6.0$ MeV and there was no indication that peaks due to these states developed an appreciable cross section for any of these angles. As the bombarding energy changed, the levels in question came into the region of interest in different ways. There was no evidence that the experimental direct peaks changed in the way that would have been expected if they had been due to Be^8 levels. We concluded, therefore, that the anomalous peaks which were observed

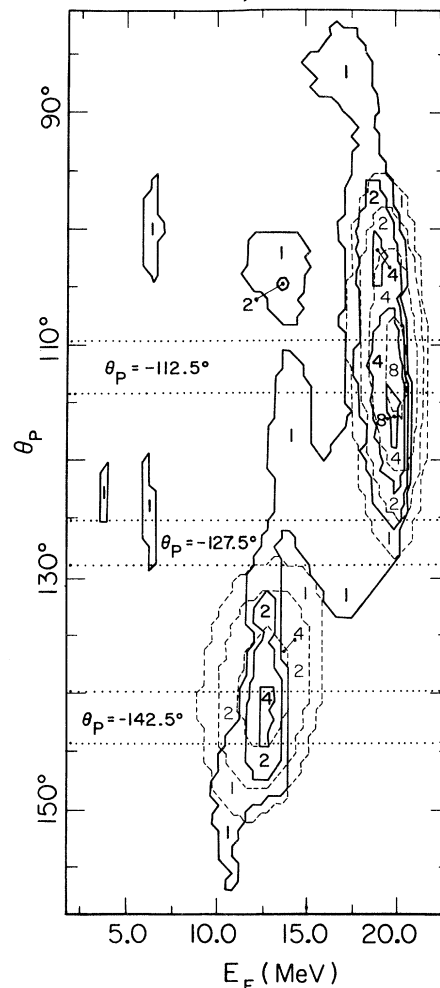


FIG. 1. Experimental (solid lines) and theoretical (dashed lines) contour maps of angle versus energy for $\theta_F = +30^\circ$ at $E_B = 6.0$ MeV. The count level of the first contour line is 91 counts. Each succeeding contour line corresponds to the indicated multiple of the first count level. The areas between the dotted lines were summed to form the energy spectra displayed in Fig. 2.

were due primarily to direct interactions and not to sequential decay processes.

The contour map for $E_B = 6.0$ MeV is shown in Fig. 1. The solid lines were drawn through the experimental data points and the dashed lines through the theoretical values. The energy scale gives the laboratory energies following the foil over the fixed detector. The angular scale gives the laboratory angles in the position detector.

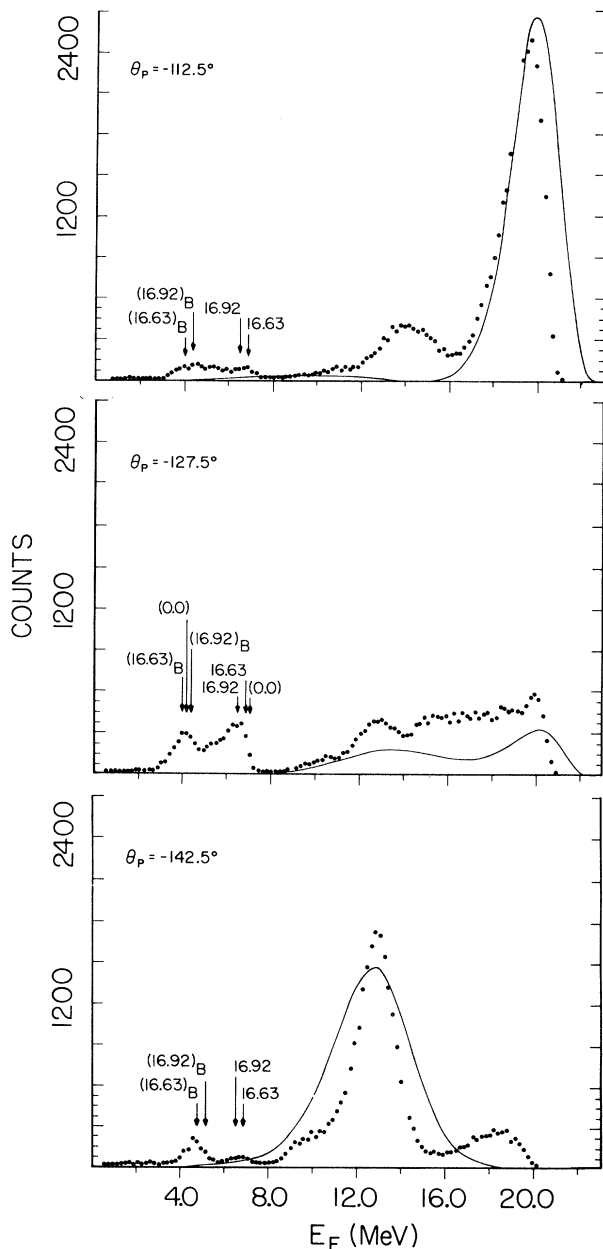


FIG. 2. Experimental (dotted lines) and theoretical (solid lines) E_F spectra. The arrows point to peaks corresponding to excited states in Be^8 .

The 5° -wide areas between the dotted lines were summed to give the spectra which are shown in Fig. 2. The experimental data points are shown together with smooth curves calculated using the theory. The locations of peaks from the sequential decay of levels in Be^8 are indicated by the excitation values. The excitation values which stand alone refer to peaks formed by events in which the initial particle was incident on the fixed detector and one of the decay particles was incident on the position detector. The values in brackets correspond to peaks formed by the initial particle striking the position detector and one of the decay particles striking the fixed detector. The subscript B on the brackets means that both decay particles were detected, one in each detector, and the initial particle was not observed.

The good agreement between experiment and theory is evident in this case, although, as noted before, the direct-interaction peaks predicted by the theory are broader in both energy and angle than the corresponding peaks in the experimental data. It is interesting to compare the relative cross sections of the sequential decay processes in the three spectra. In the center spectrum of Fig. 2 where the direct-interaction effects are small, the peaks due to sequential decay are larger by about a factor of 2 than in either of the other two spectra. Perhaps there is some sort of interference occurring between the direct mode and the sequential processes. The smaller peaks in the spectra which are not predicted by the theory are somewhat mysterious. It is thought that a more complete theory of the direct-interaction effects might explain these peaks as well.

The theory used in this paper is not at all complex; nevertheless the essential features of the direct-interaction mechanism have been included, and the theoretical computations are in reasonable agreement with the experimental data. Better results should be obtained by using more realistic nuclear potentials and by including Coulomb forces and α - α interactions in the theoretical model. Since the direct mode is so important for the production of the three-body final state in this reaction, it is reasonable to assume that it is also significant in other multiparticle nuclear reactions.

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Neutron-Proton and Neutron-Carbon Scattering Amplitudes

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Measurements of the n - p and n -C coherent atomic scattering amplitudes are described. The results $a_H = -3.740 \pm 0.003$ fm and $a_C = 6.648 \pm 0.003$ fm were obtained by reflection measurements on seventeen various liquid mirrors with the neutron gravity refractometer, for which a more accurate method of evaluation is given. The mirrors were composed of the elements hydrogen, carbon, and chlorine. The atomic scattering amplitude of chlorine was found to be $a_{Cl} = 9.580 \pm 0.002$ fm. The new a_H is of importance for describing the n - p system.

The coherent neutron-proton scattering amplitude a_H is important for determining the low-energy n - p interaction. Therefore, many experiments for measuring this amplitude have been carried out.¹ Additional precise values for the n - p free cross section at various neutron energies are necessary to fix the constants. Recently such measurements have been reported at a few electron volts.² The n - p interaction at such low energies is completely described by the singlet (a_s) and the triplet (a_t) scattering lengths. They are related to the "bound" a_H and to the free cross section σ_H at "zero energy" by $a_H = \frac{1}{2}a_s + \frac{3}{2}a_t$ (a_s and a_t are "free" values) and $\sigma_H/\pi = (a_s^2 + 3a_t^2)$. In Ref. 2 an old and a new set of values for a_s and a_t are given. The old, widely accepted set 1, $a_s = -23.680 \pm 0.028$ fm and $a_t = 5.399 \pm 0.011 \pm 0.011$ fm, is based on $\sigma_H = 20.36 \pm 0.05$ b³ and $a_H = -3.741 \pm 0.011$ fm, tabulated in Ref. 1, where a nuclear scattering amplitude for the neutron-carbon scattering $a_C = 6.622 \pm 0.017$ fm is given, which was obtained from a free carbon cross section $\sigma_C = 4.704 \pm 0.019$ b. These quantities serve as standard values.

The experiments described in Ref. 2 yielded for σ_H a new value 20.436 ± 0.023 b which is more accurate than the former one and, in addition, $\sigma_C = 4.7461 \pm 0.0045$ b, leading to $a_C = 6.650 \pm 0.009$ fm.

With an accepted but unpublished⁴ value for

$a_H = -3.721 \pm 0.004$ fm (standard value $a_C = 6.632 \pm 0.002$ fm,⁵ a new set 2 was calculated as $a_s = -23.712 \pm 0.013$ fm and $a_t = 5.423 \pm 0.005$ fm.

In Ref. 2 this set has been proposed for future use. In the following, however, it is shown that the new set is incorrect. The unpublished data for a_H and a_C corresponded to the status of running experiments in this laboratory for new determinations of scattering amplitudes with the neutron gravity refractometer.^{6,7}

These experiments were immediately started when the results of the first measurements were finished and published⁷ because it was found that the method of evaluation was incomplete. The final results of this investigation, recently completed, do not agree with the unpublished preliminary values given above. In the neutron gravity refractometer, (slow) neutrons fall in the gravitational field. When they hit a horizontal (liquid) mirror after having fallen a distance h between the culmination point and the mirror surface, they will be reflected with a reflectivity

$$r(h) = \left| \frac{1 - (1 - h_0/h + iA/h)^{1/2}}{1 + (1 - h_0/h + iA/h)^{1/2}} \right|^2,$$

wherein A is an effective absorption factor including neutron absorption and inelastic and incoherent scattering. The critical height h_0 for total reflection is determined by the bound coherent